$$X_{\alpha,\gamma}=\sqrt{\alpha}\left[1-\frac{\pi^2}{24}\left(1+\frac{\gamma-1}{\gamma}\alpha\right)\theta^2\right] \qquad (10')$$

$$(T \ll T_0);$$

$$X'_{\gamma}=1+\frac{\pi^2}{12}\,\frac{1-2\gamma}{2\gamma}\,\theta'^2; \qquad (10'')$$

$$X^{\circ}_{\gamma}=\sqrt{\gamma}\left(1+\frac{\pi^2}{24}\,\frac{\gamma-2}{\gamma^2}\,\theta'^2\right) \qquad (T \ll T'_0).$$

(10') for $\alpha \ll 1$ agrees with (7); the function (10''), depending on $\gamma$, can also increase as well as decrease. For high $T$ all three functions (10), and consequently $x_i$ as well (9), increase (for $T \gg T'_0$ they are proportional to $\sqrt{T}$, whereupon $X_{\alpha\gamma} = X^{\circ}_{\gamma}$).

Several representive curves are presented in Fig. 1 for ($X'_{\gamma}$, $X^{\circ}_{\gamma}$) and in Fig. 2 for ($X_{\alpha\gamma}$). For the sum of these curves (with corresponding coefficients) the various possibilities are even more numerous than in Case II. In particular, one can obtain a curve containing several maxima, similar to the experimental curve of Verkin (4) for Zn. For $T \lesssim T_0 \ll T'_0$ the small groups give the basic temperature dependence, as before.

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1 B. I. Verkin, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 156 (1957); Soviet Phys. JETP **5**, 117 (1957).

2 G. E. Zil'berman, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 1209 (1951).

3 G. E. Zil'berman and R. S. Kuznetskii, [Trudy Kh. P. I. (Kharkov Pedagogical Institute) No. 2, 1956].

4 B. I. Verkin and I. F. Mikhailov, J. Exptl. Theoret. Phys. (U.S.S.R.) **24**, 342 (1953).

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## Contribution to the Thermodynamical Theory of Ferroelectrics

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FROM the thermodynamical theory of ferroelectrics developed by Ginzburg[1,2], there follows a series of important conclusions concerning the dependence of the dielectric properties of barium titanate on mechanical stress. Strictly speaking, the theory applies only to single-domain monocrystals; however, in the paraelectric region it is expedient to attempt to compare experimental data obtained for polycrystals with the conclusions of the theory. Below we present data which we have obtained for polycrystalline samples of $BaTiO_3$, which corroborate certain conclusions of the thermodynamical theory.

1. The expansion of the thermodynamical potential $\Phi$ in the presence of an elastic stress $\sigma_{ik}$ differs from the analogous expansion in the absence of a stress only in the coefficients $\alpha_i$ for the polarization $p_i^2$ [3,4]. If there exists only a single compression, along the $x$-axis, for instance, then for the parallel and perpendicular directions we have respectively:

$$\alpha_1=\alpha-\varkappa_1\sigma_{xx}, \quad \alpha_2=\alpha-\varkappa_2\sigma_{xx}, \qquad (1)$$

where $\varkappa_1$, $\varkappa_2$ are the strain coefficients, and $\alpha$ the expansion coefficient in the absence of compression.

It is possible to determine the coefficients $\alpha_1$ and $\alpha_2$ from measurements of the dielectric constant:[1,4]

$$\alpha_1=2\pi/\varepsilon_{xx}, \quad \alpha_2=2\pi/\varepsilon_{yy} \quad (T>\Theta). \qquad (2)$$

In Fig. 1 is shown the experimental behavior of the coefficient $\alpha_1$ as a function of pressure (unilaterally applied) for various temperatures above the Curie point, calculated from our measurements of the dielectric permittivity for polycrystalline $BaTiO_3$ in a weak field (7 volts/cm) at high frequency (1 Mc/s). It is clear that the linear dependence of $\alpha_1$ on unilateral pressure, as required by the theory, is well realized over a wide range of pressure, in which

$$\varkappa_1 \approx +0.75\cdot 10^{-12}\ \text{cm}^2/\text{dyne} \qquad (3)$$

For measurements of $\epsilon$ in the direction perpendicular to the axis of compression, the linear dependence is violated for pressures beyond 500 kg/cm$^2$, but in the region where linearity is preserved,

$$\varkappa_2 \approx -0.23\cdot 10^{-12}\ \text{cm}^2/\text{dyne} \qquad (4)$$

The values (3) and (4) are smaller than the estimates based on x-ray measurements and the temperature dependence of the spontaneous polarization ($\varkappa_1 \approx +3 \times 10^{-12}$ cm$^2$/dyne, $\varkappa_2 \approx -1.10 \times 10^{-12}$ cm$^2$/dyne)[4], but they agree with them in sign and merely agree in magnitude, indicating the possibility of determining the coefficients $\varkappa_1$ and $\varkappa_2$ from dielectric measurements, with the aid of thermodynamical relations.

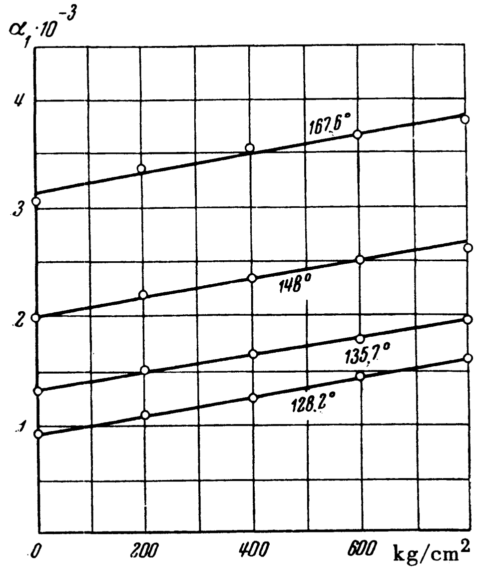

FIG. 1

2. In the thermodynamical theory it is assumed that near the Curie point it is possible to use the expansion

$$\alpha = \alpha_\theta' (T - \theta) \quad (T > \theta). \tag{5}$$

The data presented in Fig. 2 show that the coefficient $\alpha_\theta'$, determined from the slope of the straight line (5), remains constant over a wide range of temperature and pressure, and thereby confirm the validity of the expansion (5) assumed in the theory. The numerical value of $\alpha_\theta'$ for the samples which we investigated was approximately $5.8 \times 10^{-5}$ deg$^{-1}$.

3. By treating the data for $\varkappa_1$ and $\alpha_\theta'$, obtained experimentally from measurements of the dielectric permittivity, it is possible to estimate the displacement of the Curie point under unilateral compression. Theoretically this displacement is equal to the following:[4]

$$\Delta\theta \approx -\varkappa_1 \sigma_{xx} / \alpha_\theta' \approx +13 \cdot 10^{-3} \text{ deg} \cdot \text{cm}^2/\text{kg}.$$

From experimental measurements it is approximately equal to $2.8 \times 10^{-3}$ deg cm$^2$/kg;[5] that is, it has the same sign and the same order of magnitude as the value calculated from the theoretical formula.

4. Experimental results[5,6] show that the maximum relative change of $\epsilon$ under the influence of pressure is observed near the Curie point.

For unilateral compression for $T > \theta$ the theoretical dependence of the relative change of the dielectric permittivity, as measured along the axis of compression, on the temperature is given by the expression

$$\frac{\Delta\varepsilon}{(\varepsilon_{xx})_0} = \frac{\varkappa_1 \sigma_{xx}}{\alpha_\theta'(T-\theta) - \varkappa_1\sigma_{xx}} \quad \text{or} \quad \frac{(\varepsilon_{xx})_0}{\Delta\varepsilon} \tag{6}$$

$$= -\left(\frac{\alpha_\theta'\Theta}{\varkappa_1\sigma_{xx}} + 1\right) + \frac{\alpha_\theta'}{\varkappa_1\sigma_{xx}} T = A + BT,$$

where $A$ and $B$ depend on the mechanical stress. Consequently, in the para-electric region the reciprocal of the magnitude of the relative charge of $\epsilon$ at constant pressure depends linearly on temperature , but the slope of the straight line is negative ($\sigma_{xx} < 0$) and diminishes in absolute magnitude with increasing pressure. In Fig. 3 are presented the corresponding experimental data, calculated from measurements of $\epsilon$ for polycrystalline samples of $BaTiO_3$ in the paraelectric temperature region. As is obvious from these data, the kind of dependence $(\varepsilon_{xx})_0/\Delta\varepsilon = f(T)$ required by the theory is well realized.

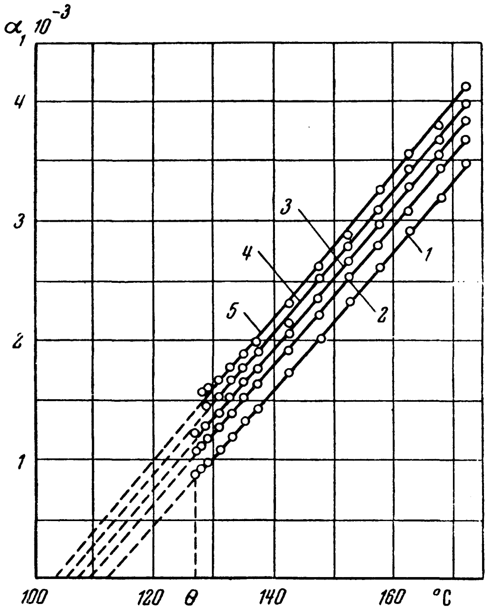

FIG. 2. 1 — $p = 0$, 2 — $p = 200$ kg/cm$^2$
3 — $p = 400$ kg/cm$^2$, 4 — $p = 600$ kg/cm$^2$
5 — $p = 800$ kg/cm$^2$

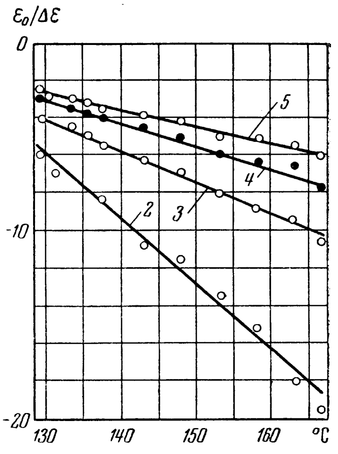

FIG. 3. *2, 3, 4, 5* —the same as in Fig. 2.

The comparison which has been made between the conclusions of the theory and the experimental data supports the possibility of applying the theory to polycrystalline $BaTiO_3$, at least in the paraelectric region. Regarding the ferroelectric region, satisfactory agreement between theory and experiment is observed only for temperatures no more than 10-12° below the Curie point.

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[1] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **15**, 739 (1945).

[2] V. L. Ginzburg, Usp. Fiz. Nauk 38, 490 (1949).

[3] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 36 (1949).

[4] L. P. Kholodenko and M. Ia. Shirobokov, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 1250 (1951).

[5] E. V. Siniakov and I. A. Izhak, Dokl. Akad. Nauk SSSR **100**, 243 (1955).

[6] I. A. Izhak, Izv. Akad. Nauk SSSR Ser. Fiz. **20**, 199 (1956).

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## On Quantum Effects Occurring on Interaction of Electrons with High Frequency Fields in Resonant Cavities

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RECENTLY, a number of papers[1-7] were published in which an analysis was made of the quantum effects occurring during the interaction of electrons with high frequency fields in resonant cavities. And in the first place, the dispersion of the kinetic energy or the electron velocity, produced during their passage through the resonator, was calculated. Erroneous deductions are contained in some of the above mentioned papers, however, and in others[4,5,7], classical results were sometimes considered as quantum; quantum-mechanical methods were used for finding expressions, which are simpler, and which in the more general case can be obtained by a classical method. At the same time, the problem of the passage of electrons through resonators is one of the most important in electronics; moreover, the region of high frequencies $\omega$ and low temperatures $T$, when the condition of classicity $\hbar\omega \ll kT$ is disturbed, is acquiring increasingly greater interest. Therefore, we considered it appropriate to discuss briefly the problem which is stated in the title (this is carried out in greater detail in Ref. 8).

We will analyze classically the following problem: a non-relativistic electron enters into the resonator at the instant $t = 0$ with a kinetic energy $K_0 = mv_0^2/2$, and emerges from the resonator at the instant $t = \tau$ with a kinetic energy $K_\tau = mv_\tau^2/2$. Moreover, for simplicity we will consider the electric field $E$ in the resonator along the electron path as homogeneous and directed along its velocity (such a case is completely real). When

$$E = E_1 \cos \omega t + (E_2 + E_0) \sin \omega t, \qquad \text{(a)}$$

we obtain:

$$m\frac{dv}{dt} = eE, \quad v_\tau = v_0 + \frac{e}{m\omega}\left[E_1 \sin \omega\tau + (E_2 + E_0)(1 - \cos \omega\tau)\right]. \qquad (1)$$

Let now $E_1$ and $E_2$ be random values, so that $\overline{E}_1 = \overline{E}_2 = 0$ and $\overline{E_1^2} = \overline{E_2^2} = \overline{V^2}/d^2$, where $d$ is the path traversed by the electron (thickness of the resonator), and $\overline{V^2}$ is the mean square of the fluctuating voltage; the averaging, which is denoted by the bar, was carried out over the corresponding ensemble of identical systems. As in the papers cited above, we shall also consider the action of the field in the resonator on the electron motion as a small effect, in the strength of which we can limit ourselves to the terms of the $e^2$ order, and for the time of the electron flight through the resonator we shall assume $\tau = d/v_0$. Then, as can be easily seen,