Alfen effect is observed is considerably less than the mass of the free electron (for which $\mu \sim 10^{-20}$). If $m^* = 10^{-2} m_0$ is taken, then it is shown that $\delta\varphi \sim 10^{-3}$ V. Moreover, it should be taken into account that $\zeta_0 \sim 10^{-12}$ erg corresponds to one electron per atom ($n \sim 1$) which, as a rule, also does not hold ($n < 1$).

If there are several electron groups then the potential difference arising is determined as

$$e\,\delta\varphi = 2\sqrt{2\pi}\left(\frac{e\hbar H}{c}\right)^{3/2}\sum\left|\frac{\partial^2 S}{\partial p_z^2}\right|_m^{-1/2} G \Big/ \sum dU_0/d\zeta\,. \quad (3)$$

Here, the summation in the numerator and denominator is made over all the electron groups. However, it should be taken into account that even the weak domain structure, which always exists in single crystals, leads to the diffusion or the total smoothing of the oscillations in components corresponding to the bands with a large number of electrons for small fields. Actually, it is easy to show[1] that it is simply necessary to average the right side of (3) over the various domains to take the domain structure into account. Here, naturally, the components containing the significant random variations of the sine argument drop out,* *i.e.*, precisely those components which correspond to bands with a large number of electrons.

2. The variation of the chemical potential of an electron gas in a magnetic field leads to a dependence of the emission current (both the cold and the thermionic emission) on the magnetic field[2]. Hence, the oscillations in the magnetic field must be observed only in the cold electron emission case**. Portions of the electrons located on the "tail" of the Fermi distribution where they behave substantially as classical particles participate in the thermionic emission. Consequently, a periodic dependence on the magnetic field naturally does not arise.

Direct calculation of the thermionic current under the assumption that the magnetic field is perpendicular to the metal surface and that the dispersion law is quadratic and isotropic, yields

$$I(H) = \frac{\beta H \cosh(\beta_0 H / kT)}{kT \sinh(\beta H / kT)} I_0,$$

where I_0 is the thermionic current in the absence of the magnetic field, $\beta = e\hbar/2m^*c$ is the electron diamagnetic moment, m is its effective mass, $\beta_0 = e\hbar/2mc$ is the electron spin magnetic moment.

It is interesting to note that the thermionic current can either increase or decrease in strong magnetic fields depending on the relation between the effective mass m^* and the mass of the free electron m. If $m^* = m$, then

$$I(H) = \frac{\beta H}{kT \tanh(\beta H / kT)} I_0,$$

that is, the thermal current increases linearly with the magnetic field $\tanh x \sim 1$, $x \gg 1$ for $\beta H \gg kT$.

It should be noted that the experimental observation of thermionic current variation in a magnetic field is extremely difficult because of the low current intensity at low temperatures.

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*We note that $S_m(\zeta)$ is a function of the angles between the magnetic field and the crystallographic axes.

**The variation of the total cold emission current in a magnetic field is extremely slight. In particular, a periodic dependence on the magnetic field is apparently not observable in practice. Consequently, we do not dwell on these questions here.

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Natural Line Widths of Microwaves

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When molecules are contained in a space whose linear dimensions are much smaller than the emitted wavelength, as often occurs in the microwave range, the molecules radiate as a single quantum-mechanical system and the line width of the spontaneous emission depends on the number of molecules. Dicke and Romer[1,2] showed that the line width is proportional to the number of molecules for "bound" states. The present note will show that this result is also valid for "unbound" states of a molecular system.

We shall consider the practical case of a Boltzmann distribution of molecular energies. For simplicity we shall assume that the molecules are in only two nondegenerate energy states E_+ and E_- ($E_+ > E_-$). The molecular radiation will be regarded as approximately dipolar. It will be assumed that direct molecular interactions can be neglected. Let $n_\pm$ be the numbers of molecules in states of energy $E_\pm$. Then when the gas is in thermal equilibrium with a thermostat at temperature T:

$$n_- - n_+ \approx nE/2kT;\; n = n_- + n_+;\; E = E_+ - E_- = \hbar\omega_0. \tag{1}$$

Here we have taken account of the smallness of E compared with kT, as is usually the case for microwaves.

According to Dicke[1], the states of the gas are characterized by the quantum number $m = (n_+ - n_-)/2$ and the "cooperation number" r. The following condition is satisfied:

$$|m| \leqslant r \leqslant n/2. \tag{2}$$

The radiation intensity from a gas in state (r, m) is[1]

$$I = I_0 (r + m)(r - m + 1), \tag{3}$$

where I_0 is the intensity of spontaneous emission from a single isolated molecule.

When the molecular system is in state (1), $\bar{m} = -nE/4kT$. The bar denotes averaging over the Boltzmann energy distribution. By Eq. (2), r cannot be smaller than $|m|$. It also cannot be much larger than $|m|$ because states with large r are of small statistical weight[1]. When $r = -m$ there is no spontaneous emission because $|m|$ then increases by unity and violates condition (2). Thus it is physically significant to consider the radiation from a gas in the state $m = \bar{m} + 1$; $r = -\bar{m}$. The radiation intensity from such a gas is*

$$I = -I_0 2\bar{m} = I_0 nE/2kT, \tag{4}$$

and thus proportional to n. We shall determine the natural line width when the initial state is $m = \bar{m} + 1$, $r = -\bar{m}$. For the probability amplitudes $b(r, m, \nu)$. where ν denotes the number of photons of frequency ω_λ) in the radiation field, the method of Weisskopf and Wigner[3,4] gives the equations

$$-i\hbar\dot{b}(-\bar{m}, \bar{m}+1, 0) = \sum_\lambda (-\bar{m}, \bar{m}+1, 0\,|H'|\,-\bar{m}, \bar{m}, 1)$$
$$\times\, b(-\bar{m}, \bar{m}, 1)\, e^{i(\omega_\lambda - \omega_0)t}, \quad -i\hbar\dot{b}(-\bar{m}, \bar{m}, 1)$$
$$= (-\bar{m}, \bar{m}, 1\,|H'|\,-\bar{m}, \bar{m}+1, 0)$$
$$\times\, b(-\bar{m}, \bar{m}+1, 0)\, e^{i(\omega_0 - \omega_\lambda)t},$$

where H' is the interaction energy operator. These equations are solved as in Refs. 3, 4 (for more details see Ref. 5). We use Dicke's relation[1]

$$(r, m\,|H'|\,r, m-1) = (E_+|H'|E_-)\,[(r+m)(r-m+1)]^{1/2},$$

where $(E_+|H'|E_-)$ is the transition matrix element for a single independent molecule. The resulting spectral intensity distribution is

$$J(\omega)\,d\omega = \frac{-2\bar{m}\gamma_0}{2\pi}\,\frac{\hbar\omega\,d\omega}{[(\omega-\omega_0)^2 + \bar{m}^2\gamma_0^2]}.$$

Thus in this case the natural line width is

$$\gamma = -2\bar{m}\gamma_0 = (\hbar\omega/2kT)n\gamma_0, \tag{5}$$

where γ_0 is the natural line width for a single molecule. We shall now estimate γ. Let $\omega_0 = 2\times 10^{11}$ sec^{-1} ($\lambda \approx 1$ cm):

$$\gamma_0 \approx \mu_{ab}^2\,\omega_0^3/\hbar c^3 \text{(cf. Ref. 4)},\ \mu_{ab} = 10^{-18}\,\text{CGSE};$$

then $\gamma_0 = 3\times 10^{-7}$ sec^{-1}. This would be the natural line width of a gas where the molecules are assumed to radiate independently. If in a distance of the order of 1 mm, $n = 10^{15}$ molecules are to be found (on levels E_- and E_+) and $T = 300$°K, then $nh\,\omega_0/2kT = 25\times 10^{13}$ and $\gamma = 10^5$ sec^{-1}. Thus the natural line width of microwaves is not negligibly small, as is usually assumed.

It is easily shown that absorption lines also have the width given by (5)[5]. This follows from general considerations.

We note in conclusion that there is a classical analogue of the effect which has been described. In a system of oscillators whose separation is small compared with the emitted wavelength there is always an indirect interaction through the total radiation field. In the dipole approximation, the radiation damping force which acts on each oscillator is[6]

$$K_{\text{rad}} = (2e^2/3c^3) \sum_{k=1}^{n} \dddot{r}_k. \tag{6}$$

By using (6), it is easily found that the width of the emitted line is $\gamma = n\gamma_0$, where n is the number of

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*We note that for spontaneous emission by a system of molecules separated by distances much greater than the emitted wavelength, we would have $I = n_+ I_0$, which disagrees with (4).

oscillators. This will be discussed in greater detail in another article.

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Double Absorption Peaks in Electron Resonance at Saturation

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SMALLER[1] observed the splitting of nuclear resonance lines when the modulation frequency of the magnitude field was of the order of the line width. We have observed the same type of double peaks of electron resonance absorption in strong fields, when the angle α between the magnetic component H_ν of the radiofrequency field and the static magnetic field H_0 differs from 90°.

Measurements were performed by Zavoiskii's[2] grid current method from 3 to 300 Mc at room temperature. The method is described in detail in Ref. 3. The oscilloscope screen showed twin resonance peaks. The splitting was produced by a 50Mc magnetic field.

For the free radical of $\alpha\alpha$-diphenyl-β-trinitrophenyl-hydrazyl in weak radiofrequency fields when α is changed from 90° to 0° only the ordinary resonance absorption peak is observed with $g_{\mathrm{eff}}=h\nu/\beta H_0=2$ (Fig. 1 *a*). This peak has maximum intensity when α=90° and vanishes when α=0. In strong fields, the free radical with $\alpha \neq 90°$ (Fig. 2) reveals additional absorption peaks (Fig. 1b) which result from modulation of the static magnetic field H_0 by the parallel component $H_{\nu\parallel}$. The number and position of these peaks depend both on the field strength and the angle α; they are given by $\nu_0=\nu+n\nu$, where $n=1, 2, 3, \ldots$ This is a special case of Smaller's condition[1].

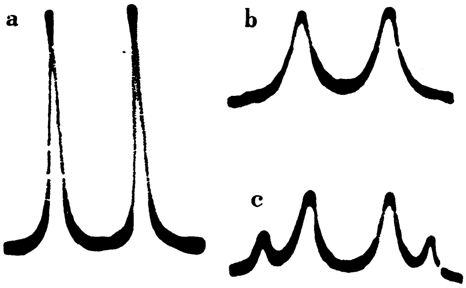

FIG. 1. Twin resonance absorption in the free radical of $\alpha\alpha$-diphenyl-β-trinitrophenyl-hydrazl at ν = 30 Mc and T = 300°*K*. *a)* α = 90° with field ($H_\nu \approx 0.1$ oersted) *b)* α = 90° and *c)* α = 45 with strong field (H_ν = 2 oersteds)

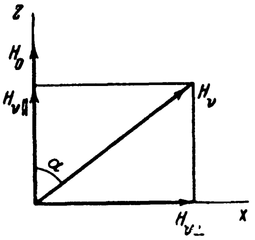

FIG. 2.

Additional peaks were also observed in anthracite from the Kuznets basin and in metallic lithium.

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