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Ultrasonic Absorption in Metals

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The absorption coefficient for metals has been computed. It is shown that if $\omega\tau \ll s/v$ (ω = frequency, τ = relaxation time, s = sound velocity, v = boundary electron velocity), the absorption coefficient is proportional to $\omega^2\tau$. For $\omega\tau \gg s/v$, the absorption is proportional to the first power of the frequency and does not depend on τ, *i.e.*, it is independent of the temperature.

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1. IN THE INVESTIGATION of the absorption of sound waves in solids, we must distinguish two cases, as is well known[1-3]: when the frequency of the sound waves ω is much greater than the reciprocal of the relaxation time τ, and when the frequency of the sound waves is much smaller than τ^{-1}. In the first case ($\omega\tau \gg 1$), we can treat the sound absorption as the absorption of sound quanta of energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$ ($\mathbf{k}$ = wave vector of the sound wave), which takes place as the result of collisions of the sound quanta with the quasi-particles which characterize the energy spectrum of the solid, *i.e.*, with phonons in ordinary dielectrics[1] and with electrons and phonons in metals.

In the second case, when $\omega\tau \ll 1$, the sound vibrations can be considered as a certain external field in which the gas of quasi-particles is located, and which modulates the energy of these particles.[2,3] This modulation leads to the result that the frequency of the phonon ω_f and the energy of the electron acquire additions proportional to the deformation tensor produced by the sound wave:

$$\omega_f = \omega_f^0(1 + \mu_{ik}u_{ik}), \quad \varepsilon = \varepsilon_0 + \lambda_{ik}u_{ik}, \tag{1}$$

where ω_f^0 and ϵ_0 are the phonon frequency and electron energy in the absence of sound, u_{ik} is the deformation tensor, μ_{ik} and λ_{ik} are tensors of second rank which characterize the solid; the tensor μ_{ik} depends only on the direction of the wave vector of the phonon f, while λ_{ik} depends both on the direction and on the magnitude of the quasi-momentum $\mathbf{p}$ of the electron.

We note that in reality the change of the energy of the electron brought about by the sound field must contain, in addition to the term $\lambda_{ik}\mu_{ik}$, an additional term connected with the Stewart-Tolman effect. This term is equal to $m(\partial\epsilon/\partial p_i)(\partial u_i/\partial t)$, where m = mass of the free electron, u_i = vector displacement. However, this term is significantly smaller than $\lambda_{ik}\mu_{ik}$ and is disregarded. Actually, in order of magnitude, λ_{ik} is equal to the limiting Fermi energy μ_0, whence

$$\lambda_{ik}u_{ik} \sim \mu_0 ku \sim mv_0^2\omega u/s,$$

$$m(\partial\varepsilon/\partial p_i)\,\partial u_i/\partial t \sim mv_0\omega u,$$

where v_0 = velocity of an electron on the Fermi surface ($\sim 10^8$ cm/sec), s = sound velocity ($\sim 10^5$ cm/sec) and, consequently,

$$\left| m\frac{\partial\varepsilon}{\partial p_i}\frac{\partial u_i}{\partial t}\right| \Big/ \left|\lambda_{ik}u_{ik}\right| \sim \frac{s}{v_0} \ll 1.$$

2. In the present paper, we shall consider the sound absorption in metals at low temperatures. In this case, the role of the phonons is important,[3] since their number tends to zero with decrease in temperature (proportional to T^3) and the sound absorption is determined by the interaction of the sound waves with the conduction electrons. We note that this is possibly connected with the experimentally observed difference in the absorption coefficients of ultrasound in metals in the normal and superconducting states.

We first consider the case in which $\omega\tau \ll 1$. Here the sound wavelength $\lambda\!\!\!^- = s/\omega$ can be larger or smaller than the mean free path of the electron $l = v_0\tau$. Therefore, both the time and space changes in the electron distribution function brought about by the sound field are important.

We denote the electron distribution function by $n(\mathbf{p}, \mathbf{r}, t)$. For simplicity, we shall assume that the collision integral has the form

$$(\partial n/\partial t)_c = -(n - \bar{n})/\tau, \tag{2}$$

where $\bar{n} = \bar{n}(\epsilon, \mathbf{r}, t)$.is the mean value of the distribution function over a surface of equal energy

$$\bar{n} = \oint n \frac{ds}{v} \Big/ \oint \frac{ds}{v}, \qquad (3)$$

ds is the element of area in momentum space of the isoenergetic surface and $v = |\partial t/\partial \mathbf{p}|$. The kinetic (Boltzmann) equation in this case can be written in the following manner:

$$\frac{\partial n}{\partial t} + \mathbf{v}\frac{\partial n}{\partial \mathbf{r}} + \dot{\mathbf{p}}\frac{\partial n}{\partial \mathbf{p}} + \frac{n - \bar{n}}{\tau} = 0. \qquad (4)$$

Setting

$$n = n_0 - \chi \frac{\partial n_0}{\partial \varepsilon}, \qquad n_0 = \left(\exp\frac{\varepsilon - \mu}{T} + 1\right)^{-1}, \qquad (5)$$

where μ is the limiting energy, and noting that at low temperatures

$$-\partial n_0/\partial \varepsilon = \delta(\varepsilon - \mu),$$

it is easy to show that the energy dissipation per unit volume is equal to the product of the temperature and the time derivative of the entropy, and has the form

$$T\dot{S} = (2\pi\hbar)^{-3}\left\{\frac{1}{\tau}\left(\overline{|\chi|^2} - |\bar{\chi}|^2\right)\oint\frac{ds}{v}\right\}_{\varepsilon=\mu}, \qquad (6)$$

where the values of all the quantities entering into the formula are taken on the Fermi surface.

The sound absorption coefficient is connected with the dissipation function by the relation[4]

$$\gamma = T\dot{S}/E_s \qquad (7)$$

where E_s is the energy of the sound wave per unit volume.

Thus the computation of the sound absorption coefficient reduces to the determination of the function $\chi(\mathbf{p}, \mathbf{r}, t)$.

3. As has been shown, the sound field leads to a change in the energy of the electrons given by Eq. (1). In turn, this brings about a change in the chemical potential μ and temperature T:

$$\mu = \mu_0 + \mu', \quad T = T_0 + T',$$

where μ_0 and T_0 are the values of μ and T at $u_{ik} = 0$, while μ' and T' are the additions to μ and T, and are proportional to u_{ik}. They can be found from the conservation laws of the number of the electrons and the energy. Taking it into consideration that the sound vibrations represent a plane monochromatic wave

$$u_{ij} = {}^1/_2\,[\partial u_i/\partial x_j + \partial u_j/\partial x_i], \quad u_i = u_i^0 e^{i(\omega t - \mathbf{kr})}, \qquad (8)$$

we can show that

$$\mu' = \bar{\lambda}_{ik} u_{ik}, \quad T' = T\rho_{ik} u_{ik}, \qquad (9)$$

where the bar over the λ_{ik} denotes an averaging over the Fermi surface, and ρ_{ik} is a certain tensor of second rank whose explicit form was derived in Ref. 3. Since a temperature change enters the kinetic equation in the form

$$-\frac{\partial n_0}{\partial \varepsilon}\frac{\varepsilon - \mu}{T}\dot{T}',$$

then the contribution brought about by this change in the dissipation function is proportional to the $(T/\mu_0)^2$, which is significantly less than unity. This means that at low temperatures in metals the thermal conductivity does not play an important role in the dissipation of energy, which is determined chiefly by the "friction" of the electron gas. Therefore we do not have to take the temperature change into consideration.

The force $\dot{\mathbf{p}}$ (which acts on the electron) entering the kinetic equation (4) is obviously equal to

$$\dot{\mathbf{p}} = e\mathbf{E} - \nabla\varepsilon = e\mathbf{E} - \lambda_{ik}\nabla u_{ik}, \qquad (10)$$

where **E** is the electric field which arises as the result of the redistribution of charges in the metal under the action of the sound wave. This field can be found from the simultaneous solution of the kinetic equation and the Maxwell equation. Since the velocity of the electrons and the sound velocity are significantly smaller than the velocity of light, one does not have to take the resulting magnetic field into account, and we regard the electric field as longitudinal. On the other hand, thanks to the high conductivity of the metal, the volume density of the charge ρ is practically equal to zero.

Inasmuch as

$$\rho = \frac{2e}{(2\pi\hbar)^3}\oint\frac{ds}{v}\cdot\bar{\chi},$$

then, to find the field, we must start from the condition

$$\bar{\chi} = 0. \qquad (11)$$

Substituting the expression for n in the form (5) in the kinetic equation (4), and making use of the second relation of (1), and also (8)-(10), we obtain the following equation for the function χ (in the linear approximation in u_{ik}):

$$\frac{\partial \chi}{\partial t} + (\mathbf{v}\nabla)\chi + \frac{\chi}{\tau}$$

$$= e\mathbf{v}\mathbf{E} + (\lambda_{ik} - \overline{\lambda}_{ik})\,\dot{u}_{ik} - \mathbf{v}(\nabla)\,\lambda_{ik}\,\dot{u}_{ik}, \qquad (12)$$

whence

$$\chi = \frac{\tau'(\lambda_{ik} - \overline{\lambda}_{ik})\,\dot{u}_{ik} + l'\varkappa\nu\,(eE + \overline{\lambda}_{ik}\,\dot{u}_{ik}/s)}{1 - ikl'\varkappa\nu}, \qquad (13)$$

$$\varkappa = \mathbf{k}/k,\ \ \nu = \mathbf{v}/v,\ \ \tau' = \tau/(1 + i\omega\tau),\ \ l' = v\tau'.$$

It follows from (13) and (11) that

$$eE = -\frac{1}{s}\overline{\lambda}_{ik}\dot{u}_{ik} + i\frac{\omega\tau'}{s(1-J)}\Lambda_{ik}\,\dot{u}_{ik}. \qquad (14)$$

Here

$$J = \overline{\frac{1}{1 + z\varkappa\nu}},\quad \Lambda_{ik} = \overline{\frac{\lambda_{ik} - \overline{\lambda}_{ik}}{1 + z\varkappa\nu}},\quad z = -ikl'. \qquad (15)$$

Substituting (14) in (13), we get

$$\chi = \tau'\left\{\frac{(\lambda_{ik} - \overline{\lambda}_{ik}) + \Lambda_{ik}/(1-J)}{1 + z\varkappa\nu} - \frac{1}{1-J}\Lambda_{ik}\right\}\dot{u}_{ik}.$$

Then, by Eq. (6) we can find the energy dissipation. The quantity $\overline{|\chi|^2}$ ($\overline{|\chi|}^2 = 0$) entering into Eq. (6) has a complicated form in the general case of arbitrary z. We therefore consider only the main term in the equation for $\overline{|\chi|^2}$, which gives the principal contribution in the limiting cases of large and small $|z|$:

$$\overline{|\chi|^2} = |\tau'|^2\dot{u}_{ik}\,\dot{u}_{lm}$$

$$\times \oint\frac{ds}{v}\,\frac{(\lambda_{ik} - \overline{\lambda}_{ik})(\lambda_{lm} - \overline{\lambda}_{lm})}{|1 + z\varkappa\nu|^2} \Big/ \oint\frac{ds}{v}. \qquad (16)$$

Noting that the tensor λ_{ik} is of the order of magnitude of μ_0, we obtain, in the limiting case of small $|z|$ (*i.e.*, for $\omega\tau \ll s/v$), the following formula for the sound absorption coefficient:

$$\gamma \sim \tau\omega^2 N\mu_0/\rho s^2,\quad \omega\tau \lll s/v, \qquad (17)$$

where ρ is the density of the metal and N is the number of electrons per unit volume. This formula coincides with the result obtained in Ref. 3. In this case, Eq. (17) is applicable if $\omega\tau \ll s/v$ (the inexact criterion $\omega\tau \ll 1$ is given in Ref. 3). We note that we can represent the quantity τ in terms of the electronic conductivity.

In the case of large $|z|$, we can represent (16) in the following form:

$$\overline{|\chi|^2} = -\pi|\tau'|^2\dot{u}_{ik}\,\dot{u}^*_{lm}$$

$$\int_0^{2\pi}\frac{(\lambda_{ik} - \overline{\lambda}_{ik})(\lambda_{lm} - \overline{\lambda}_{lm})}{vK(\varphi)\,\mathrm{Im}\,z}\,d\varphi \Big/ \oint\frac{ds}{v},$$

where $K(\varphi)$ is the Gaussian curve of the Fermi surface and the integration is carried out along the line where $\nu \perp \kappa$. (This circumstance is connected with the fact that the electrons moving in the planes of equal phase of the sound wave play the principal role in the energy dissipation.)

The absorption coefficient for $|z| \gg 1$, *i.e.*, for $\omega\tau \gg s/v$, is equal in magnitude to

$$\gamma \sim \omega N\mu_0/v\rho s,\quad \omega\tau \ggg s/v. \qquad (18)$$

We see that in this case the absorption coefficient is proportional not to the second, but to the first power of the frequency, and does not depend on τ, *i.e.*, on the temperature. This expression can be obtained formally from (17), by substituting for the path l the wavelength $\lambda = s/\omega$.

We note that all considerations are valid if $\gamma \ll \omega$. As it is easy to see, this condition reduces, in the case $\omega\tau \gg s/v$, to the inequality $m/M \ll s/v$, where M is the atomic mass.

4. Up to the present we have used the classical description of the sound wave, regarding it as a certain field in which the electrons are located. Such a consideration is known to be valid if $\omega\tau \ll 1$. On the other hand, for $\omega\tau \gg s/v$, the relaxation time τ does not enter into the absorption coefficient, for the computation of which we have not used the inequality $\omega\tau \ll 1$. This shows that Eq. (18) ought to be valid for $\omega\tau \gg 1$.

We can establish this by considering the damping of the sound as the absorption of a sound quantum which takes place as the result of its collision with the conduction electrons. The probability of the absorption of a sound quantum is given by

$$\gamma = \frac{2\pi}{\hbar}\int |U_{12}|^2[n_1(1-n_2) - n_2(1-n_1)]$$

$$\times\,\delta(\varepsilon_1 + \hbar\omega - \varepsilon_2)\frac{2d\mathbf{p}}{(2\pi\hbar)^3}, \qquad (19)$$

where $n_{1,2}$ are Fermi functions corresponding to the energy values $\epsilon_1 \equiv \epsilon(\mathbf{p}_1)$ and $\epsilon_2 \equiv \epsilon(\mathbf{p}_1 + \hbar\mathbf{k})$, and U_{12} is the matrix element of the transition, equal to

$$u_i = e_i\sqrt{\hbar/2\rho\omega}\; e^{i\mathbf{kr}}$$

(**e** is the unit vector of the polarization of the sound wave). We note that the change in the chemical potential is taken into account by having $\lambda_{ij} - \bar{\lambda}_{ij}$, and not $\bar{\lambda}_{ij}$, enter into U_{12}.

Since $\hbar\omega \ll \mu_0$, $\hbar k \ll p_0$, then

$$n_1(1 - n_2) - n_2(1 - n_1) \approx -(\partial n/\partial \varepsilon)\,\hbar\omega,$$

$$\delta(\varepsilon_1 + \hbar\omega - \varepsilon_2) \approx \frac{s}{\hbar\omega v}\,\delta\left(\cos\theta - \frac{s}{v}\right),$$

where θ is the angle between the vectors **k** and $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$. Substituting this expression in Eq. (19), we get the value of γ given by Eq. (18). We note that the coincidence of the results of classical and quantum theory in the case $\omega\tau \gg s/v$ is essentially connected with the small value of s/v in comparison with unity.

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Excitation of Rotational States in the Interaction Between Neutrons and Nuclei

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The excitation of rotational states in nuclei by neutrons is studied in the energy range from the threshold up to 1.5–2 Mev.

1. STATEMENT OF THE PROBLEM

WE SHALL INVESTIGATE interactions between neutrons and nonspherical nuclei and the excitation of rotational states, using an optical model which has been modified to take into account the nuclear deformation caused by the existence of rotational states.

On the usual optical model[1] the total scattering cross section is divided into two parts:

$$\sigma = \sigma_{se} + \sigma_c, \tag{1}$$

where σ_{se} is the elastic scattering cross section for a spherically symmetrical complex potential, and σ_c includes both the cross section σ_{ce} for the formation of a compound nucleus with subsequent emission of particles of the same energy and the reaction cross section (Feshbach, Porter and Weisskopf call σ_c the cross section of compound nucleus formation, which is not quite correct because it includes the cross sections of direct expulsion processes and the excitation of collective motions which will be considered below). All these processes are described by the imaginary part of the complex potential.

For nonspherical nuclei this model must be modified as follows: 1) the complex potential must be nonspherical; 2) since the deformed nucleus is capable of rotational motion, the nonspherical potential of the optical model must be capable of a change of orientation.

In the interaction between a neutron and a nucleus different rotational levels can be excited, *i.e.*, the rotational velocity in the potential can change. We thus have a problem in which the variables which characterize the orientation of the nonspherical potential must be regarded as dynamical