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On the Mechanism of Fission of Heavy Nuclei

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The effect of the state of individual nucleons on the shape of the nucleus prior to fission is studied. It is shown that the presence of excess nucleons with large values of the angular momentum projection on the symmetry axis of the nucleus may lead to loss of stability of the nucleus with respect to asymmetric deformations in the saddle point. This facilitates the explanation of some of the experimental facts.

OUR PRESENT IDEAS about the fission of heavy nuclei at low excitation, based on the liquid drop model[1], are connected with the fact that, for a sufficient elongation of an incompressible drop, the sum of the Coulomb and of the surface energies attains a maximum equal to the fission threshold, further elongation of the drop being energetically favorable. It was shown by various authors[2] that, at the critical elongation, the nucleus retains its stability with respect to asymmetric deformations. The energy of the nucleus expressed in terms of the deformation parameters possesses therefore a saddle point at the critical elongation, the loss of stability depending only on the one deformation parameters that characterizes the elongation. The shape of the nucleus in the saddle point remains symmetric.

The quantitative comparison of calculations based on the liquid drop model with experimental data encounters a number of difficulties. The theoretically predicted strong dependence of the fission threshold $U \sim (1-x)^3$ on the parameter $x \sim Z^2/A$ has not been confirmed experimentally[3,4]. In fact, the threshold was found to be almost identical for a number of elements. Difficulties are also encountered in attempts to explain the observed asymmetry in the mass distribution of fission fragments. It has been shown in recent works[5,6] that it possible to explain this asymmetry on the basis of the liquid drop model. The authors indicate that upon further elongation of the nucleus, after the saddle point has been passed, the stability with respect to asymmetric deformations is lost and there may be a fast increase in the asymmetry of the nucleus. It seems very probable that their estimate of the mean ratio of the masses of the fission fragments is correct. The calculations pertaining to the dynamics of such systems, however, have not been done as yet and the correctness of the above explanations cannot, therefore, be regarded as sufficiently established.

It is easier to explain many of the singularities of the fission process by assuming that the loss of stability with respect to asymmetric deformations during elongation of the nucleus occurs before the energy maximum of the symmetric shape has been passed. In fact, instead of a single saddle point we have in this case two saddle points with an asymmetric configuration of the nucleus. The inequality of masses of the fission fragments is therefore basically explained. If the saddle points of the different nuclei correspond to elongations of the same order, the value of the fission threshold should be proportional to $(1-x)$ and, therefore, the dependence on A and Z should be much weaker than the one predicted by the Bohr-Wheeler theory. Still another experimental fact can be explained more easily by assuming that the shape of the nucleus is asymmetric at the saddle point. Fraser and Milton[7] measured the ratio of secondary neutrons emitted in the direction of motion of the light and of the heavy fission fragments respectively. It was found that this ratio differs considerably from unity and, which is of special interest, does not approach unity (remains of the order of 3) even when the fragments have almost equal masses, namely $m_H/m_L = 1.1$. This fact indicates that the fission of the nucleus into fragments of similar mass is not really a symmetric event and the excitation energies of the two fragments differ considerably. Were the symmetric shape of nucleus to correspond to the saddle point, the fission into two equal fragments would correspond to the not very probable case of the nuclear shape changing by a sequence of completely symmetric deformations and there would not be any serious reason for the occurrence of such a case. On the other hand, for an asymmetric saddle point the inequality of the light and heavy fragments is inherent from the very beginning and there is no reason to expect all the parameters characterizing the fragments (charge, excitation energy) to be equal in case of a change equality of masses.

The above considerations add interest to the study of possible conditions for the occurrence of asymmetric saddle points, despite the fact that such a notion does not lie within the classical framework of the liquid drop model. The explanation of this effect, given below, is based upon the study of the states of individual nucleons in the deformed nucleus in the spirit of the collective model, and is of a qualitative character.

We shall consider only such states of deformation of the nucleus for which the nucleus possesses rotational symmetry with respect to the OZ axis. Such states are most favorable energetically for a given elongation of the nucleus and there are all reasons to suppose that they lead more easily to fission. For such a symmetry of the nucleus the wave functions of free nucleons can be classified in terms of the quantum number $\Omega = l_z + s_z$, equal to the projection of the total angular momentum of the nucleon on the nuclear axis of symmetry[8]. The quantum numbers Ω represent the approximate integrals of motion and are adiabatically invariant with respect to slow changes in the shape of the nucleus. Out of the variables r, z, φ and s_z which determine the position and spin of a nucleon in the cylindrical system of coordinates, the latter two can be separated by introducing the functions $\exp\{i(\Omega \pm ½)\varphi\}$. The order variables, as a rule, cannot be separated. For a symmetric deformation of the nucleus we have to account also for the classification of the wave functions of individual nucleons with respect to their parity, which is not conserved for asymmetric deformations. Evidently, for excitation energies of the order of 5–6 Mev, corresponding to fission resulting from slow neutron capture, the quantum number will not represent the integrals of motion so accurately as in the case when the nucleus is in the ground state. Strongly elongated states that are prone to fission should, however, approach the unexcited state since the greater part of the energy surplus has been already used up for the elongation. The nucleus in this state has so to speak cooled down, which renders transitions between various states of individual nucleons more difficult in view of the Pauli principle. Besides, departures from rotational symmetry which would facilitate the mixing of states with different Ω, are reduced, owing to the lack of a sufficient amount of energy for the excitation of the corresponding degrees of freedom of the nucleus.

With the progress of the elongation of the nucleus the states with high Ω become energetically unfavorable because of the centrifugal energy $\hbar^2 l_z^2/2mr^2$, the mean value of which rises sharply with diminishing radial dimensions of the nucleus. At the same time, the number of energetically favorable states with small values of Ω increases with increasing length of the nucleus. Consequently, a reorganization of the filling of nucleonic levels occurs during the elongation of the nucleus. As mentioned above, for the largest energetically permissible elongations such reorganization is difficult and there is every reason to believe that the nucleus in such a state will posses an excess of nucleons with large values of the angular momentum projection Ω, while there will be a deficiency in the filling of levels with low Ω.

In the asymptotic approximation of a large number of particles, the energy of the degenerated Fermi gas depends only on the volume and not on the shape of the container. This, however, is true first for the case of the nucleus only for a very large number of nucleons, and secondly for the equilibrium distribution of levels with different values of angular momentum. As it is well known[9], it is indeed the deviations of the state distribution of nuclei from the asymptotic laws that lead to a marked elongation of nuclei with unfilled shells in the ground state. An analogous influence of the nucleon distribution on the shape of the nucleus should take place in fission as well.

It is easy to show that the presence of excess nucleons with high values of Ω in a strongly elongated nucleus should sharply diminish its stability with respect to asymmetric deformations and should slightly increase the stability with respect to symmetric deformations. This effect can be approximately estimated in the following way: let the elongated nucleus be of the shape of an axially symmetric body with a·symmetric generator

$r_1(z) = r_1(-z)$, the maximum $r_1(z)_{\max} = b$ being attained for $z = 0$. We shall consider a small asymmetric deformation of the nucleus $\Delta r = \beta\eta(z) = -\beta\eta(-z)$. The maximum of the cross-section of the nucleus will be shifted in the direction of positive Δr and will be equal

$$r_{1\,\max} = b + {}^1/_2\, \beta^2\, (d\eta\,/\,dz)^2_{z=0}\, |\, d^2 r_1\,/\,dz^2\, |^{-1}_{z=0}.$$

The energy of a nucleon with the largest value of angular momentum Ω can be estimated from the maximum cross-section of the nucleus r_m:

$$W \sim \hbar^2\, l_z^2 / 2 m r_m^2.$$

For an asymmetric deformation the energy increases by the amount

$$\Delta W \sim -\beta^2\, (\hbar^2\, l_z^2\,/\,2mb^3)\, (d\eta/dz)^2_{z=0}\, |\, d^2 r_1\,/\,dz^2\, |^{-1}_{z=0},$$

the sign of which indicates the decrease in stability with respect to the deformation $\beta\eta(z)$. We shall note that, even for $d^2 r_1/dz^2 > 0$ the sign of the effect is invariant, although the change of the maximum radius cannot be estimated in such a simple way. For the shape of the nucleus approximating an ellipsoid of revolution with semiaxes a_0 and $b_0 = \sqrt{a_0^2 - e^2}$:

$$r = \sqrt{a^2 - e^2}\,\sqrt{1 - \mu^2},$$

$$z = a\mu, \quad a = a_0\,[1 + \beta P_3\,(\mu)],$$

we obtain

$$d^2 r\,/\,dz^2 = b\,/\,a^2; \quad \Delta r \approx 3az\beta\,/\,2b,$$

$$\Delta W \sim -9\hbar^2 l_z^2\, a^4 \beta^2\,/\,8mb^6.$$

The sum of these values for the excess nucleons with the largest Ω has to be equated to the deformation parameter distribution of the Coulomb and of the surface energies, separating the terms proportional to β^2 which determine the shape stability with respect to asymmetric deformations. For a spherical nucleus this term equals

$$\Delta E = \frac{2}{7}\left(\frac{5}{2} - \frac{10}{7}\,x\right) E_s \beta^2,$$

where β is the coefficient of the third-order Legendre polynomial, E_s is the surface energy of the nucleus and x is the fission parameter. For the case of elongated nuclei with the axis ratio equal to 1.5 – 2 this value, according to the estimates of Ref. 5 and 6, is reduced by a factor of ½ at least. Assuming for our estimate $x = 0.7$ and $E_s = 500$ Mev, we obtain, for the case of elongated nuclei

$$\Delta E \approx \beta^2 \cdot 100 \text{ Mev}.$$

Assuming for the case of the uranium nucleus $ab^3 = (1.3 \times 10^{-13})^3\, A$ and $b = 0.5\, a$, we shall estimate the effect of one excess nucleon with orbital angular momentum $l_z = 5$:

$$\Delta W \approx -\beta^2 \cdot 350 \text{ Mev}.$$

For the case of a strongly elongated nucleus, therefore, the effect of a single additional nucleon with a large value of angular momentum is very large and the shape stability with respect to asymmetric deformations is, evidently, lost much sooner. It is possible that this occurs at elongations only slightly larger than the initial elongation of the nucleus in the ground state.

It should be noted that the loss of stability with respect to small asymmetric deformations, due to such a mechanism, should not cause an unlimited increase of asymmetry. When a pear-like shape is attained by the nucleus, any further dilatation of the wider end should stop as soon as the maximum radius attains such a value that the energy of nucleons with the largest value of angular momentum equals the Fermi limit. This makes it possible to explain qualitatively the experimentally observed fact that the mass of the larger fission fragment is is equal for different elements. As it is well known, the mass of the lighter fragment varies within much wider limits. Evidently, for all studied fissile nuclei, the maximum values of the nucleonic angular momentum coincide prior to fission. Most probably, all of them then posses a pair of neutrons with an angular momentum of the order of 5–6. The angular momentum of these nucleons determines the cross-section of the wider end of the nucleus which subsequently forms the heavier fission fragment. It follows from this approximate quantization of the size of the heavy fragment that the variations in its mass are smaller than is the case for the lighter fragment.

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Moment of Inertia of a System of Interacting Particles

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The problem of singling out the collective degrees of freedom of a system consisting of N interacting particles is considered. It is shown that for some special states of internal motion, the energy of the system in the center of mass system can be represented as the sum of the energy of internal motion and the rotational energy. The concept of the moment of inertia of a system of N interacting particles is introduced.

INTRODUCTION

AT PRESENT it has been established that the lowest excited states of nuclei in the mass number range $150 < A < 190$ and $A > 225$ are rotational states. Such states arise in Coulombic excitation of the nuclei, in processes of radioactive decay, and also in inelastic collisions of particles with the nucleus.

An explanation of rotational states of the nucleus in the quasi-molecular model of the nucleus proposed by A. Bohr[1] is related to the motion of a wave around the nucleus. The nuclear matter is regarded as an incompressible, irrotational fluid (the hydrodynamical model). The part of the nuclear matter which participates in the rotation, according to the hydrodynamical model, is proportional to the square of the deviation of the form of the nucleus from a sphere. If we assume that the nucleus has the form of an ellipsoid of rotation with semiaxes c and a, then the moment of inertia of the nucleus is given by

$$J = {}^1/_5\, mA\, (c^2 - a^2)^2 / (c^2 + a^2),$$