On the Absorption of High Energy Nuclear-Active Particles

S. Z. BELEN'KII AND N. M. GERASIMOVA
P. N. Lebedev Physical Institute, Academy of Science, USSR
(Submitted to JETP editor December 29, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 547-551 (March, 1957)

The propagation of nuclear-active particles in matter is computed from the cross sections for the elementary collision act, obtained on the basis of the hydrodynamic theory of multiple particle production.

THE experimental data[1] show that the process of multiple particle production occurs in nuclear collisions in the high energy region ($\sim 10^{12}$ ev and higher). The secondary particles of high enough energy can, in their collision with the nuclei of the matter, lead to additional multiple production (we will call such particles nuclear-active). In this fashion not only absorption, but also multiplication, of the nuclear-active particles occurs as they propagate through matter. For the solution of the problem of propagation of nuclear-active particles, we will start from the expression for the energy distribution in the elementary act, obtained on the basis of the hydrodynamic theory of multiple particle production[2,3]. We use the following formulas[3] for the collision of a pion or a nucleon with a nucleus:

$$dN = C_1 (2\pi L)^{-1/2} \exp\{-L/2 + \sqrt{L^2 - \lambda^2}\}\, d\lambda; \quad (1)$$

$$E = C_2 \exp\left\{\frac{5}{6} L + \lambda + \frac{1}{3}\sqrt{L^2 - \lambda^2}\right\}. \quad (2)$$

where dN is the number of particles produced in the interval $d\lambda$; λ is a parameter related to the energy E of the particle by the relationship (2). Formulas (1) and (2) give the particle distribution in a parametric form. The parameter λ varies in the range $-\sqrt{3}L/2 < \lambda < \sqrt{3}L/2$. The magnitude L is related to the energy of the primary particle.

As shown in Refs. 1 and 3, the collision of the particle with the nucleus can be considered as the collision of the particle with a "tube" cut out from the nucleus, of cross section equal to that of the nucleon; the length of this tube varies between the nuclear diameter and a length of the order of the nucleon's size. In the center-of-mass system, the dimensions of the impinging particle and of the tube are subject to a Lorentz contraction in the direction of motion. The quantity e^{-L} represents the ratio of the transverse dimensions of the system at the first instant after the collision, to the longitudinal dimensions:

$$e^{-L} = [2Mc^2\chi(\bar{l})/E_0]^{1/2}, \quad \chi(\bar{l}) = \bar{l}\,(1/\bar{l} + \bar{l})^2. \quad (3)$$

where M is the nucleon mass; E_0 is the energy of the impinging particle in the laboratory system; l is the average length of the tube in units of the nucleonic radius;

$$\bar{l} = 2[A - (2A^{1/3} - 1)^{3/2}]/3(A^{1/3} - 1)^2, \quad (4)$$

where A is the atomic number. For air, for instance, $A = 14.8$, $l = 2.25$. In what follows we will use this value of l.

The coefficients C_1 and C_2 in Eqs. (1) and (2) are determined by the energy conservation law and by the normalization of the number of particles. The total number of particles, in the case of the collision of a nucleon (or a π-meson) with a nucleus, is taken to be[3]:

$$N = kA^{0,19}(E_0/2Mc^2)^{1/4}, \quad (5)$$

where $k \approx 2$. The coefficients C_1 and C_2 are equal to:

$$C_1 = kA^{0,19}[\chi(\bar{l})]^{1/4};$$

$$C_2 = 5\sqrt{5}\,Mc^2[\chi(\bar{l})]^{3/4}/2\sqrt{3}\,kA^{0,19}. \quad (6)$$

Making use of Eqs. (1) and (2), we assume that the energy distribution in the collision with the tube does not differ much from the energy distribution in the collision with a nucleon. Equations (1) and (2) also do not take into account the fact that not only π-mesons, but also particles of different kinds, occur in the stars. Furthermore, the nuclons present in the tube which collide with the primary particle acquire a substantial energy, and are active in the further nuclear-cascade process. A production of a certain number of antinucleons is also not impossible. In the present paper we shall not account for the nucleons, which do not play a very substantial role at the very high energies ($>10^{14}$); we also shall not consider the influence of the heavy mesons and hyperons — this will be the object of another report.

Let us consider now the kinetic equation which determines the distribution of the nuclear-active particles. If we assume that only π-mesons are produced in the elementary process, we have the following equation:

$$\frac{\partial P(y,t)}{\partial t} = -P(y,t) + \frac{2}{3}\int_y^\infty P(y_0,t)\frac{\partial N}{\partial y}dy_0. \quad (7)$$

where $P(y, t)$ is the number of particles in the interval dy, $y=\ln E$; t is the quantity of matter traversed by the particles. The quantity of matter is measured in units of t, where t is the "nuclear mean free path" of the particles in the considered matter; $y_0=\ln E_0$, where E_0 is the energy of the primary particle. The factor 2/3 is present because not all the π-mesons are nuclear-active. A number of π^0-mesons, equal on the average to 1/3 of the total number of mesons, decay before any interaction with nuclei. The decay of charged π-mesons is already negligible at energies of $\sim 10^{12}$ ev, and is therefore not accounted for by our equations.

We choose the following for the boundary condition for Eq. (7):

$$P(y,0) = Be^{-\gamma y}, \quad (8)$$

where $\gamma=1.5-1.75$. This corresponds to the fact that the nucleons falling on the atmospheric boundary have a spectrum of power form[4]. In addition, the atmospheric boundary is subject to a cascade of nuclei of different elements, with a predominant part of α-particles.

The solution of equations of the form (7) have been obtained numerically by Zatsepin and Guzhavin[5] by a modified method of successive approximations[6]. Analogous numerical calculations, starting from the expression for the elementary act obtained by Fermi, have been performed later[7]. In the present paper we propose to obtain an analytic expression for the dependence of the number of nuclear-active particles on the energy and depth.

In the first place, we make the substitution

$$P(y,t) = e^{-t}\varphi(y,t). \quad (9)$$

Equation (7) then reduces to

$$\frac{\partial\varphi(y,t)}{\partial t} = \int_y^\infty \varphi(y_0,t)\frac{\partial N}{\partial y}dy_0, \quad (10)$$

which can be written in the form

$$\partial\varphi(y,t)/\partial t = \mathscr{L}[\varphi(y,t)], \quad (10)$$

where $\mathfrak{L}$ is a linear integral operator. Let us examine some of the properties of the operator $\mathfrak{L}$. Let us assume that the operator $\mathfrak{L}$ operates on a function which has a power dependence on E and an exponential dependence on y, i.e., a function $\varphi(y,t)=Be^{\gamma y}$. Substituting the expression for dN/dy under the integral sign [see Eqs. (1) and (2)], we get

$$\mathscr{L}[Be^{-\gamma y}] =$$

$$B_1\int_{L_{\min}}^{L_{\max}}\frac{\sqrt{L^2-\lambda^2}\exp\{-(2\gamma+1/2)L+\sqrt{L^2-\lambda^2}\}}{\sqrt{2\pi L}(\sqrt{L^2-\lambda^2})-\lambda/3)}dL,$$

$$B_1 = {}^4/_3B[2MC^2\chi(\bar{l})]^{-\gamma}C_1; \quad (11)$$

$$L_{\min} = 0{,}536(y-\ln C_2), \quad L_{\max} = 7{,}4(y-\ln C_2).$$

Equation (11) obviously, has to be completed by Eq. (2), which we write in the following form

$$y = \ln C_2 + 5L/6 + \lambda + \sqrt{L^2-\lambda^2}/3.$$

It is not difficult to see that the function in the exponent under the integral sign of (11) has a sharp maximum. Let us recall that the formulas giving the energy distribution in the elementary act are themselves derived with exponential accuracy, and the factors of the exponentials are determined by the normalization conditions. It is natural, therefore, to limit the evaluation of the integral (11) to the exponential accuracy. Let us expand the function in the exponent of formula (11) in a series, around the maximum, including terms up to the second derivative; the factor of the exponential will be put equal to its value at the maximum. We obtain:

$$\mathcal{L}[Be^{-\gamma y}] = Be^{-\gamma y} S(\gamma)\, e^{-\delta(\gamma)\, y}, \tag{12}$$

$$S(\gamma) = \frac{{}^4/_3\, kA^{0,19}\, [\chi(\bar{l})]^{1/4} \sqrt{1-\nu^2(\gamma)}\, [\sqrt{1-\nu^2(\gamma)} - \nu(\gamma)/3]}{{}^5/_6 + \nu(\gamma) + \sqrt{1-\nu^2(\gamma)}/3}\, C_2^{\gamma+\delta(\gamma)} [2Mc^2\chi(\bar{l})]^{-\gamma}, \tag{13}$$

$$\nu(\gamma) = \frac{-(2\gamma/3+1) + (2\gamma+{}^1/_2)\sqrt{(2\gamma/3+1)^2 + (2\gamma+{}^1/_2)^2 - 1}}{(2\gamma+{}^1/_2)^2 + (2\gamma/3+1)^2}, \tag{14}$$

$$\delta(\gamma) = \frac{(2\gamma+{}^1/_2) - \sqrt{1-\nu^2(\gamma)}}{{}^5/_6 + \nu(\gamma) + \sqrt{1-\nu^2(\gamma)}/3} - \gamma. \tag{15}$$

In the evaluation of the integral in (10), L_{min} and L_{max} have been replaced by $-\infty$ and $+\infty$. The calculation shows that the quantity $\delta(\gamma)$ for $\gamma=1.5-2$ is small, and therefore the factor $e^{\delta(\gamma)y}$ leads to a y dependence, small compared to $e^{-\gamma y}$ (see Table 1)

TABLE I

γ	1,5	1,6	1,7	1,8	1,9	2
$\delta(\gamma)$	0,07	0,08	0,09	0,1	0,11	0,12

It follows from (12) that the operator $\mathcal{L}$ operating on a function having a power dependence on the energy again gives a power spectrum, only slightly smoothed with respect to the primary one. Bearing this result in mind, we seek a solution of Eq. (10) in the form

$$\varphi(y, t) = Be^{-\gamma y} f(y, t), \tag{16}$$

we will consider $f(y, t)$ as depending weakly on y. Let us substitute the expression (16) in Eq. (10).

For the evaluation of the integral, let us make use of the assumption on $f(y, t)$, and take the value of this function at the point where the function in the exponent has a maximum. We obtain the following equation for $f(y, t)$:

$$\partial f(y, t)/\partial t = \Delta(y, \gamma) f(y_1, t), \tag{17}$$

$$\Delta(y, \gamma) = S(\gamma)\, e^{-\delta(\gamma)\, y}, \tag{18}$$

y_1 is the point at which the function in the exponent has a maximum.

y_1 is related to y by the relationship

$$y_1 = 2d(\gamma)(y - \ln C_2) + \ln 2Mc^2\chi(\bar{l}), \tag{19}$$

$$d(\gamma) = [{}^5/_6 + \nu(\gamma) + \sqrt{1-\nu^2(\gamma)}/3]^{-1}. \tag{20}$$

Table 2 gives the values of y_1 and $\Delta(y, \gamma)$ for $\gamma=1.75$ and 1.5 and for different y's

TABLE II

y	32,2		34,5		36,8		39,2		41,4	
γ	1,5	1,75	1,5	1,75	1,5	1,75	1,5	1,75	1,5	1,75
y_1	34,8	34,8	37,4	37,4	39,9	39,9	42,5	42,5	45	45
$\Delta(y, \gamma)$	0,131	0,06	0,111	0,048	0,095	0,038	0,08	0,03	0,07	0,024

For the solution of (17), let us make a Laplace transformation with respect to the variable t:

$$f(y, t) = \frac{1}{2\pi i}\int_{\varepsilon - i\infty}^{\varepsilon + i\infty} e^{pt} f(y, p)\, dp;\quad f(y, p) = \int_0^\infty e^{-pt} f(y, t)\, dt. \tag{21}$$

Taking the boundary condition into account we obtain, instead of (17),

$$pf(y, p) - \Delta(y, \gamma) f(y_1, p) = 1. \qquad (22)$$

It is easy to see that the solutions of this equation can be written in the form of the following series

$$f(y, p) = \frac{1}{p - \Delta(y, \gamma)} \qquad (23)$$

$$+ \frac{\Delta(y, \gamma)}{p} \frac{[\Delta(y_1, \gamma) - \Delta(y, \gamma)]}{[p - \Delta(y, \gamma)][p - \Delta(y_1, \gamma)]}$$

$$+ \frac{\Delta(y, \gamma)\Delta(y_1, \gamma)}{p^2} \frac{[\Delta(y_2, \gamma) - \Delta(y_1, \gamma)]}{[p - \Delta(y_1, \gamma)][p - \Delta(y_2, \gamma)]} + \ldots +$$

$$\frac{\Delta(y, \gamma)\Delta(y_1, \gamma)\ldots\Delta(y_n, \gamma)[\Delta(y_{n+1}, \gamma) - \Delta(y_n, \gamma)]}{p^{n+1}[p - \Delta(y_n, \gamma)][p - \Delta(y_{n+1}, \gamma)]} + \ldots$$

Here y_2 is related to y_1 the same way as y_1 is related to y. In this fashion, if $y_1 = \psi(y)$ according to (19)

$$y_2 = \psi(y_1), \quad y_3 = \psi(y_2), \ldots y_n = \psi(y_{n-1}).$$

Using the relationship (21) one can compute

$$f(y, t) = f_0(y, t) + f_1(y, t) + f_2(y, t) + \ldots \qquad (24)$$

$$f_0(y, t) = e^{\Delta(y, \gamma)t}; \; f_1(y, t) =$$

$$\Delta(y, \gamma)\left[\frac{e^{\Delta(y_1, \gamma)t} - 1}{\Delta(y_1, \gamma)} - \frac{e^{\Delta(y, \gamma)t} - 1}{\Delta(y, \gamma)}\right]$$

$$f_2(y, t) = \Delta(y, \gamma)\Delta(y_1, \gamma) \qquad (25)$$

$$\times\left[\frac{e^{\Delta(y_2, \gamma)t} - 1}{\Delta^2(y_2, \gamma)} - \frac{e^{\Delta(y_1, \gamma)t} - 1}{\Delta^2(y_1, \gamma)}\right]$$

$$- t\Delta(y, \gamma)\left[\frac{\Delta(y_1, \gamma)}{\Delta(y_2, \gamma)} - 1\right].$$

It is easy to see that the series (2) converges. It can also be shown that $f(y, t)$ does indeed change slowly with y; this justifies the transition from Eq. (10) to Eq. (17) for which the function $f(y, t)$ was evaluated at the point where the function in the exponent has a maximum.

Let $y = 32.2$ and 36.8 ($E = 10^{14}$ ev and 10^{16} ev and $t = 10$. In the first case ($y = \ln 10^{14}$)

$$f_1(y,t)/f_0(y, t) = -0{,}09;$$

$$f_2(y, t)/f_0(y, t) = -0{,}02.$$

In the second case ($y = \ln 10^{16}$):

$$f_1(y, t)/f_0(y, t) = -0{,}065;$$

$$f_2(y,t)/f_0(y, t) = -0{,}005.$$

The function $P(y, t)$ can be written in the following form [see (9)]:

$$P(y, t) = Be^{-\gamma y}\exp\{-[1 - D(\gamma, y, t)]t\};$$

$$D(\gamma, y, t) = t^{-1}\ln f(y, t). \; (26) \qquad (26)$$

TABLE III

t	$y = 32{,}2$	$y = 36{,}8$
2	0,13	0,09
10	0,12	0,08
15	0,11	0,07

Table 3 gives values of $D(\gamma, y, t)$ for different t, y and $\gamma = 1.5$. For $\gamma = 1.75$ and different y and t: $D(\gamma, y, t) \simeq 0.05$. For energies smaller than 10^{14} ev the results obtained are already erroneous. The investigation of the absoprtion of nuclear-active particles for these energies will be given in a further report.

[1] I. L. Rozental'and D. C. Chernavskii, Usp. Fiz. Nauk **52**, 185 (1954)

[2] L. D. Landau, Izv. Akad. Nauk SSR ser fiz. **17**, 51 (1953)

[3] S. Z. Belenkii and L. D. Landau, Usp. Fiz. Nauk **56**, 309 (1955)

[4] Bartlett, Bollinger, Cocconi, Eisenberg and Greisen, Rev. Mod. Phys. **24**, 133 (1952)

[5] Guzhavina, V. V. Guzhavin and G. T. Zatsepin, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 819 (1956); Soviet Phys. JETP **4**, 690 (1957)

[6] I. L. Rosental', J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 440 (1952) G. T. Zatsepin and I. L. Rozental' Dokl. Akad. Nauk SSSR **99**, 369 (1954)

[7] V. Haber-Schaim, Nuovo Cimento **11**, 172 (1954); Amaldi, L. Mezzetti and Soppini, Nuovo Cimento **10**, 803 (1953)

Translated by E. S. Troubetzkoy
125