Electron-Loss Probabilities for Multiply-Charged Ions

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The electron-loss probability for fast multiply-charged ions moving in a medium is considered. It is assumed that the loss probability depends only on the ratio of ion velocity to electron orbital velocity. Using this assumption, it is possible to compute the effects of electronic interactions between fast ions and matter by summing over individual electrons of the ion. Results of calculations of the mean charge, equilibrium charge distribution, specific ionization and range in air are presented for helium, lithium, beryllium, and nitrogen ions. The computed values are found to be in good agreement with the available experimental data.

IN analyzing the effects which arise in the passage of multiply-charged ions through matter, it is usually assumed that those electrons, the velocity of which is below a certain given value, are completely stripped while the remaining electrons remain with the ion, that is, the probability for electrons loss changes sharply at a given electron velocity. In the original theory, as given by Bohr[1], it was assumed that $v/v_e = 1$. Here v is the ion velocity for which the probabilities for electron capture and electron loss are equal and v_e is the orbital electron velocity, calculated on the basis of the Fermi-Thomas model. As more experimental data became available, the original Bohr criteria were refined. Thus, Branning, Knipp and Teller[2,3] have taken $v/v_e = \gamma$. However, γ is determined experimentally, and the values of this quantity in different ions and in a given ion are a function of velocity.

It is of great interest to find more general criteria for determining the electron-loss probabilities for fast multiply-charged ions. Starting from the fact that the interaction energy between individual electrons within the atom is small compared with the electronic binding energy, and neglecting the screening of inner electrons (this last assumption is somewhat debatable, especially for K-electrons, but is justifiable for strong ionization), we propose the following.

1. The probability for the loss of a given electron P, depends only on the ion velocity v and is independent of the loss of other electrons:

$$P = P(v)$$

2. The function $P(v/v_I)$ is the same for inner and outer electrons of all atoms. Here, v_I is the electron orbital velocity. Although the velocity of the electron depends on the degree of ionization, in what follows we will use the constant value $v_I = \sqrt{2I/\mu}$ derived from the ionization potential for a given electron I.

The form of the function $P(v/v_I)$ can be obtained from measurements of the mean charge of ions of atomic hydrogen as a function of velocity, that is, the case of ions with a single electron. The function $P(v/v_I)$ for hydrogen ions is somewhat different in different media. In the following analysis use will be made of the function $P(v/v_I)$ measured in air[4-6]. For the hydrogen ion $v_I = v_0 = e^2/\hbar = 2.19 \times 10^8$ cm/sec.

The ratio of the electron-loss cross section for a given level to the capture cross section for the same level may be written in the form

$$\sigma_{\text{loss}}/\sigma_{\text{cap}} = P/(1-P), \tag{1}$$

that is, this quantity also depends only on the velocity of the ion. Gluckstern[7] has determined the electron-capture and loss cross-sections for multiply-charged ions experimentally and has also calculated these quantities on the basis of the Fermi-Thomas model; however, in the work of this author these quantities appear as cross-sections summed over all electrons in the loss case and over all unfilled levels in the capture case and the present analysis does not apply directly.

In a multiply-charged ion the loss probability for each electron, P, can be obtained as a function of velocity by multiplying the abscissa $P(v/v_0)$ by the corresponding orbital velocity for the electron v_I. In this case, the density of the medium must be such that the time between charge-exchange collisions does not allow reforming of the electron shells[8].

Using these assumptions it is possible to calculate the mean charge $\bar{z}$, the equilibrium charge distribution Φ, the specific ionization dE/dx and the range R of multiply-charged ions as a function of velocity. Below, we present the results of these calculations for helium, lithium, beryllium and nitrogen ions and certain values for oxygen and fluorine ions.

For a given velocity the mean ion charge is found by summing the loss-probabilities over all electrons, that is,

$$\bar{z} = \sum_{i=1}^{z} P_i . \qquad (2)$$

In Fig. 1 is shown the dependence of $\bar{z}$ on velocity for the ions He, Li, Be, and N. In this same figure, we plotted the experimental values of $\bar{z}$ for helium in hydrogen, air, and mica and for nitrogen in nickel, in the thin organic film, in Formvar and in nitrogen.

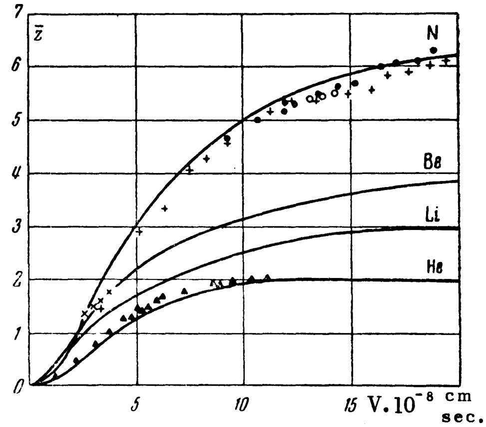

FIG. 1. Dependence of $\bar{z}$ on velocity for He, Li, Be and N ions. ▲ measured values of $\bar{z}$ for helium ions in hydrogen, air and mica[9,10]. Measurements of $\bar{z}$ for nitrogen ions: + in nickel[11], o in thin organic films[12], ● in Formvar[13], × in nitrogen[14].

TABLE I

Ion	N^{14}			O^{16}		F^{19}
V.10^{-8} cm/sec.	14,3	13,7	13,2	14,3	8,6	14,3
$\bar{z}$ experimental.	5,49	5,40	5,35	6,22	4,86	6,63
$\bar{z}$ calculated from Eq.(2)	5,73	5,65	5,58	6,38	4,86	6,94
$\bar{z}$ calculated in Ref. 12 using assumption "a".	5,88	5,86	5,84	6,78	5,68	7,63
The same using assuption "b".	5,26	5,18	5,12	5,83	4,52	6,36

Table 1 shows values of $\bar{z}$ as measured in a thin organic film for N^{14}, O^{16} and F^{19} ions[12] and as calculated from Eq. (2). For comparison purposes, in this same table are given values of $\bar{z}$ calculated in Ref. 12 on the basis of the Fermi-Thomas statistical model under the assumptions that a) the electrons with smallest binding are stripped and b) the outermost electrons are stripped.

The equilibrium charge distribution Φ_i, that is, the fraction of ions with a charge i, is found by summing the quantities φ_{ij} – the fractions of ions with charge i for different fixed configurations of the remaining electrons j.

$$\Phi_i = \sum_{j=1}^{C_z^i} \varphi_{ij} = \sum_{j=1}^{C_z^i} \Big[\prod^{i} P_k \prod_{k \neq q}^{z-i} (1-P_q)\Big]. \qquad (3)$$

For example, we obtain the following expressions for the equilibrium charge distribution for lithium ions

$$\Phi_0 = (1-P_1)(1-P_2)(1-P_3),$$
$$\Phi_1 = P_1(1-P_2)(1-P_3) + P_2(1-P_1)(1-P_3) + P_3(1-P_1)(1-P_2),$$
$$\Phi_2 = P_1P_2(1-P_3) + P_1P_3(1-P_2) + P_2P_3(1-P_1),$$
$$\Phi_3 = P_1P_2P_3 .$$

The number of terms in the sum for Φ_i is equal to the number of components C_z^i; however, some of the P_i are unity and the corresponding φ_{ij} become zero.

In Table 2 are shown the values of the equilibrium distributions obtained with thin films using nitrogen, oxygen and fluorine ions[12]. The experimental values of Φ (expressed in percent) are listed in the columns marked "a" and those calculated from Eq. (3) are in the columns marked "b".

The equilibrium charge distribution as a function of velocity is shown in Fig. 2 a–b for helium, lithium, beryllium and nitrogen ions. (The absence of

TABLE II

Ion	N^{14}						O^{16}				F^{19}	
$V\cdot10^{-8}$ cm/sec.	14,3		13,7		13,2		14,3		8,6		14,3	
i	a	b	a	b	a	b	a	b	a	b	a	b
1		0		0		0		0		0,06		0
2		0		0		0		0		2,1		0
3	0,4	0,2	0,2	0,2	0,5	0,3		0,04	4,5	8,6		0,02
4	6,0	4,1	7,5	5,6	8,6	6,6	0,5	0,1	26,4	24,8		0,5
5	45,6	35,8	50,1	38,0	51,7	39,9	14,0	13,6	47,5	36,4	8,8	5,9
6	40,5	44,6	36,6	42,7	34,3	41,1	51,8	41,9	21,6	22,7	32,6	24,2
7	7,5	15,3	5,6	13,5	4,9	12,1	30,3	35,6	0	4,9	46,5	41,7
8							3,4	8,8	0	0,3	10,8	23,6
9											1,3	4,1

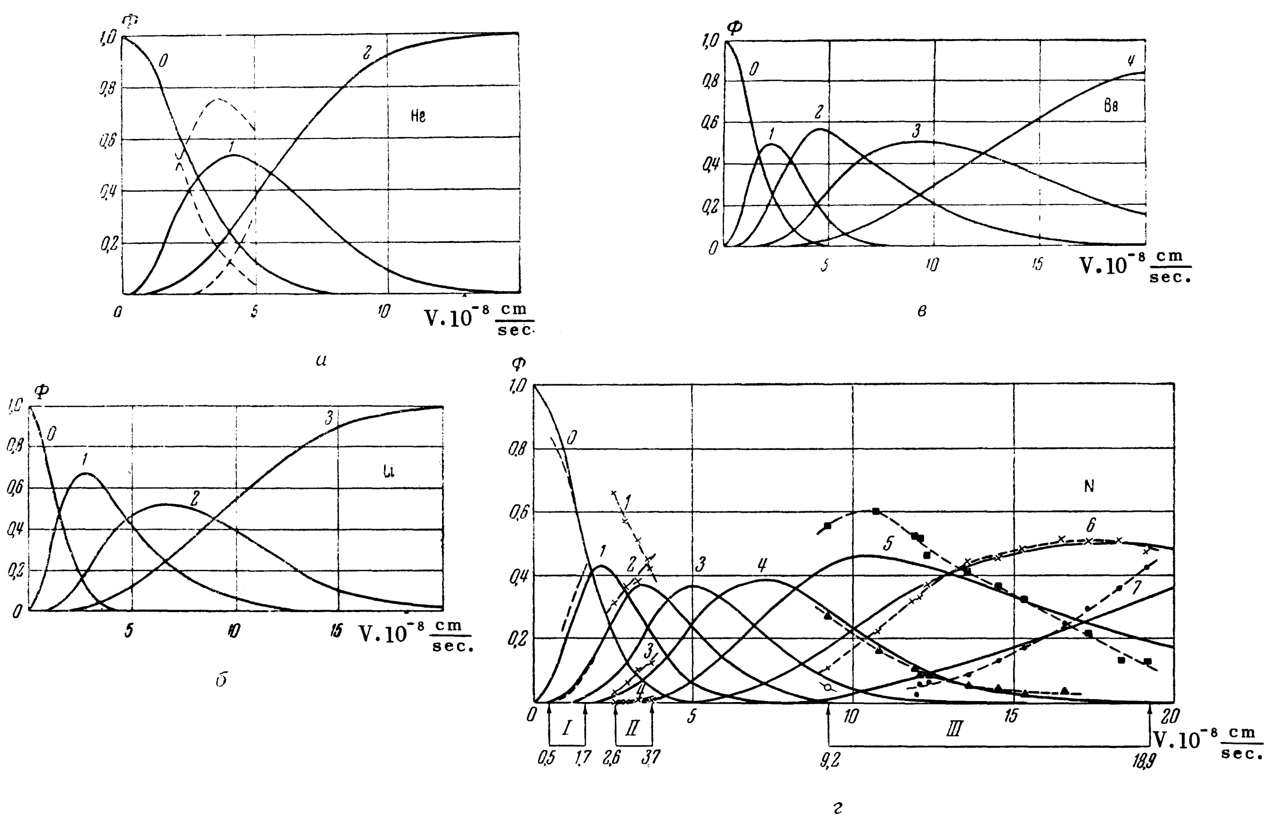

FIG. 2. Equilibrium charge distribution as a function of velocity: a) helium ions; the dashed curves pertain to the experimental values of Φ measured in air[15]; b) lithium ions; c) beryllium ions; d) nitrogen ions; the dashed curves denote the experimental values of Φ: I) $0.5 \leqslant v \leqslant 1.7 \times 10^8$ cm/sec. Measurements in nitrogen[6]. In calculating the equilibrium distribution it has been assumed that $\Phi_3/\Phi_2 = 0$; II) $2.6 \leqslant v \leqslant 3.7 \times 10^8$ cm/sec. Measurements in nitrogen; it is assumed that at these velocities $\Phi_0 = 0^{14}$; III) $9.2 \leqslant v \leqslant 18.9 \times 10^8$ cm/sec. Measurements in Formvar[13].

data on the magnitude of the neutral component of the nitrogen ion beam[14] obviously leads to a high value for the singly and doubly-charged components.)

The experimental curves for the equilibrium distribution start systematically at ion velocities higher than the calculated values. This may be explained by the fact that the inner electrons are shielded by the outer electrons; for equal values of the ratio v/v_I it should be more difficult to remove inner electrons.

Inasmuch as a beam of multiply-charged ions, moving in a medium, consists of component beams with charge $i=0, 1, 2 \ldots z$, between which dynamic equilibrium obtains, the specific ionization is found by summing over all component beams, that is

$$\frac{dE}{dx} = N\left(\sum_{i=1}^{z} \Phi_i i^2\right)\sigma_e, \quad (4)$$

where the sum of squares of the charges, taken with appropiate weighting factors Φ_i is multiplied by the quantity σ_e the specific electron stopping cross-section calculated by Knipp and Teller[2], and by N – the number of atoms per cm^3 of the medium (in this procedure, no account is taken of energy losses due to excitation of the atoms and to nuclear collisions).

The results of calculations of the specific ionization as a function of velocity for He, Li, Be and N ions are shown in Fig. 3. The dashed curve indicates the specific ionization for α-particles given by Bethe and Livingston[16].

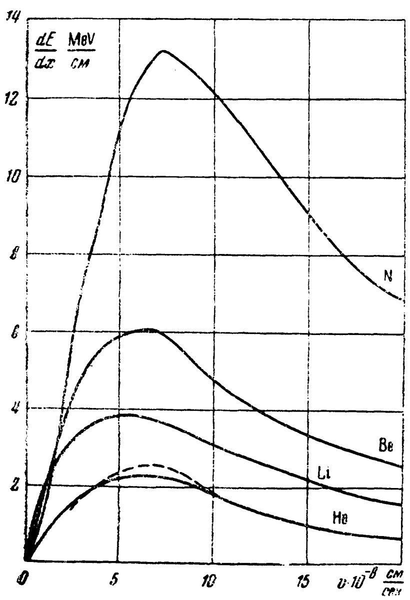

FIG. 3. Specific ionization due to ions as a function of ion velocity. The dashed curve pertains to the specific ionization of α-particles given in Ref. 16.

Numerical integration of the specific ionization gives the range of the ion in the medium

$$R = \int_0^E \frac{1}{dE/dx} dE. \quad (5)$$

In Figs. 4 and 5 are shown the range-energy relations for He^4, Li^8, Be^9 and N^{14} ions in air. The points

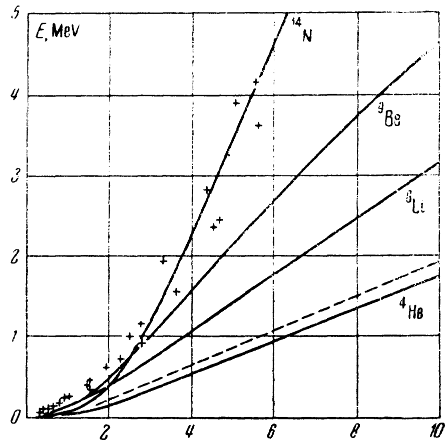

FIG. 4. Range-energy relation for ions (in *mm* of air). The dashed curve denotes the experimental values for helium[16], + the same for nitrogen[17].

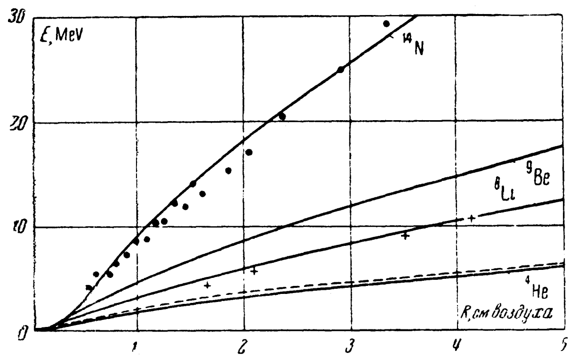

FIG. 5. Range-energy relation for ions (in *cm* of air). Experimental values: dashed curve) for helium[16] + for Li^8 in a photo-emulsion[18]. ● for N^{14} in a photo-emulsion[19]. The magnitude of the stopping power in both cases is taken as 1800.

denote the experimental value of the range for α-particles, which differs from the calculated value by approximately 10 percent. In Fig. 4 are shown range values for nitrogen ions obtained by Blackett and Lees in a Wilson cloud chamber[17]. In Fig. 5 are also shown range values calculated for Li^8 ions in air[18] and N^{14} ions in a photo-emulsion[19]. The range-energy curve for Li^8 given by Kuznetsov, Lukirskii, and Perfilov[20] lies considerably below the calculated curve.

In conclusion it should be noted that the calculated results, in general, are in satisfactory agreement with the available experimental data; the discrepancies which are observed in a number of cases can be attributed to limitations imposed by the assumptions which have been used and to experimental errors.

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Inelastic Proton-Proton Scattering

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The problem of inelastic proton-proton scattering at 690 Mev is considered. It is assumed that in the intermediate state an isobar is formed which decays into a nucleon and π-meson. Limiting the consideration to isobars in S-states, using the laws of conservation of angular momentum and parity, and taking into account the Pauli principle, it was found possible to obtain the angular distribution of scattered nucleons, with the introduction of only one arbitrary constant.

INTRODUCTION

WE base our consideration of proton-proton scattering on the experimental fact[1] of the existence of an excited state of the nucleon (isobar) with ordinary and isotopic spins equal to 3/2. We consider further that in the collision process an isobar with a definite mass $M=1.31$* is formed in an S-state. Neglect of P- and D-state isobars is possible for energies of the incident nucleon not greatly exceeding the 650 Mev threshold energy for formation on an isobar with mass $M=1.31$. In connection with this, all calculations were carried out for an incident proton energy of 690 Mev, which is the energy obtained on the accelerator of the Institute for Nuclear Problems, Academy of Sciences. The assumptions made proved to be sufficient to obtain an angular distribution, containing one arbitrary constant, for the scattered protons.

It must be noted, however, that because of the finite lifetime of the isobar ($\sim 10^{-23}$sec) the energy spectrum and angular distribution of the scattered protons at 690 Mev should be rather strongly smeared out, and only upon increasing the energy of the incident protons to 800 Mev and above does the picture become more clear cut.

1. KINEMATICAL CALCULATION

We consider collisions of two particles of mass m_1 and m_2 such that two new particles of mass M_1 and M_2 are formed in place of the initial ones. Let

*An absolute mass unit corresponding to 931 m_e is taken as mass and energy unit.