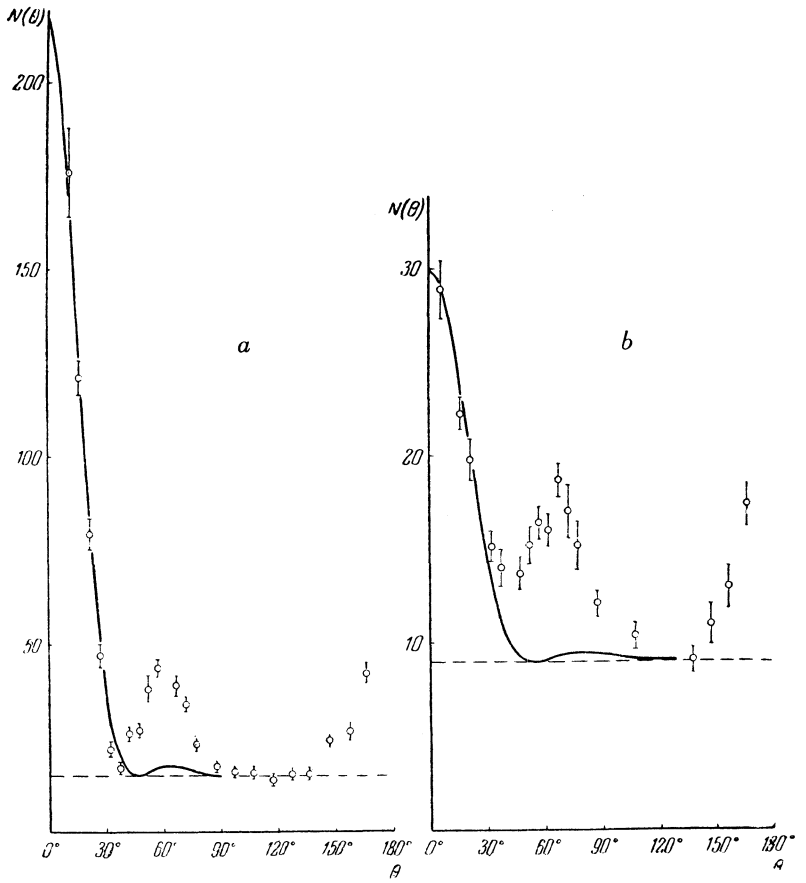

FIG. 2. Angular distributions of the p_1 proton group. Deuteron energy: *a*—3.8 mev; *b*—1.8 mev. Continuous curves correspond to $l = 0$. The scale of the ordinate axis of Figs. 1a and 2a is identical, the same is true for Figs. 1b and 2b.

1 I. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) A66, 467 (1953).
2 Bhatia, Huang, Huby and Newns, Phil. Mag. 43, 485 (1952).
3 S. T. Butler and N. Austern, Phys. Rev. 93, 355 (1954).
4 J. Yoccoz, Proc. Phys. Soc. (London) A67, 813 (1954).
5 W. Tobocman, Phys. Rev. 94, 1655 (1954).
6 W. Tobocman and N. H. Kalos, Phys. Rev. 97, 132 (1955).
7 J. Horowitz and A. M. L. Messiah, J. Phys. Radium 14, 695 (1953).

Translated by H. Kasha
35

On a Method of Direct Computation of the Nucleon–Nucleon Interaction on the Basis of Experimental Values for the Levels of Light Nuclei

IU. M. SHIROKOV, V. V. BALASHOV AND K. A. TUMANOV

Moscow State University

(Submitted to JETP editor September 29, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 167–168 (January, 1957)

A METHOD is given below for the study of the nucleon–nucleon interaction in nuclei based on the following assumptions: *A*) the forces in nucleus act between pairs of nucleons; *B*) the mean velocity of a nucleon in the nucleus is of the order of 0.1 c or less. Besides these assumptions which are essential for the application of the method, we assume the isotopic invariance of the proper nuclear interaction and neglect the difference in the masses of the proton and the neutron.

The wave function of the nucleus with mass number A is expanded in terms of the products A of single particle eigenfunctions of nucleons in a three–dimensional oscillator well. In this oscillator representation, Schrödinger equations are written down for different nuclei, in which the

matrix elements of nuclear interaction and the coefficients can be eliminated by writing secular equations for the different levels. In the secular equations, only the matrix elements of the nuclear interaction between pairs of particles are unknown, since energy eigenvalues are known from experiment. If the expansion of the wavefunction in terms of the oscillator function converges rapidly, the secular equations can be cut off. The resulting system of the cut–off secular equations is then solved simultaneously for these matrix elements.

The fulfillment of the condition B corresponds to the rapid convergence of the nuclear wave function expansion in terms of the oscillator functions. The oscillator problem consists of finding the minimum value of the expression

$$W = \langle \Delta p^2 \rangle r_0^2 / \hbar^2 + \langle \Delta x^2 \rangle r_0^{-2},$$

proportional to the oscillator energy, where <. . .> denote the mean values in a given state,

$$r_0 = (\hbar / m\omega)^{1/2},$$

ω is the basic oscillator frequency. In a pure oscillator state with the quantum number n, we have

$$W = 2(n + 1/2).$$

If a given state represents a mixture of oscillator states with quantum number n and higher, then

$$W \geqslant 2(n + 1/2).$$

If, on the other hand, it is known that in a certain state $W \approx 1$, then in the expansion of the wave function in terms of oscillator functions, the coefficients with $n > 0$ are small. In general, if

$$W \approx 2(n + 1/2),$$

and all states with quantum number less than n are occupied (in accordance with the Pauli principle), then the only important term in the expansion is the one with quantum number n. The estimation of W for a given nucleus is straightforward. Δx is the nuclear radius, known from experiment, and Δp is obtained from the assumption $v = 0.1$ c (it should be noted that these values of Δx and Δp are subjected to the indetermination relation). The parameter r_0 is chosen in such a way that W is minimum. Calculations for He^4 with

$$\Delta x = 1.2 \cdot 10^{-13} \text{ cm}.$$

yield the value $W \approx 1$. Analogous calculations for heavier elements up to oxygen yield $W \approx 3$. It follows that, in the expansion of the wave functions of H^3, He^3 and He^4 nuclei, only the first oscillator state with $n = 0$ is essential while, for the heavier nuclei up to oxygen, the two first states with $n=0$ and $n = 1$ are essential. The contribution of the other exicted states is small.

The convergence rate of the expansion of the wave function in terms of oscillator functions decreases with the deviation of r_0 from the optimal value for a given nucleus, corresponding to the minimum of W. This optimal value of r_0 varies only slightly for different nuclei:

$$r_0 \sim \sqrt{\Delta x} \sim A^{1/6};$$

the expansion of the wave functions of close nuclei can be effected for an oscillator well of the same average width.

Three out of the $3A$ coordinates of the nucleons in a nucleus describe the free motion of the nucleus as a whole. The expansion of a plane wave in terms of oscillator functions diverges and therefore the motion of the system as a whole should be separated before passing to the oscillator representation. The use of Jacobi coordinates makes it difficult to take advantage of the symmetry properties of the wave function. It is more convenient to proceeed as follows: subtract the kinetic energy of the movement of the nucleus as a whole from the total Hamiltonian; the Hamiltonian in the center–of–mass system is then obtained; its eigenfunctions represent the energy levels of the nucleus. In the doubly quantized form, this Hamiltonian is

$$H = \int \psi^+(\mathrm{x}) \frac{\mathrm{p}^2}{2m} \psi(\mathrm{x})\, d\mathrm{x} - \frac{1}{2mA} \left\{ \int \psi^+(\mathrm{x})\, \mathrm{p}\, \psi(\mathrm{x})\, d\mathrm{x} \right\}^2 - \frac{1}{2} \int \psi^+(\mathrm{x})\, \psi^+(\mathrm{x}')\, V(\mathrm{x} - \mathrm{x}')\, \psi(\mathrm{x}')\, \psi(\mathrm{x})\, d\mathrm{x}\, d\mathrm{x}'.$$

where ψ and ψ^+ are the usual nucleon destruction and creation operators. They are spinors both in the ordinary and in the isotopic space; the interaction

$$V(\mathrm{x} - \mathrm{x}')$$

may contain matrices of both spins. The first two terms in the expression for $\hat{H}$ together represent the kinetic energy operator in the center–of–mass system and can be written in the form

$$-\frac{1}{2A} \int \psi^+(\mathrm{x})\, \psi^+(\mathrm{x}') \frac{(\mathrm{p} - \mathrm{p}')^2}{2m} \psi(\mathrm{x})\, \psi(\mathrm{x}')\, d\mathrm{x}\, d\mathrm{x}'.$$

It should be noted that the operator H commutes with the operators of total momentum and of coordinates of the center–of–mass.

The operators ψ, ψ^+ can be expanded in terms of the oscillator functions

$$\psi_{\mu\nu}(\mathrm{x}) = \sum_{nlm\sigma\tau} b_{nlm\sigma\tau} R_{nl}\left(\frac{r}{r_0}\right) Y_{lm}(\theta, \varphi)\, \chi_\sigma(\mu)\, \chi_\tau(\nu).$$

where n, l and m are the principal, orbital, and magnetic quantum numbers, σ and τ are the projections of the ordinary and isotopic spins. The operators

$$b_{nlm\sigma\tau} \equiv b(q),\ b^+(q)$$

destroy and create nucleons in the state q. In the new representation, the vector of state is given by the sets of functions

$$C_A(q_1, q_2 \cdots q_A),$$

depending on a sets of indices; the Hamiltonian is of the form

$$\hat{H} = \sum_{q_1 q_2 q_1' q_2'} -\frac{1}{2} b^+(q_1)\, b^+(q_2) \langle q_1 q_2 | \hat{H} | q_1' q_2' \rangle\, b(q_1')\, b(q_2').$$

No assumptions are made about the convergence of the matrix elements

$$\langle q_1 q_2 | \hat{H} | q_1' q_2' \rangle$$

in terms of the oscillator quantum number n. Only a small part of these elements will be independent and non–vanishing; in an interaction between a pair of particles, the total momentum of both particles, the coordinates of their center of gravity, the total moment and the isotopic spin, their projections, and parity are conserved. In order to make use of the above conservation laws, one can express the matrix elements through the matrix elements

$$\langle Q | \hat{H} | Q' \rangle$$

in terms of the oscillator wave functions of the relative motion of the two particles (Q and Q' are the sets of quantum numbers describing the relative motion of the two particles).

The operator $\hat{H}$ (or $\hat{V}$) does not act upon the coordinate of the center–of–mass of two particles

$$(r_1 + r_2)/2.$$

The matrix element

$$\langle Q | \hat{V} | Q' \rangle$$

is diagonal in respect to the total moment and the isotopic spin of the two particles and is independent of their projections. If, in the expansion of the wave functions, we limit ourselves to the first two oscillator states with $n = 0$ and $n = 1$, then there will be only 16 different matrix elements

$$\langle Q | V | Q' \rangle$$

We can hope that it will be possible to describe, by means of these 16 values, the ground and the lower excited states of nuclei up to oxygen. The actual computations in this approximation require the use of computers.

We shall present the results of computation in the most crude approximation ($n = 0$, i.e., all nucleons in the ls state) for the H^3, He^3 and He^4 nuclei. In these approximations, there are only two matrix elements

$$\langle {}^3S_1,\ T = 0 | V | {}^3S_1,\ T = 0 \rangle = A_1;$$

$$\langle {}^1S_0, T = 1 | V | {}^1S_0,\ T = 1 \rangle = A_0,$$

for which we obtained the following system of equations:

$$(3\hbar^2/mr_0^2) - 3(A_1 + A_0) = -8.49 \text{ mev}, \qquad (H^3)$$

$$(3\hbar^2/mr_0^2) + (2e^2/\sqrt{\pi}r_0) - 3(A_1 + A_0) = -7.73 \text{ mev}, \qquad (He^3)$$

$$(9\hbar^2/2mr_0^2) + (2e^2/\sqrt{\pi}r_0) - 6(A_1 + A_0) = -28{,}27 \text{ mev}, \qquad (He^4)$$

This system of three equations contains two unknowns r_0 and $A_1 + A_0$. The equations are satisfied for

$$A_1 + A_2 = 10.83 \text{ mev}, \text{ and } r_0 = 2.27 \cdot 10^{-13} \text{ cm}.$$

The Coulomb energy

$$2e^2/\sqrt{\pi}r_0$$

equals to 0.716 mev, while the experimental value is 0.764 mev.

1 J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics.*

Translated by H. Kasha
36

Scale Transformation and the Virial Theorem in Quantum Field Theory

Iu. V. Novozhilov
Leningrad State University

(Submitted to JETP editor October 4, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 171–173
(January, 1957)

UNDER the term "scale transformation" we shall understand the transformation of the scale of coordinates, accompanied by an inverse change of the mass scale