are contained in Ref. 9. No satisfactory explanation has yet been obtained of the origin of the low energy peak in the $n-\gamma$ coincidence curve (Fig. 2). The 1.5 mev $\gamma$– rays for Po–Li previously reported in Ref. 8, have not been observed by us.

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Translated by J. L. Herson
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## Concerning the Temperature Dependence of the Magnetic Susceptibility of the Elements

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THE experimentally-observed magnetization of a substance is made up of the diamagnetism and paramagnetism of the ionic lattice and the diamagnetism and paramagnetism of the free carriers of charge (for instance the conduction electrons of the metals). Accordingly, one can assume that for the susceptibilities of the elements

$$\chi_{exp} = \chi^{-}_{ion} + \chi^{+}_{ion} + \chi^{-}_{elect} + \chi^{+}_{elect} \qquad (1)$$

(the signs – and + designate the dia- and paramagnetic contributions to the measured susceptibility).

The accepted classification of magnetic substances is based on the sign of the susceptibility measured experimentally. Examination of the experimental data concerning the temperature dependence of the susceptibility of the elements permits us to suggest another method for classifying magnetic materials, based on the character of the temperature dependence of the susceptibility in weak magnetic fields. In line with this proposal, one has to consider four groups of elements, exhibiting 1) paramagnetism which is practically independent of temperature (the alkali and alkali-earth metals), 2) paramagnetism which depends on temperature (the rare earths and the transition metals), 3) diamagnetism which depends on temperature (Be, Mg, Zn, Cd, Hg, Al, Ga. In, Tl, C, Sn, Pb, As, Sb, Bi), and 4) diamagnetism which is independent of temperature (the noble gases, etc.). The character of the temperature dependence of the susceptibility within each group is determined by the predominance of one of the terms in (1). Thus, paramagnetism which is practically temperature-independent indicates the predominance of the paramagnetic contribution of the conduction electrons; temperature-dependent paramagnetism is related to the predominance of the paramagnetic contribution of the ionic lattice; temperature-dependent diamagnetism (as will be shown below) reflects the predominance of the diamagnetic contribution of the conduction electrons; and finally, the magnetic properties of the fourth group of elements indicates the predominance of the diamagnetic contribution of the ionic lattice.

One should pay particular attention to the fact that the elements of the four groups indicated occupy a definite place in the so-called long periodic system of the elements (Fig. 1).* For all metals of the third group, the de Haas-van Alphen effect is observed at low temperatures. With respect to their atomic spectra and the magnetic properties of their crystals, Be and Mg are closer to Zn and Cd than to the alkali-earth metals, and consequently they are situated in the 26th, rather than in the 2nd column of the periodic system. Copper, silver and gold occupy an intermediate position between the transition metals and the elements of the third group; the magnetic properties of sulfur, selenium, tellurium and polonium have been inadequately and insufficiently studied.

The existing theory of the magnetic properties of electrons in metals,[1,3] which includes an explanation of the essential features of the de Haas-van Alphen effect in assuming the existence in these metals of anomalously small groups of electrons, gives the following expression for the constant component of the magnetic susceptibility:[2,3]

$$\chi = \chi_0 (1 - \pi^2/12)(T/T_0)^2 \quad \text{for } T \ll T_0, \qquad (2)$$

$$\chi = {}^2/_3 \chi_0 T_0/T \quad \text{for } T \gg T_0, \qquad (3)$$

where

$$\chi_0 = \frac{\sqrt{2}}{4\pi} \cdot \frac{e^2}{hc^2} \frac{M_i}{\sqrt{m_1 m_2 m_3}} E_0^{1/2} \left[ 3 \left( \frac{m_{эфф}}{m_0} \right)^2 - 1 \right],$$

and $T_0$ is the degeneracy-temperature of the electrons comprising the small groups, responsible for the fundamental frequency of the de Haas-van Alphen effect. If $m_{eff} < m_0$, then $x_0 < 0$ and correspondingly $x < 0$.

The temperature dependence of $x/x_0$ predicted by these relations is presented graphically in Fig. 2. The curves *a* and *b* correspond to equations (2) and (3); the curve *b* is a typical experimental curve for the temperature dependence of $x/x_0$ for metals of the third group. It is obvious that for $T \ll T_0$ and $T \gg T_0$ the existing theory qualitatively conveys the character of the temperature dependence for the third-group metals. The dependence of their diamagnetic susceptibility on temperature, as well as the de Haas-van Alphen effect, is explained by the presence of the anomalously small groups of electrons with low degeneracy-temperatures. Since $m_{eff}$ is anisotropic, it is clear

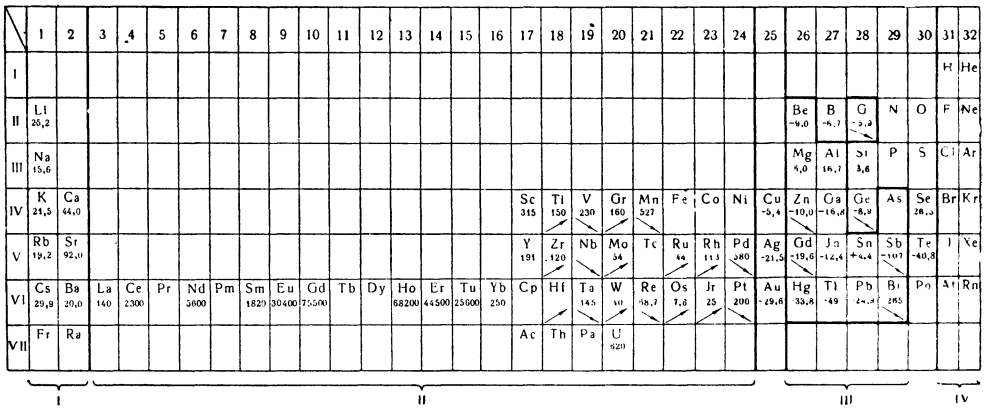


FIG. 1. The location in the periodic table of elements whose susceptibility varies in different ways with temperature. I – paramagnetic substances whose paramagnetism is practically independent of temperature; II – elements whose paramagnetism depends on temperature; III – elements whose diamagnetism increases with decreasing temperature; metals exhibiting the de Haas-van Alphen effect at low temperatures are encircled with heavy lines; IV – diamagnetic materials whose susceptibility is temperature-independent.

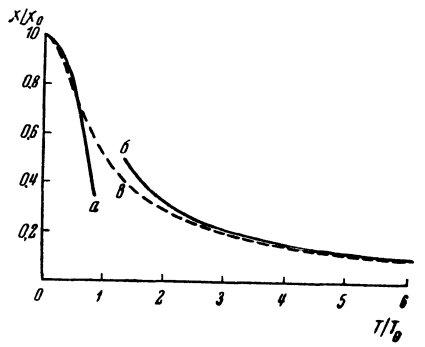


FIG. 2. The temperature dependence of the susceptibility of the conduction electrons in metals: *a* – theoretical curve for $T \ll T_0$; *b* – theoretical curve for $T \gg T_0$.

that the temperature dependence of $x_{||}$ and $x_{\perp}$ can be different. We note that the calculated values of the quantity $x_0$, taken from data for $m_1$, $m_2$, $m_3$, and $E_0$, obtained from investigations of the de Haas-van Alphen effect, are close to the experimental values. It is necessary to point out that the principle (large) group of electrons with a high degeneracy-temperature gives only an additive constant of the same order of magnitude.

Thus a systematic investigation of the temperature dependence of the magnetic susceptibility of the metals of the third group represents another possible method of studying the energy spectrum of electrons in metals experimentally.

It would also be of interest to look for the de Haas-van Alphen effect in crystals of Ge and in the $\gamma$-phase of a series of alloys (for example $Cu_5Zn_8$) whose susceptibilities exhibit the same temperature dependence as the metals of the third group.

* In Fig. 1 the numerals represent the values of the atomic susceptibilities of the elements at room temperature; the arrows indicate the change of the susceptibility coefficient of each element as the temperature increases.

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## Temperature Dependence of the Magnetic Susceptibility of Electrons in Metals

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WE have investigated the temperature dependence of the magnetic susceptibility $x$ of electrons over a wide range of temperatures in weak magnetic fields, where $x$ is practically independent of $H$. The necessity of this investigation for the classification of magnetic materials was pointed out to us by Verkin[1]. The following cases were studied: I) only small groups of electrons are present; II) there is, in addition to these small groups, a large group of electrons; III) besides the foregoing there is a large group of holes. The calculation is carried out for a quadratic law of dispersion with respect to the spin paramagnetism and to the anisotropy of the effective masses.

We consider hexagonal crystals like bismuth (for other types of symmetry the results do not differ qualitatively). We make the usual assumption that these crystals have three identical small groups in the form of ellipsoids, the axes of which make an angle of 120° in the plane of the binary axes.

Case I. Carrying out our calculations as in Refs. 2 and 3, we obtain the component $x_i$ (3 is the index of the principal axis) of the three groups referred to:

$$\chi_i = -{}^1/_2 AB_i (kT)^{1/2} F_{-1/2}(\zeta / kT) = -AB_i \sqrt{\zeta_0}\, \mathrm{X} \quad (1)$$

(since $m_i \ll m_0$, where $m_0$ is the mass of a free electron, we can ignore the spin paramagnetism). In this equation,

$$A = \sqrt{2}\, e^2/6\pi hc^2, B_1 = B_2 = 3(m_1 + m_2)/2\sqrt{m_1 m_2 m_3}, B_3 = 3m_3/\sqrt{m_1 m_2 m_3}; \quad (2)$$

$$\chi = {}^1/_2 \sqrt{\theta}\, F_{-1/2}(u); \quad (3)$$

$$\theta = \frac{T}{T_0}, \quad T_0 = \frac{\zeta_0}{k}, \quad (4)$$

$$\zeta_0 = \zeta|_{T=0} = \left(\frac{nh^3}{16\pi \sqrt{2 m_1 m_2 m_3}}\right)^{2/3};$$

$$F_s(u) = \int_0^\infty \frac{x^s dx}{1 + e^{x-u}}, \quad u = \frac{\zeta}{kT}.$$

The dependence of the chemical potential $\zeta$ on temperature is determined from the condition that the concentration $n$ of electrons be constant, which gives

$$\theta = [{}^3/_2 F_{1/2}(u)]^{-2/3} \quad (5)$$

By calculating the function $F_{\pm 1/2}$, we determine from Eqs. (3) and (5) the function $X(\theta)$, which is the desired dependence $X(T)$ in generalized coordinates, and also $\zeta(\theta)/\zeta_0$. For the limiting cases — strong degeneracy and Boltzmann statistics — we obtain

$$\begin{aligned} T \ll T_0 &: \mathrm{X} = 1 - \pi^2 \theta^2 / 12, \\ &\zeta / \zeta_0 = 1 - \pi^2 \theta^2 / 12; \\ T \gg T_0 &: \mathrm{X} = 2 / 3\theta, \\ &\zeta / \zeta_0 = ({}^3/_2)\, \theta \ln [(16/9\pi)^{1/3}\, \theta^{-1}]. \end{aligned} \quad (3')$$

Curves of $X(\theta)$ and $\zeta(\theta)/\zeta_0$, and also their asymptotic expressions (broken curve), are presented in Fig. 1. A series of metals has just this temperature dependence for $x$.

Case II. The dependence of the chemical potential $\zeta'$ of a large group (we consider its ellipsoid of revolution with $m_1' = m_2' \sim m_3' \sim m_0$) on $T$ is represented here, as it turns out, by the same curve (Fig. 1) as in the preceding case (only the scale is changed; the new reduced temperature $\theta' = T/T_0'$, where $T_0' = \zeta_0'/k$; generally $\zeta_0' \gg \zeta_0$). From the definition of $\zeta(T)$ (for a small group) it turns out that the parameter $\alpha = \zeta_0/\zeta_0' \lesssim 1$.

The overall magnetic susceptibility of the small ($X_\alpha$) and large groups is equal to

$$\chi_i = -A\sqrt{\zeta_0'}\, [B_i \mathrm{X}_\alpha(\theta') + B_i' \mathrm{X}(\theta')], \quad (6)$$

$$\begin{aligned} B_1' = B_2' &= 1/\sqrt{m_3'} - 3m_1'\sqrt{m_3'}/m_0^2, \\ B_3' &= \sqrt{m_3'}/m_1' - 3m_1'\sqrt{m_3'}/m_0^2, \\ \mathrm{X}_\alpha(\theta') &= \frac{1}{2}\sqrt{\theta'}\, F_{-1/2}\left[u'(\theta') - \frac{1-\alpha}{\theta'}\right]; \end{aligned} \quad (6')$$