Charge and Momentum Analysis of Relativistic Particles by the Nuclear Emulsion Technique in Pulsed Magnetic Fields

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Strong pulsed magnetic fields (1.2×10^5 gauss) were used in experiments to observe particles from an electron accelerator in nuclear emulsions. The method was applied to measure the energy spectrum of bremsstrahlung from a synchrotron target, and to observe the annihilation of positrons in flight.

As was reported earlier[1], strong pulsed magnetic fields synchronized with the working cycle of an accelerator can be used to obtain a charge and momentum analysis of particles observed in nuclear emulsions.

We have exposed electron-sensitive emulsions [NIKFI type "R"] to γ-rays in a magnetic field of 1.2×10^5 gauss, the emulsions being situated inside a pulsed magnetic field generator (Fig. 1).

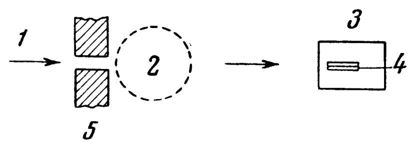

FIG. 1. Sketch of the experiment. *1*, γ-rays. *2*, Magnetic field of 7000 gauss. *3*, Pulsed magnetic field generator. *4*, Nuclear emulsion. *5*, Collimator.

The electron-positron pairs were analyzed by momentum and charge, the annihilation of positrons in flight was studied, and the energy spectrum of bremsstrahlung from the synchrotron of the Physical Institute of the Academy of Sciences was measured.

1. METHODOLOGICAL INVESTIGATIONS

1. *Method of measuring magnetic curvature and multiple scattering of particles in emulsion.*

There exist several methods[2-4] for measuring the scattering of particles in emulsion. We adapted and used one of the variants of the angle method.

The track of the particle in the emulsion was divided, as in other methods, into equal intervals (boxes) of length 100μ. Using the eye-piece scale shown in Fig. 2, the angles between consecutive chords were measured. The procedure was as follows: by fine adjustment of the microscope stage, the center of one grain in the particle track is placed at the center of the scale 0. Let the reading α_1 on the scale correspond to the track position 1-2 shown in Fig. 2. After the point 2 on

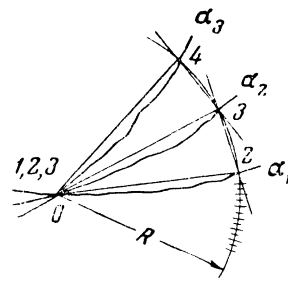

FIG. 2. Eye-piece scale, used for measuring the angles of magnetic deflection and multiple scattering of particles.

the track is moved to the center of the scale, the track occupies position 2-3 and the reading on the scale is α_2, and so on. In this way the track is divided into equal intervals of length R_μ equal to the radius of the scale, and the difference $\alpha_{i+1} - \alpha_i = \Delta_i$ between two consecutive readings represents the angle between consecutive chords. This method is simple to use and can be applied successfully to particles with large scattering. To measure the scattering of high-energy particles, when the intervals must be chosen to be longer than R_μ, the procedure is slightly modified. In this case we consider differences of the form $\alpha_{i+2} - \alpha_i = {}^{(2)}\Delta_i$, or more generally $\alpha_{i+k} - \alpha_i = {}^{(k)}\Delta_i$. If the track is divided into n intervals ot length R, then the angle of magnetic deflection in each box is*

$$M = \bar{\Delta}_i \pm (\pi/2)^{1/2} \langle |S_i| \rangle / \sqrt{n}. \quad (1)$$

* All readings are supposed statistically independent.

Here $\langle|S_i|\rangle = (1/n)\Sigma_i n_i|\Delta_i - \bar{\Delta}_i|$ is the mean value of the random deflection, and the mean square deviation of $<|s_i|>$ is

$$\sigma_{<|S_i|>} = <|S_i|> \sqrt{(\pi - 2)/2n}.$$

The results of a calculation of the ratio of the magnetic deflection M to the multiple scattering angle $<|s_i|>$, for electrons and positrons, are shown in Fig. 3. The abscissa is the length of the observed track, and results are shown for various values of H. The average number of boxes ($R \approx 100\mu$) per track in our experiment was about 15. From Eq. (1) the error in the determination of the energy of a particle from the magnetic curvature at $H = 1.2 \times 10^5$ gauss was about 30 percent, while the error in a determination by multiple scattering was about 20 per cent. The error in a measurement of the energy of an electron-positron pair was smaller by roughly a factor $2^{-1/2}$, i.e., equal to 21 per cent and 14 per cent by the two methods.

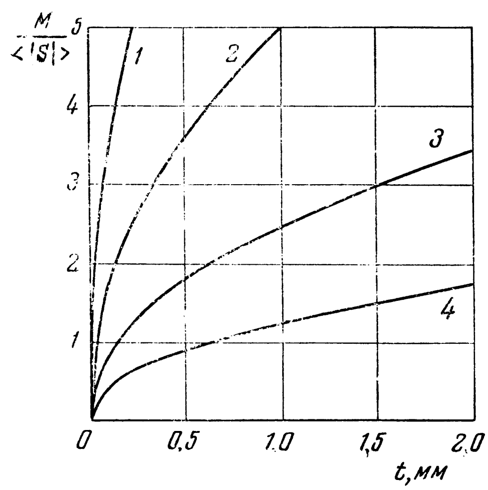

FIG. 3. Results of a calculation of the ratio of the magnetic deflection M to the multiple scattering angle $<|S|>$ for an electron or positron. The abscissa is the length of the observed track. Curves are shown for various values of H. *1*, $H = 4.8 \times 10^5$. *2*, $H = 2.4 \times 10^5$. *3*, $H = 1.2 \times 10^5$. *4*, $H = 0.6 \times 10^5$ gauss.

There are two ways to reduce the error in a measurement of magnetic curvature. One is to increase the strength of the magnetic field. The other is to use diluted emulsions**. A rough estimate of the increase in accuracy obtainable from diluted emulsions is made as follows. An emulsion which is diluted twofold gives a reduction of 15 per cent in the average angle of multiple scattering, while the density of grains along a track is hardly affected. An eightfold diluted emulsion gives a reduction of 50 per cent in multiple scattering and a reduction by a factor of 3 in the grain-density[5].

** By diluted emulsions we mean emulsions in which the ratio of gelatin to silver halide is higher than normal.

2. *Questions of Distortion and Spurious Scattering.* The experiments were carried out with NIKFI type "R" emulsions, 200 microns thick. The development was done by the NIKFI staff using the temperature method. After drying, the surfaces of the emulsion were coated with shellac. The following steps were taken to reveal and to measure distortions:

a). The emulsions were exposed twice, first with and then without magnetic field. Between the two exposures the plates were rotated through 180° about an axis perpendicular to their own plane. The tracks of electron-positron pairs produced in the second exposure were used to detect and measure distortions. The scattering of 100 tracks of average length 1.6 mm was measured. In each case the algebraic mean deflection $<S_i>$ and the mean absolute deflection $<|S_i|>$ were determined. It was found that the algebraic mean deflections lay within the limits of their statistical uncertainty $\pm(\pi/2)^{1/2}\langle|S_i|\rangle/\sqrt{n}$. A part of the results of these measurements is shown in the following table.

b). From the theory of multiple scattering it is known that, in the absence of distortion, the probability $P(+)$ of a plus sign and the probability $P(-)$ of a minus sign in the first (or second) differences are equal to 0.5. We define the quantity K, which we call the coefficient of distortion, by

$$K = 1 - [P(+)/P(-)]. \qquad (2)$$

In the absence of distortion, and for a track with a sufficiently large number of measurements, $P(+) = P(-) = 0.5$ and $K = 0$. If there is a C-shaped distortion, $P(+) \neq P(-)$, and the probability for a plus or minus sign to appear takes the form

$$P(+) = \int_0^\infty \frac{1}{\sqrt{2\pi D(\theta)}} \exp\left\{-\frac{(\theta-\theta_0)^2}{2D(\theta)}\right\} d\theta = g \neq 0.5;$$

$$P(-) = 1 - g,$$

№	t, μ	$<\|S_i\|>°$	$<S_i>°$	$\pm(\pi/2)^{1/2}<\|S_i\|>/\sqrt{n}$
1	1720	0.30	0.02	±0.09
2	1260	0.37	0.05	±0.13
3	2300	0.43	0.06	±0.11
4	1150	0.48	0.01	±0.19
5	1380	0.77	0.01	±0.27
97	1610	0.36	0.08	±0.12
98	1960	0.25	0.03	±0.07
99	1000	1.25	0.05	±0.45
100	1000	0.95	0.10	±0.37

and the coefficient of distortion is

$$K = 1 - [g/(1-g)] \neq 0.$$

If track number i contains N_i measurements (boxes), then the probability that the number of observed positive deflections should be n_i is, in the absence of distortion,

$$P[n_i, (N_i - n_i)] = \left(\frac{1}{2}\right)^{N_i} C_{N_i}^{n_i}, \quad (3)$$

and in the presence of distortion

$$P[n_i, (N_i - n_i)] = g^{n_i}(1-g)^{N_i - n_i} C_{N_i}^{n_i}. \quad (4)$$

In Fig. 4, the theoretical distribution curve of the number of plus signs per track is shown by the continuous line. The experimental points are shown as crosses. The theoretical curve is obtained by adding together curves of the form of Eq. (3), one corresponding to each of the 100 tracks upon which measurements were made. The resultant curve is unsymmetrical because the different tracks have different lengths and different values of N_i.

The theoretical expectation for the probability (or the number of cases) that the number of plus signs in a track should not exceed 5 is 0.54 ± 0.05, i.e., 54 tracks in the observed sample. In this estimate the variance is calculated according to the formula $D(n_i) = Np(1-p)$, with $N = 100$, $p = 0.54$. The experimental value is 53 tracks.

c). "Spurious scattering" includes errors σ_1 arising from non-parallel movement of the microscope stage, the so-called "stage-noise", errors σ_2 arising from the scatter of grains in a particle track away from the true trajectory, and errors of measurement σ_3. Our measurements were made with a MBI-8 microscope. The variance due to "spurious scattering" is the sum of the variances σ_1^2, σ_2^2, σ_3^2. The mean square error produced by spurious scattering in our experiment did not exceed 0.07°.

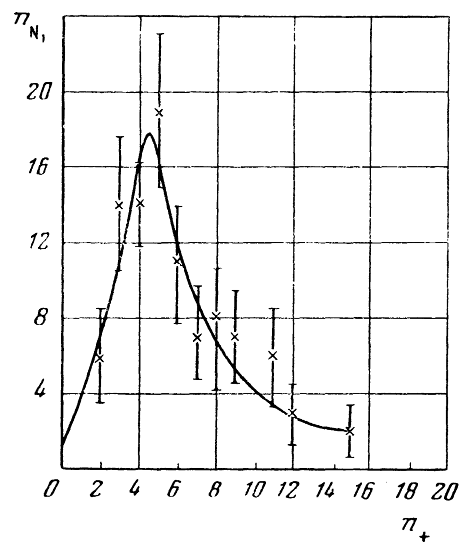

FIG. 4. Theoretical distribution curve for the number of plus signs per track in the 100 tracks which were measured. The crosses are the experimental values.

3. *Charge-Analysis of Particles.*

If the direction of motion of a particle is known, the determination of the sign of the charge depends on measuring the deflection in the magnetic field. Figure 5 shows the probability that a positively charged particle traversing t microns of emulsion in a magnetic field of strength H should be identified as negative. It is assumed that positive and negative particles have the same *a priori* probability and the same momentum distribution.

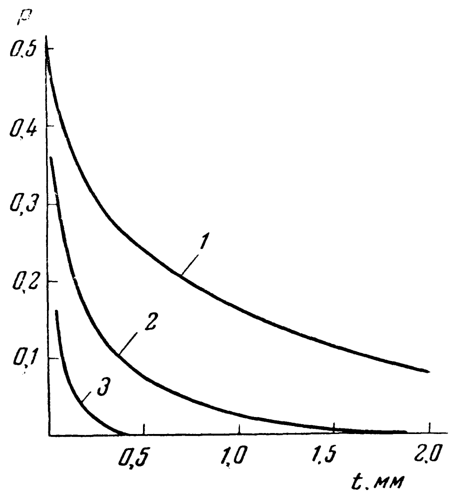

FIG. 5. Curves showing the probability of an incorrect determination of the sign of the charge. *1*, $H = 0.6 \times 10^5$ gauss; *2*, $H = 1.2 \times 10^5$; *3*, $H = 2.4 \times 10^5$.

The determination of the sign of the charge from magnetic deflection in weaker fields (10-30 kilogauss) is made difficult by the presence of strong multiple scattering. However, if the momentum of the particle can be measured, either from the multiple scattering itself or otherwise, then the sign of the charge can be fixed with reasonable certainty[6].

4. *Methodological Deductions.*

Some 800 electron-positron pairs, in which the track-length of each particle was greater than 540μ, were chosen for measurement. The total track-length measured was 230 cm. Figure 6 shows microprojections of electron-positron pairs seen in the emulsion exposed in a magnetic field of 1.2×10^5 gauss. By measuring simultaneously the angle of magnetic deflection M and the angle of multiple scattering S, we can deduce the scattering constant for electrons and positrons of NIKFI type R emulsion. We used the formula

$$K_{100\mu} = \overline{|S|E_M},$$

where E_M is the particle energy determined from the magnetic deflection. The scattering constant derived from 2700 measurements was

$$K_{100\mu} = 23.4 \pm 0.7 \text{ degree MeV} / (100\ \mu)^{1/2}.$$

In measuring the scattering constant we cut off the large-angle single scattering by requiring that each observed scattering angle should not exceed four times the average angle. The mean energy per particle deduced from the magnetic deflection of the 1600 tracks was 43 ±3 mev, and from the multiple scattering measurements 46 ±2 mev.

2. BREMSSTRAHLUNG SPECTRUM AND POSITRON ANNIHILATION IN FLIGHT

The continuous curve in Fig. 7 shows the result of a calculation of the energy-spectrum of bremsstrahlung produced by the synchrotron of the Physical Institute of the Academy of Sciences. The synchrotron target was a tungsten

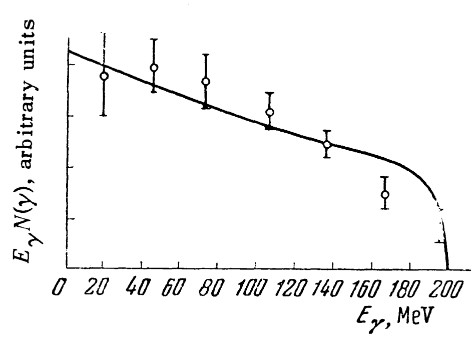

FIG. 7. Experimental values of photon intensity $E_\gamma N(\gamma)$ plotted against photon energy E_γ.

wire of diameter 1 mm. The effective target thickness was 0.16 radiation lengths. The calculation was made from the Bethe-Heitler formula, including effects of absorption of γ-rays in the target and of double radiation by electrons. At such thicknesses the spectrum of photons emitted in the forward direction is rather accurately the same as the complete spectrum from a single atom.

To deduce the number of photons from the number of observed electron-positron pairs, one needs to know the effective cross-section for pair creation by nuclei of the emulsion. We calculated the effective cross-section from the Bethe-Heitler formulas. We used the correction factor*

$$[1 + 0.12(Z/82)]^{1/2}.$$

* *Translators note.* This formula is incorrectly quoted and should read $[1 - 0.12\,(Z/82)^2]$.

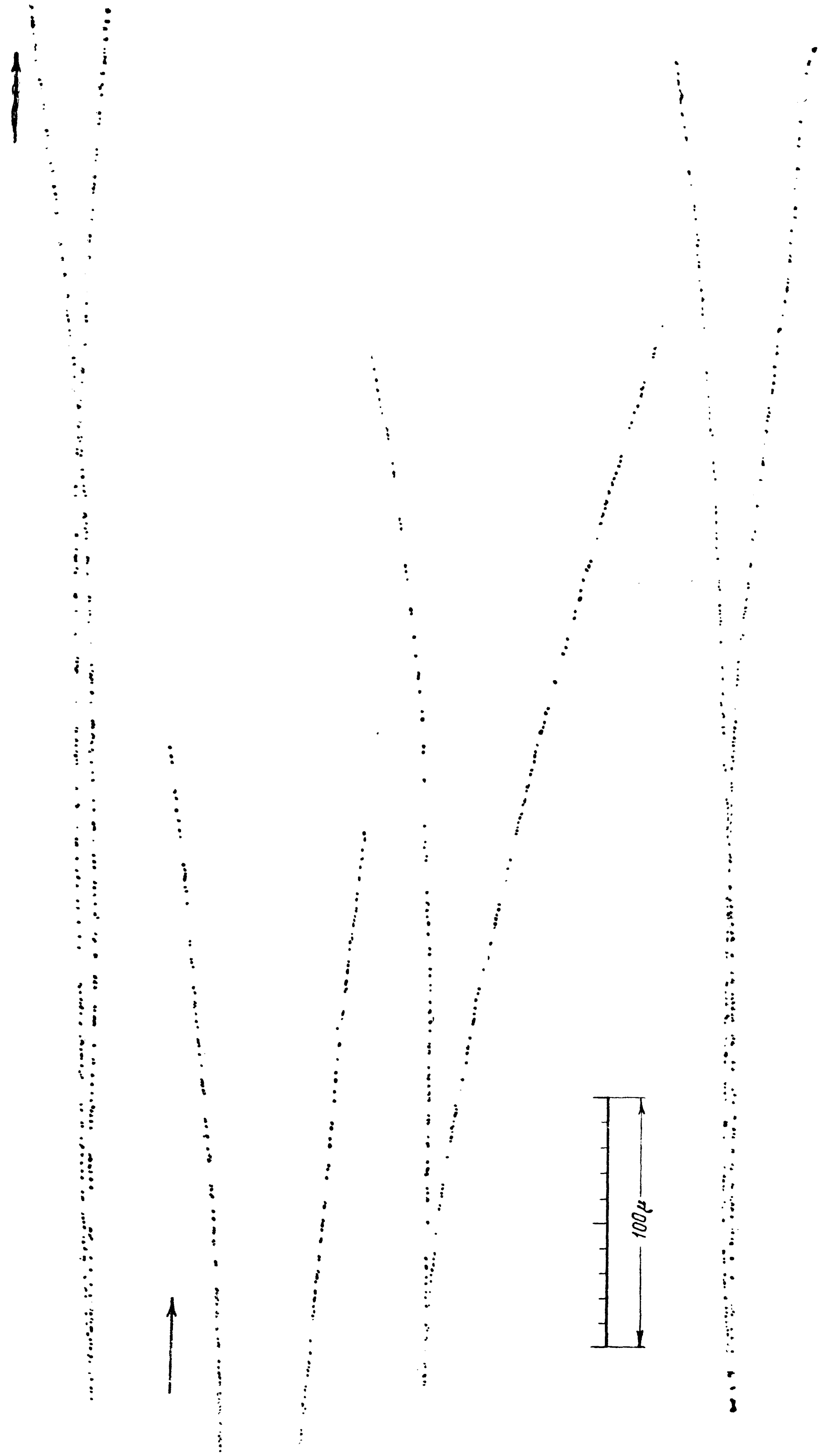

FIG. 6. Microprojections of electron-positron pairs observed in emulsions which were exposed in a magnetic field of 1.2×10^5 gauss.

which has been shown by various authors[7-10] to be required in order to convert the Born approximation Bethe-Heitler cross-sections to true cross-sections. For Ag($Z = 47$) and Br($Z = 35$), which are the worst emulsion nuclei for using the Born approximation, the correction does not exceed 4 per cent.

In making a histogram of the spectral distribution of electron-positron pairs, we accepted only those pairs in which each particle had a track-length in emulsion greater than 540μ. This selection discriminated against low-energy pairs, because the components of low-energy pairs are more likely to leave the emulsion, and it was necessary to correct the observed spectrum accordingly.

The probability that neither component of a positron-electron pair of energy E_0 should leave the emulsion before travelling a distance t microns is

$$Q(E_1, E_0 - E_1, t, d) = [1 - P(E_0, E_1, t, d)]\,[1 - P(E_0, E_0 - E_1, t, d)],$$

Here $P(E_0, E_1, t, d)$ is the probability that one component (electron or positron) with energy E_1 should leave the emulsion of thickness d before travelling t microns. To find the value of Q averaged over the energies of the components we must evaluate an expression of the form

$$\int_0^{E_0 - 2mc^2} \Psi(E_0, E_1)\, Q(E_1, E_0 - E_1, t, d)\, dE_1 \Big/ \int_0^{E_0 - 2mc^2} \Psi(E_0, E_1)\, dE_1,$$

where $\Psi(E_0, E_1)$ is the probability for a photon of energy E_0 to produce a pair with component energies E_1, $E_0 - E_1$. The integration was performed graphically. This correction was calculated, including the effects of multiple scattering and of the deviation of the angles of emission of positron and electron from the direction of the photon[11]. The results of the calculation* are shown in Fig. 8.

The determination of the energy of a pair from the magnetic deflection is made with an average error of 21 per cent. This large experimental error causes a redistribution of points in the observed pair spectrum. The redistribution must be taken into account when the experimental results are compared with theory. Figure 9 shows a histogram of the experimental pair spectrum (full lines), and a histogram obtained from the theoretical spectrum by introducing random "errors of measurement" by a Monte Carlo technique (dotted lines). The two histograms coincide within the statistics.

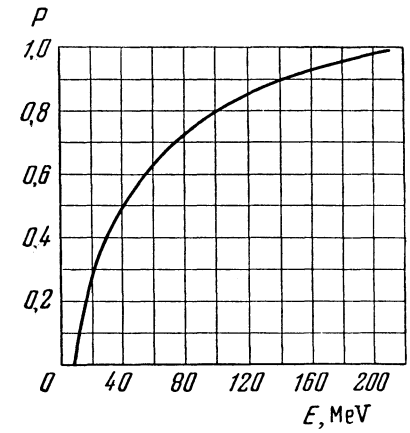

FIG. 8. Calculated correction factor to determine the "true" number of pairs from the number observed in emulsion.

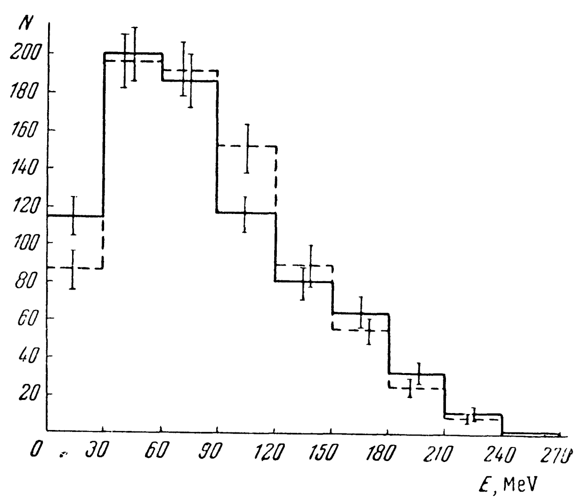

FIG. 9. Histograms of the pair energy spectrum. The full lines are the experimental values. The dotted lines are theoretical values with "errors of measurement" introduced by a Monte Carlo technique.

Experimental values of the photon intensity as a function of photon energy are shown in Fig. 7. These are deduced from the pair spectrum. The experimental errors in the measurement of the pair energies prevent a detailed investigation of the form of the spectrum, especially at the high-energy end where a departure from the Bethe-Heitler theory might most reasonably be expected.

In scanning the emulsions we observed four cases of particle annihilation in flight. A charge measurement showed that in each case the vanishing particle was a positron. In the extreme relativistic limit, the probability for this process is[12]

$$\Phi = \pi r_0^2 \frac{mc^2}{E} [\lg (2E_+/mc^2) - 1].$$

The annihilation probability for a particle of average energy (43 mev) passing through a length 1400 μ (the average track length) of emulsion is 1.7×10^{-3}. The experimental frequency of annihilation is 1.5×10^{-3}, obtained by dividing the total track-length of the annihilating particles (1795μ) by the total track-length of all positrons ($118 \times 10^4 \mu$).

In conclusion the authors express their deep gratitude to Professors V. I. Veksler and M. P. Podgoretskii for their constant attention and help.

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* *Note added in proof.* More detailed calculations of the correction were completed after this manuscript went to press. The maximum deviation from the preliminary results was not greater than 15 per cent, and for positron-electron pairs with energy above 100 mev the deviation was not greater than 4 per cent.

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