corrections proportional to $\alpha^4$. This is at the borderline of present-day experimental errors. Nonelectromagnetic effects should likewise contribute significantly to corrections to the (ordinary) fine structure of $\mu$-mesic hydrogen ($\alpha^3$ approximation).

Let us examine, for example, the polarization of the vacuum in $\mu$-mesic hydrogen. Generalizing the results of Vehling[4] for polarization corrections to arbitrary atomic and vacuum particles having masses $m_0$ and $m_V$ in the case where $m_0/m_V$ is much less than $1/\alpha$, we have:

$$\Delta E = -\frac{8}{15\,\pi}\left(\frac{m_0}{m_v}\right)^2 \frac{\mathrm{Ry}',\alpha^3}{n^3}\left(1+\frac{m_0}{M}\right)^{-3} G. \qquad (3)$$

The factor $G$ in Eq. (3) is determined by the spins of the particles examined and their interaction with the vacuum fields. It follows from Eq. (3) that the polarization of the $\pi$-mesic vacuum introduces into $\mu$-mesic atoms terms of the same order as arise from the polarization of the $\mu$-mesic vacuum. For ordinary hydrogen the polarization of the $\pi$-mesic vacuum introduces terms of the same order as electromagnetic corrections of order $\alpha^5$. These are beyond present-day experimental techniques.

At the same time, because the polarization of the $\pi$-mesic vacuum by the external electromagnetic field is seriously affected by the strong interaction of virtual $\pi$-mesic and nucleon pairs, the nonelectromagnetic additions to the fine structure of the $\mu$-mesic hydrogen have a first order effect. The isolation of these effects might serve to establish the limits of applicability of pure electrodynamics and possibly lead to some tests of present-day ideas of the interactions of nucleons with $\pi$-mesons.

In conclusion, I thank Prof. V. L. Ginzburg for discussions of this note and for a series of critical comments.

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Translated by A. Turkevich
198

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# The Photoproduction Cross Section for Positronium in an External Field Taking into Account Radiative Corrections

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THE differential cross section $d\sigma$ for the photoproduction of positronoum in an external field, taking into account radiative corrections of any order, is connected with the differential cross section $d\sigma_f$ for the photoproduction of the free products with zero relative velocity by the following relation:[1]

$$d\sigma = [(\bar{\psi}(0)\,\psi(0)) / (\bar{\psi}_f(0)\,\psi_f(0))]\, d\sigma_f. \qquad (1)$$

where $\psi(x)$ is a wave function (in relative coordinates) of the ground state of positronium (with energy $E$), satisfying a Bethe-Salpeter equation which takes into account the possibility of annihilation[1,2], $\psi_f(x)$ is a wave function of the free particles and is a proper function of the same complete set (but with an energy $E_f = E + \epsilon$, where $\epsilon > 0$ is the binding energy), as is $\psi(x)$. In Eq. (1) it is necessary to evaluate $(\bar{\psi}(0)\psi(0)) = \mathrm{Sp}[\bar{\psi}(0)\psi(0)]$ with the same accuracy as is taken in the calculation of $d\sigma_f$.

If only the first radiative corrections are examined, the multiplier $(\bar{\psi}(0)\psi(0)]$ must be replaced by its nonrelativistic value, since the next correction to $|\psi_{\mathrm{nonrel}}(0)|^2$ is of the order of $(e^2/\hbar c)^2$ [3]. In addition, when positronium is produced in a ground state, the factor $(\bar{\psi}_f(0) \times \psi_f(0))$ can be taken equal to one[3]. Thus, the calculation of the photoproduction cross section of positronium in an external field, taking into account first order radiative corrections, is reduced to the problem of finding the cross sections of either the photoproduction of the free particles or of bremsstrahlung in an external field. This latter has been investigated by Myamlin.[4] However, from his results we cannot immediately write down the cross section $d\sigma_f$, since the author's purpose was to find the radiative corrections to bremsstrahlung, and so he did not examine three diagrams corresponding to radiation with double scattering of the electron in an external field (higher Born approximation).

In view of the tedious character of the evaluation $d\sigma_f$ (taking into account the first radiative corrections), we have confined ourselves in what follows to evaluating the order of magnitude of the cross section for small velocities $v$ of positronium, large enough, however, so that the Born approximation is still justified ($Ze^2/\hbar c \ll v/c \ll 1$). Calculations lead to the following result for the photoproduction cross section of positronium in the ground state:

$$d\sigma = Z^2 r_0^2 \alpha^4 [\Phi + \alpha (v/c)^2 (\Phi_1 + Z\Phi_2)]\, d\Omega, \quad (2)$$

where $\alpha = e^2/\hbar c$, $r_0 = e^2/mc^2$, $Ze$ is the charge of the external field, and the element of the solid angle $d\Omega$ indicates the direction of the total momentum $\mathbf{P}$ of the positronium. $\Phi_1$ and $\Phi_2$ are dimensionless functions of $\mathbf{P}$ and the photon momentum $\mathbf{k}$, having magnitudes of order of unity. Further, $Z^2 r_0^2 \alpha^4 \Phi\, d\Omega \equiv d\sigma_0$ is the cross section in the first nonvanishing approximation having the form

$$d\sigma_0 = 4Z^2 r_0^2 \alpha^4 [(mc)^3 P [\mathbf{kP}]^2 / k^2 (\mathbf{P} - \mathbf{k})^4 (\mathbf{kP} - k^2)]\, d\Omega. \quad (3)$$

It is interesting to notice that, in contrast to Eq. (2), the first radiative corrections to the photoproduction cross section of positronium with two quanta* do not contain the multiplier $(v/c)^2$ at low velocities $v$.

At high velocities, the Feynman diagrams with $Z^2$, corresponding to a higher Born approximation, do not contribute to $d\sigma_f$ [6] and the photoproduction cross section of positronium $d\sigma$ is determined by the formula

$$d\sigma = |\psi_{\text{nonrel}}(0)|^2 [d\sigma_f^r (\mathbf{p}, \mathbf{P}) / d^3p\, (2\pi\hbar)^{-3}]_{\mathbf{p}=0}, \quad (4)$$

where $\mathbf{p}$ is a relative momentum, and the cross section $d\sigma_f^r(\mathbf{p}, \mathbf{P})$ is the photoproduction cross section for free particles, taking into account the radiative corrections calculated by Myamlin[4].

The infrared divergences in the radiative corrections [not only in first order (Eqs. (2) and (4)), but in any order in $e^2/\hbar c$] cancel each other in the photoproduction of positronium. This is seen as follows. The amplitude of the photoproduction of positronium, taking into account radiative corrections of any order, is the same even to the multiplier with the amplitude for the photoproduction of free particles with zero relative velocity[1]. However, in the case of free particles, the radiative corrections to the process of order $(e^2/\hbar c)^n$, which bring in the infrared divergence, can be written as (see Ref. 7): $2(\rho_1 + \rho_2)M$ where $M$ is the amplitude of the given process to order $(e^2/\hbar c)^n$, and $\rho_1$ and $\rho_2$ are numerical coefficients proportional to $e^2/\hbar c$ and having the form**

$$\rho_1 = -\frac{e^2}{\hbar c}\frac{i}{(2\pi)^4}\int \frac{(p_i p_f)}{(p_i k)(p_f k)}\frac{d^4k}{k^2}, \quad (5)$$

$$\rho_2 = \frac{e^2}{\hbar c}\frac{1}{(2\pi)^2}\int \frac{dk_0}{k_0},$$

where $p_i$ and $p_f$ are momenta corresponding to the initial and final states of the electron. Since in the case of photoproduction, $p_i = -p_2$ ($p_2$ is the momentum of the positron), $p_f = p_1$ ($p_1$ is the electron momentum), and $p_1 = p_2$, then $\rho_1 + \rho_2 = 0$; therefore, the terms with the infrared divergence in the radiative correction to the photoproduction of positronium vanish. [In an analogous fashion it can be shown (see Ref. 1 and 3) that the infrared divergences in the radiative corrections to the annihilation of positronium likewise cancel to any order of $e^2/\hbar c$.]

At high energies, the more probable process is not the formation of positronium, but the formation of interacting particles. In fact, on one hand, at high energies of the $\gamma$-quanta, the angle $\theta$ between the particles formed is small: $\theta \sim mc^2/\omega$ [8] ($\omega$ being the energy of the quantum). On the other hand, the angle $\theta$ is of the order $v_{\text{rel}}/v$, where $v_{\text{rel}}$ is the relative velocity of the particles and $v$ is the velocity of their center of mass ($v \approx c$). It follows that the particles formed go off with low relative velocity $v_{\text{rel}}/c \sim mc^2/\omega \ll 1$. In this case, taking into account the interaction between the two particles according to a formula analogous to Eq. (1) (see Ref. 1), leads to a significant increase in the differential cross section [relative to Eq. (4)] due to the factor $(\bar{\psi}(0)\psi(0))$ which, in first approximation, is equal to

$$(\bar{\psi}(0)\psi(0)) = (2\pi e^2/\hbar v_{\text{rel}})(1 - \exp\{-2\pi e^2/\hbar v_{\text{rel}}\})^{-1}. \quad (6)$$

In conclusion, I would like to thank A. D. Galanin for discussions of the results of this work.

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* This cross section is evaluated (see Ref. 1) by a formula analogous to Eq. (1), where the cross section $d\sigma_f$ for the photoproduction of the free particles by two quanta is calculated, together with the first radiative corrections, by Brown and Feynman[5].

** The following summation rule has been used:

$$(ab) = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3.$$


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Translated by A. Turkevich
199

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## A Bubble Chamber for the Study of Cosmic Rays

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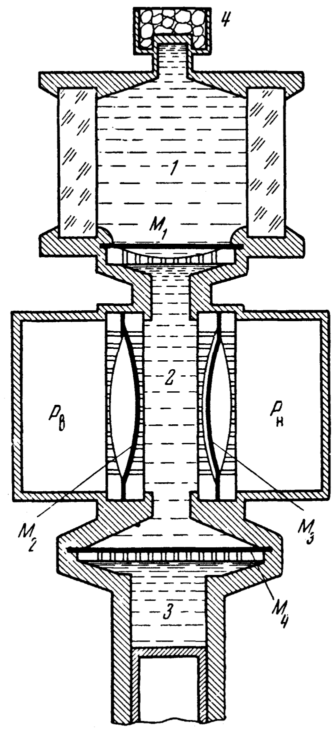
UP until now bubble chambers have not been used for the study of cosmic rays. The reason for this lies in the lack of control of such chambers, since the life–time of the embryonic bubbles formed by ionizing particles in going through the chamber is significantly less than the time in which decompression occurs. However, as will be seen in this work, bubble chambers can be used to study cosmic rays. This can be accomplished by increasing the efficiency of the chamber, that is, by increasing the fraction of the time in which the chamber is sensitive. The efficiency is determined by the sensitive time in each cycle as well as by the length of each cycle. By shortening the cycling time and taking certain other precautions, we have increased the efficiency of the chamber so that it has become practical to work with cosmic rays.




A schematic plan of the set–up is given in the Figure. It consists of the chamber *1*, a limiter *2*, and compressor *3*. The chamber *1* has a cylindrical form and is filled with freon–13 ($CClF_3$). The volume of the chamber is 1 liter. The viewing windows are discs of organic glass mounted without gaskets. The membrane $M_1$ is constructed out of rubber made from domestic synthetic nayrite.

The compressor *3* is made like the air-compressor type KVD and produces a periodic compression and decompression of the chamber at a rate of 10 cycles/sec. The horespower used in such a compression process is not large since the energy expended by the rotor during the compression part of the cycle is returned upon decompression. In this way, the method described is advantageously different from one using a gas. The cylinder of the compressor is filled with oil. The pressure exerted on it by the plunger is transmitted to water occupying the central volume of the limiter and through it to the working fluid in the chamber *1*.

The limiter *2* determines the limits in which the pressure in the chamber can vary. It gives the