is screened by atomic electrons.

These results can also be used as a correction to polarization in nuclear scattering. In this case

$$d\sigma_{1/2} - d\sigma_{-1/2} \sim |A_{\text{nuc}\,1/2} + A_{q\,1/2}|^2 - |A_{\text{яд}\,-1/2} + A_{q\,-1/2}|^2,$$

where A_{nuc} and A_q are the nuclear and Coulomb scattering amplitudes, respectively. In the interference term it is sufficient to take A_q in the first order perturbation approximation.

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Isotopic Invariance and the Creation of Particles

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The consequences of conservation of isotopic spin are investigated. Relations between different cross sections are found which are valid if in the meson nucleon interaction a state with a particular value of isotopic spin predominates. The relations of Smorodinskii and Jacobson for elastic nucleon-nucleon cross sections are generalized for the case of meson and nucleon–antinucleon pair production. Furthermore, the consequences of isotopic spin conservation are given for the following cases: meson production on nuclei, creation of heavy meson pairs and nucleon antinucleon pairs in π–nucleon collisions, and for some processes of nucleon annihilation in collisions with deuterons.

1. IN connection with the increase in the number of possible high energy experiments on nucleons and mesons it is interesting to investigate the consequences of the so called hypothesis of charge independence or isotopic invariance.

The meson creation processes should allow the most direct experimental verification of the conservation of isotopic spin. Besides the relation given by Yang[1]

$$d\sigma\,(p + p \to \pi^+ + d) = 2d\sigma\,(n + p \to \pi^0 + d),$$

one can show,[2,3] using just one condition derived from isotopic invariance, that the following relation also holds

$$d\sigma\,(p + d \to \mathrm{H}^3 + \pi^+) = 2d\sigma\,(p + d \to \mathrm{He}^3 + \pi^0).$$

Several reactions are forbidden by isotopic invariance. Among them is the following curious case: in d–d collisions leaving the deuterons intact, only even numbers of mesons can be created. The forbidden character of the reactions

$$d + d \to d + d + \pi^0;\quad d + d \to \mathrm{He}^4 + \pi^0$$

is clear. The case of three π^0–mesons can be proven as follows. Calling Ψ_{T,T_z} a function with definite T^2 and T_z, one has for the wave function of two π^0–mesons

$$(\pi^0\pi^0) = \{\sqrt{2}\psi_{2,0} - \psi_{0,0}\} / \sqrt{3},$$

which does not contain $T = 1$ components. Therefore a system of three π^0–mesons cannot have a part with $T = 0$. This shows that three π^0–mesons cannot be created since the nuclear system has $T = 0$ before and after the collision.

2. The investigation of the consequences of isotopic spin conservation furthermore allows one to obtain information on the meson–nucleon and nucleon–nucleon interaction in states of definite isotopic spin. For example, as is well known, the elastic and the charge exchange scattering cross section of mesons is given in terms of the amplitudes of the states with $T = 3/2$ and $T = 1/2$, a_3

and a_1, respectively, in the following way:

$$d\sigma(\pi^+p \to \pi^+) = |a_3|^2, \qquad (1)$$
$$d\sigma(\pi^-p \to \pi^-) = {}^1/_9 |a_3 + 2a_1|^2,$$
$$d\sigma(\pi^-p \to \pi^0) = {}^2/_9 |a_3 - a_1|^2.$$

One sees from (1) that the elastic cross sections in the $T = 3/2$ and $T = 1/2$ states are given by

$$d\sigma_{^3/_2} = d\sigma(\pi^+p \to \pi^+),$$
$$d\sigma_{^1/_2} = {}^1/_2 \{3[d\sigma(\pi^-p \to \pi^-)$$
$$+ d\sigma(\pi^-p \to \pi^0)] - d\sigma(\pi^+p \to \pi^+)\},$$

which yield for the total cross sections

$$\sigma_{^3/_2} = \sigma(\pi^+p \to \pi^+) = \sigma^+, \qquad (2)$$
$$\sigma_{^1/_2} = {}^3/_2 \{[\sigma(\pi^-p \to \pi^-)$$
$$+ \sigma(\pi^-p \to \pi^0)] - \sigma^+\} = {}^1/_2 (3\sigma^- - \sigma^+).$$

For meson production in π–N collisions, one can obtain

$$d\sigma_{^3/_2}(\pi N \to N\pi\pi)$$
$$= d\sigma(\pi^+p \to \pi^+\pi^+) + d\sigma_+(\pi^+p \to \pi^+\pi^0),$$
$$d\sigma_{^1/_2}(\pi N \to N\pi\pi) \qquad (3)$$
$$= {}^1/_2 \{3[d\sigma(\pi^-p \to \pi^0\pi^0) + d\sigma_+(\pi^-p \to \pi^-\pi^0)$$
$$+ d\sigma_+(\pi^-p \to \pi^+\pi^-)] - d\sigma_{^3/_2}(\pi N \to N\pi\pi)\},$$

where, for example

$$d\sigma_+(\pi^-p \to \pi^+\pi^-)$$
$$= d\sigma(\pi^-p \to n\pi^+\pi^-) + d\sigma(\pi^-p \to n\pi^-\pi^+),$$

The yield for the total cross sections

$$\sigma_{^3/_2}(\pi N \to N\pi\pi) = \sigma(\pi^+p \to \pi\pi), \qquad (4)$$
$$\sigma_{^1/_2}(\pi N \to N\pi\pi) = {}^1/_2 \{3\sigma(\pi^-p \to \pi\pi) - \sigma(\pi^+p \to \pi\pi)\}.$$

Let σ^+, σ°, and σ^- be the cross section for multiple production of mesons by positive, uncharged and negative meson respectively. Then it can be shown that expressions of the form (2) are valid for any number of produced mesons; therefore, they remain valid for the total meson–nucleon interaction cross section. The validity of these relations is connected with the fact that the isotopic spin of the meson system can be only 0, 1, or 2 if the total isotopic spin of the meson–nucleon system is $T = 1/2$ or $T = 3/2$.

If the elastic cross section predominates, one obtains for a state with $T_{\pi T} = 3/2$ (as is well known)

$$d\sigma(\pi^+p \to \pi^+) : d\sigma(\pi^-p \to \pi^0) : d\sigma(\pi^-p \to \pi^-)$$
$$= 9:2:1.$$

In order to obtain the quantitative consequences of the predominance of the $T = 3/2$ state in the meson–nucleon interaction in meson production, one has to expand the final system into subsystems. In the analysis of meson production processes by nucleons, the final state of the system is usually classified in the way that the isotopic spin is a sum of the isotopic spin of the nucleons and of the meson.[4,5] This way one can follow through transitions taking place in the nucleon system.[6] However, expanding into meson–nucleon subsystems, one can explore the transitions undergone by subsystems with different $T_{\pi N}$ and as a result establish relations between different cross sections which are due to the predominance of certain values of $T_{\pi N}$ in the meson nucleon interaction.

If isotopic spin is conserved, one can express the cross section of single meson production in N–N collisions in terms of three independent matrix elements. Two of them correspond to transitions with $T = 1$, and one with $T = 0$. Usually the two $T = 1$ states of the final system are classified according to their symmetry with respect to nucleon exhange. In the scheme adopted in the present paper, the total isotopic spin $T = 1$ is obtained by combining the spin of the nucleon $T_N = 1/2$ with the πN subsystem in either the $T_{\pi N} = 1/2$ or the $T_{\pi N} = 3/2$ state.

Denoting the transition amplitudes into the T=1 state with A_{13} and A_{11}, and into the $T = 0$ state with A_{01} the cross sections for the different processes of single meson production can be expressed as follows:

$$M(pp \to \pi^+pn) = 3^{1/2}2^{-1}A_{13},$$
$$M(pp \to \pi^+np) = -12^{-1/2}A_{13} + 2^{1/2}3^{-1/2}A_{11},$$
$$M(pp \to \pi^0pp) = -6^{-1/2}A_{13} - 3^{-1/2}A_{11},$$
$$M(np \to \pi^+nn) = 2^{-1/2}\{6^{-1/2}A_{13} + 3^{-1/2}A_{11} + 3^{-1/2}A_{01}\},$$
$$M(np \to \pi^-pp)$$
$$= -2^{-1/2}\{6^{-1/2}A_{13} + 3^{-1/2}A_{11} - 3^{-1/2}A_{01}\},$$
$$M(np \to \pi^0pn) \qquad (5)$$
$$= 2^{-1/2}\{3^{-1/2}A_{13} - 6^{-1/2}A_{11} - 6^{-1/2}A_{01}\},$$
$$M(np \to \pi^0np)$$
$$= -2^{-1/2}\{3^{-1/2}A_{13} - 6^{-1/2}A_{11} + 6^{-1/2}A_{01}\}.$$

The two nucleon—meson states ($\pi\, pn$) and (πnp) are obtained from each other by exchanging the nucleons. Writing $d\sigma_+ (pp \to \pi^+) = d\sigma\, (pp \to \pi^+ np) + d\sigma\, (pp \to \pi^+ pn)\ l + c$ we obtain immediately

$$d\sigma_+ (pp \to \pi^+) + d\sigma\, (np \to \pi^+) + d\sigma\, (np \to \pi^-) = 2\, [d\sigma\, (pp \to \pi^0) + d\sigma_+ (np \to \pi^0)]. \tag{6}$$

Furthermore, one can obtain relations from (5) between cross sections for different processes if the meson nucleon interaction of one $T_{\pi N}$ predominates. Thus, if π meson—nucleon interaction occurs only in the $T_{\pi N} = 3/2$ state, then the following relation obtains

$$d\sigma\, (pp \to \pi^+) = 5 d\sigma\, (pp \to \pi^0) = 2{,}5\, d\sigma\, (np \to \pi^0) = 10\, d\sigma\, (np \to \pi^-) = 10\, d\sigma\, (np \to \pi^+). \tag{7}$$

In the complementary case where only the $T_{\pi N} = 1/2$ state contributes, (7) changes into

$$d\sigma_+ (pp \to \pi^+) = 2 d\sigma\, (pp \to \pi^0), \tag{8}$$

$$d\sigma_+ (np \to \pi^-) + d\sigma\, (np + \pi^+) = 2 d\sigma_+ (np \to \pi^0). \tag{9}$$

The relations (6) to (9) are true forcertain parts of the meson spectrum. These relations can only be approximately true forthe total cross sections even if the main part of the spectrum consists of meson energies where the interaction in a particular $T_{\pi N}$ state predominates.

It is known from experimental results on the meson-nucleon interaction that for meson energies of 150 to 200 mev, the interaction in the $T_{\pi N} = 3/2$ state predominates. The experimental total cross sections for single meson production at a nuclear energy of ≈ 650 mev, where the center of mass of the spectrum of outgoing mesons lies at 150 to 200 mev, do not contradict Eq. (7).[7,8,9]

From (5) one can express A_{13}, A_{11}, and A_{01} in terms of cross sections. Thus, $d\sigma_0 = |A_{01}|^2$, the cross section for meson production in the $T = 0$ state takes the form

$$d\sigma_0\, (NN \to \pi) = 3\, \{d\sigma\, (np \to \pi^+) + d\sigma\, (np \to \pi^-) - d\sigma\, (pp \to \pi^0)\}. \tag{10}$$

Close to the meson production threshold, all quantities on the right hand side of (10) are small in comparison to $\sigma\, (pp \to \pi^+)$. This leads to the well known predominance of the $T_{NN} = 1$ state for meson creation.[10] In the range of large energies, where the meson—nucleon interaction occurs in the $T_{NN} = 3/2$ state, the expression (10) due to (7) alsotends to zero.

From the experimental data on the nucleon—nucleon interaction one can obtain information on the interaction in states of definite isotopic spin in a fashion analogous to the way in which Eqs. (2) gave similar information on the meson—nucleon interaction.

Let $\sigma^{y}_{np}(\theta)$, $\sigma^{y}_{pp}(\theta) = \sigma^{y}_{1}(\theta)$ and $\sigma^{y}_{0}(\theta)$ be the differential nucleon scattering cross sections. Then, as shown by Smorodinskii[11] and Jacobson[12] holds

$$d\sigma^{y}_{0}(\theta) = 2\, [d\sigma^{y}_{np}(\theta) + d\sigma^{y}_{np}(\pi - \theta)] - d\sigma^{y}_{pp}(\theta),$$

For the total cross sections, this becomes

$$\sigma^{y}_{0} = 2\sigma^{y}_{np} - \sigma^{y}_{pp}. \tag{11}$$

Using (10) one can obtain another expression from (5) for single meson production in the $T = 0$ state:

$$\sigma_0\, (NN \to \pi) = \sigma_0 = 2\sigma\, (np \to \pi) - \sigma\, (pp \to \pi) = 2\sigma_{np} - \sigma_{pp}, \tag{12}$$

Here $\sigma\, (np \to \pi)$ is the total single meson production cross section in n—p collisions and $\sigma(pp \to \pi) = \sigma_1$ is the total single meson production cross section in p—p collisions. The relation (6) appears as the result of the existence oftwo expressions, (9) and (12) for σ_0. From the last two relations it follows that (12) holds for the total cross sections of nucleon interactions, including both elastic scattering and meson production. It remains valid even in presence of multiple meson production if the contribution from the latter processes is subtracted.

3. In the analysis of meson creation in π—N collisions[2,5] the final state was expressed such that the total isotopic spin was given as the sum of the nucleon isotopic spin and the isotopic spin of the meson system. Thereby the possibility arose to investigate the transitions into states of different meson exchange symmetries. When dividing the system into meson—nucleon subsystems the remarkable fact emerged that the meson—nucleon subsystem of the final state can have $T_{\pi N} = 3/2$ or $1/2$ even if the total system had the same T respectively.

In the case where both $T = 3/2$ and $T_{\pi N} = 3/2$

the cross sections of the different processes obey the relations

$$d\sigma_+(\pi^+p\to\pi^+\pi^0) \tag{13}$$
$$=\frac{13}{2}d\sigma(\pi^+p\to\pi^+\pi^+)=\frac{117}{2}d\sigma(\pi^-p\to\pi^0\pi^0)$$
$$=\frac{117}{17}d\sigma_+(\pi^-p\to\pi^-\pi^0)=\frac{117}{10}d\sigma(\pi^-p\to\pi^-\pi^+),$$

Clearly the first of these processes predominates over all meson–proton processes.

For $T=3/2$ and the isotopic spin for the subsystem $T_{\pi N}=1/2$, one obtains, instead of (13),

$$d\sigma(\pi^+p\to\pi^+\pi^+) \tag{14}$$
$$=2d\sigma_+(\pi^+p\to\pi^+\pi^0)=9d\sigma(\pi^-p\to\pi^0\pi^0)$$
$$={}^{18}/_5\,d\sigma_+(\pi^-p\to\pi^0\pi^-)=9d\sigma_+(\pi^-p\to\pi^-\pi^+).$$

In both these expressions, i.e., if the total isotopic spin is $T=3/2$ with arbitrary isotopic spin of the subsystems, the following relations hold:

$$d\sigma(\pi^+p\to\pi^+\pi^+):d\sigma(\pi^0p\to\pi^0\pi^0):d\sigma(\pi^-p\to\pi^0\pi^0)=9:2:1, \tag{15}$$
$$2\,[d\sigma(\pi^+p\to\pi^+\pi^+)+d\sigma_+(\pi^+p\to\pi^+\pi^0)]=3\,[d\sigma(\pi^0p\to\pi^0\pi^0)+d\sigma_+(\pi^0p\to\pi^+\pi^-)+d\sigma_+(\pi^0p\to\pi^+\pi^0)],$$
$$2d\sigma_+(\pi^-p\to\pi^-\pi^0)=d\sigma_+(\pi^-p\to\pi^-\pi^+)+4d\sigma(\pi^-p\to\pi^0\pi^0).$$

In the case when the interaction predominates for $T=1/2$ of the total system, nothing happens for incoming positive mesons. Besides the known expressions

$$4d\sigma(\pi^-p\to\pi^0\pi^0)+d\sigma_+(\pi^-p\to\pi^-\pi^0)=2d\sigma_+(\pi^-p\to\pi^+\pi^-), \tag{16}$$
$$d\sigma(\pi^-p\to\pi^0\pi^0)=2d\sigma(\pi^0p\to\pi^0\pi^0),$$
$$d\sigma_+(\pi^-p\to\pi^+\pi^-)=2d\sigma_+(\pi^0p\to\pi^+\pi^0)$$

there exist still further relations when $T_{\pi N}$ is specified. For $T_{\pi N}=3/2$ ($T=1/2$), one obtains, instead of (12), the relation

$$d\sigma_+(\pi^-p\to\pi^-\pi^+)=2{,}5\,d\sigma_+(\pi^-p\to\pi^-\pi^0)=2{,}5\,d\sigma(\pi^-p\to\pi^0\pi^0), \tag{17}$$

and for $T_{\pi N}=1/2$),

$$d\sigma_+(\pi^-p\to\pi^-\pi^+)=d\sigma_+(\pi^-p\to\pi^0\pi^-)=4d\sigma(\pi^-p\to\pi^0\pi^0). \tag{18}$$

In the energy range where the interaction of the mesons with the nucleon in the final state occurs only in the $T_{\pi N}=3/2$ state, but where the total isotopic spin is both 3/2 and 1/2, the expressions (13) change only for interactions involving negative mesons and protons since $T=1/2$ states do not participate in the (π^+p) system. Denoting the total cross section of the change of a positive meson into two mesons by $d\sigma(\pi^+p\to\pi\pi)$, we obtain the relation in the form

$$65d\sigma(\pi^-p\to\pi^0\pi^0)+6d\sigma(\pi^+p\to\pi\pi)=40d\sigma_+(\pi^-p\to\pi^-\pi^0)+10d\sigma_+(\pi^-p\to\pi^-\pi^+). \tag{19}$$

4. In analyzing the nonproduction of mesons from the point of view of isotopic spin invariance[13,14] one has to consider (in addition to the operator S which transforms like a scalar) an operator which transforms like the third component V_z of a vector in isotopic spin space. The only nonvanishing matrix elements of such an operator are (see for Example Ref. 15).

$$(V_z)_{n'TT_z}^{nTT_z}=V_{n'T}^{nT}T_z, \tag{20}$$

$$(V_z)_{n'TT_z}^{nT-1\,T_z}=(V_z)_{n'T-1\,T_z}^{nTT_z}=V_{n'T-1}^{nT}(T^2-T_z^2)^{1/2}. \tag{21}$$

Here n and n' are all other indices specifying the state of the system.

We shall now investigate the process of single meson photoproduction on deuterium and look for relations between different cross sections, considering the meson–nucleon system to be in definite $T_{\pi N}$ states. Because of (20), $(V_Z)_{n\,T_0}^{n\,T\,0}=0$; hence the different matrix elements of the photoprocess have the form

$$\begin{aligned}M(\gamma d\to\pi^+nn)&=3^{-1/2}S_0+6^{-1/2}V_{13}+3^{-1/2}V_{11},\\ M(\gamma d\to\pi^-pp)&=3^{-1/2}S_0-6^{-1/2}V_{13}-3^{-1/2}V_{11},\\ M(\gamma d\to\pi^0pn)&=-6^{-1/2}S_0+3^{-1/2}V_{13}-6^{-1/2}V_{11},\\ M(\gamma d\to\pi^0np)&=-6^{-1/2}S_0-3^{-1/2}V_{13}+6^{-1/2}V_{11},\end{aligned} \tag{22}$$

Here S_0 is the transition amplitude connected with the operator S, and V_{11} and V_{13} are the corresponding quantities for V_z; V_{13} is the amplitude connected with transitions into states with $T_{\pi N}=3/2$ and V_{11} into states with $T_{\pi N}=1/2$, when the isotopic spin of the total system equals one.

In the energy range where the $T_{\pi N}=3/2$ state predominates in the meson—nucleon interaction the processes involving V_{13} will also predominate. Then the following relations will hold between the cross sections:

$$d\sigma(\gamma d \to \pi^+) = d\sigma(\gamma d \to \pi^-) = {}^1/_4\, d\sigma(\gamma d \to \pi^0). \tag{23}$$

In the energy range when the interaction in the $T_{\pi N} = 1/2$ state predominates one obtains from (22)

$$d\sigma(\gamma d \to \pi^+) + d\sigma(\gamma d \to \pi^-) = 2d\sigma(\gamma d \to \pi^0). \tag{24}$$

In the analysis of meson photoprocesses the smallness of S as compared with V_z is sometimes taken as a starting point.[16] This is based on the smallness of S_0 as compared with V_{13}. However, the smallness of the matrix element S_0 may be due to the predominance of the meson—nucleon interaction in the $T_{\pi N} = 3/2$ state. The rather large cross section for the "elastic" π^0 photoproduction[17,18] also may be due solely to the above mentioned characteristics of the meson-nucleon interaction (the quantity S_0 does not make any contribution to the $\gamma d \to d + \pi^0$ process).

As shown above in many examples, the predominance of the nucleon—meson interaction in states of definite T can result in relations between the cross sections of different but related meson production processes. The derivation of these relations did not involve any details of the meson—nucleon interaction besides the assumption of conservation of isotopic spin. These relations must therefore also follow from the more detailed models if they include the strong interaction in the $T = 3/2$ state, independently of the other aspects of the model. Relations of the type (7) must also appear in the different "isobaric models".[19] However in the same way as the experimental verification of the 9:2:1 ratios in meson scattering did not decide the question of the resonance character of the state, the experimental verification of relation of the type (7) does not confirm the "isobaric model" but only points up the significantly larger importance of the $T_{\pi N} = 3/2$ state as compared to the doublet state. This clearly holds for all processes of real meson production.

5. The majority of the published relations between cross sections for scattering or production of π-mesons concern interactions with nucleons or deuterons. However, there exist similar relations even for nuclei, if they are in a definite isotopic spin state. Such nuclei are the light nuclei, as is well known. We shall now investigate the consequences of isotopic invariance for meson production in nucleon—nucleus collisions. We shall consider specifically nuclei with a difference between neutron and proton numbers of one and two. The relations which we will thus obtain are useful for checking the hypothesis of isotopic invariance, or for obtaining information on the purity of the isotopic spin in the states of the nuclei, and also for obtaining information regarding difficult reactions.

As is well known, the cross sections for production of positive (σ^+), negative (σ^-), and neutral (σ^0) mesons in nucleon—deuteron collisions in case of isotopic invariance have to obey the relation

$$\sigma^+ + \sigma^- = 2\sigma^0. \tag{25}$$

The same relation clearly holds for all nuclei with $T = 0$.

We shall now investigate the production of mesons by nucleon collision with mirror nuclei like H^3, He^3, and Li^7, Be^7. They have all isotopic spin $T = 1/2$. The total isotopic spin of nucleus plus nucleon therefore can be 0 or 1. The final state consists of an even number of nucleons and a single meson. The case $T = 0$ of the total system requires a nucleon isotopic spin $T_N = 1$ because of $T_\pi = 1$. The case $T = 1$ can be obtained with $T_N = 2, 1$ or 0. Since no states with $T_N > 2$ can participate, it is clear that the results obtained for He^3 and H^3 hold for all similar pairs of mirror nuclei. On the other hand, one sees immediately that the results of the present section have to go over into the case of meson production in nucleon—nucleon collisions when taking into account that there the state with $T_N = 2$ is absent.

As a result of the usual considerations, one obtains the following relations

$$\begin{aligned}[\sigma_{T_z=-1/2}(n,\pi^-) + \sigma_{T_z=-1/2}(p,\pi^-)] \\ + [\sigma_{T_z=-1/2}(n,\pi^+) + \sigma_{T_z=-1/2}(p,\pi^+)] \\ = 2[\sigma_{T_z=-1/2}(p,\pi^0) + \sigma_{T_z=-1/2}(n,\pi^0)],\end{aligned} \tag{26}$$

Here, for example, $\sigma_{T_z=-1\text{-}2}(p,\pi^+)$ denotes the cross section for π^+-meson production by protons summed over all nucleons of a $T_z = -1/2$ nucleus like H^3 or Be^9.

Because of charge symmetry,

$$\sigma_{T_z=-1/2}(n,\pi^\pm) = \sigma_{T_z=+1/2}(p,\pi^\mp), \tag{27}$$

$$\sigma_{T_z=-1/2}(n,\pi^0) = \sigma_{T_z=+1/2}(p,\pi^0), \tag{28}$$

One therefore can transform (26) into

$$[\sigma_{T_z=-1/2}(p,\pi^+)+\sigma_{T_z=+1/2}(p,\pi^+)] \quad (29)$$
$$+[\sigma_{T_z=-1/2}(p,\pi^-)+\sigma_{T_z=+1/2}(p,\pi^-)]$$
$$=2[\sigma_{T_z=-1/2}(p,\pi^0)+\sigma_{T_z=+1/2}(p,\pi^0)].$$

One can easily check that for the case of the simplest mirror "nuclei," the proton and the neutron, (29) changes into the well known formula

$$\tfrac{1}{2}\sigma(pp\to\pi^+)=\sigma(np\to\pi^0)$$
$$+\sigma(pp\to\pi^0)-\sigma(np\to\pi^+),$$

if we note that

$$\sigma_{T_z=+1/2}(p,\pi^-)=\sigma(pp\to\pi^-)=0,$$
$$\sigma(np\to\pi^+)=\sigma(np\to\pi^-).$$

In addition to Eq. (29), it is possible to establish several inequalities, as, for example,

$$\sigma_{T_z=+1/2}(p,\pi^0)\geqslant\tfrac{1}{2}\sigma_{T_z=+1/2}(p,\pi^-), \quad (30)$$
$$\sigma_{T_z=+1/2}(p,\pi^+)\geqslant\tfrac{1}{6}\sigma_{T_z=+1/2}(p,\pi^-),$$
$$\tfrac{1}{3}\sigma_{T_z=+1/2}(p,\pi^-)+\sigma_{T_z=+1/2}(p,\pi^+)$$
$$\geqslant\sigma_{T_z=+1/2}(p,\pi^0)\geqslant\tfrac{1}{3}\sigma_{T_z=+1/2}(p,\pi^-).$$

One can apply a similar analysis to nuclei with $T_N = 1$ forming an isobaric triplet. The result is

$$[\sigma_{T_z=+1}(p,\pi^+)+\sigma_{T_z=0}(p,\pi^+) \quad (31)$$
$$+\sigma_{T_z=-1}(p,\pi^+)]+[\sigma_{T_z=+1}(p,\pi^-)$$
$$+\sigma_{T_z=0}(p,\pi^-)+\sigma_{T_z=-1}(p,\pi^-)]$$
$$=2[\sigma_{T_z=+1}(p,\pi^0)+\sigma_{T_z=0}(p,\pi^0)$$
$$+\sigma_{T_z=-1}(p,\pi^0)].$$

Similarly one can obtain inequalities such as

$$\sigma_{T_z=+1}(p,\pi^+)\geqslant\tfrac{1}{10}\sigma_{T_z=+1}(p,\pi^-); \quad (32)$$
$$\sigma_{T_z=+1}(p,\pi^+)\geqslant\tfrac{1}{4}\sigma_{T_z=+1}(p,\pi^0);$$
$$\sigma_{T_z=+1}(p,\pi^+)+\sigma_{T_z=-1}(p,\pi^-)\geqslant\tfrac{2}{3}\sigma_{T_z=+1}(p,\pi^0)$$

and a number of others.

5. We shall now illustrate the characteristic peculiarities of the relation between the meson-creation cross sections in case of predominance of certain $T_{\pi N}$ in the meson–nucleon interactions, and choose as an example, a very simple nucleus, the deuteron. The final state in the case of meson production in a d–p collision can be split into subsystems considering the meson and nucleon as one, and the remaining two nucleons as the other, subsystem. The matrix elements for the production of mesons of different charge, M^+, M^-, and M^0, can be expressed in terms of three quantities R_t, N_t and N_s, which characterize the transitions in the different possible isotopic spin states of the subsystems. Here $R_t = 3\sqrt{2}\, r_t$ corresponds to transitions with $T_{\pi N} = 3/2$, $T_{NN} = 1$; for $T_{\pi N} = 1/2$, $N_t = 3\, n_t$ corresponds to transitions with $T_{NN} = 1$ and $N_s = \sqrt{3}\, n_s$ to transitions with $T_{NN} = 0$. The different matrix elements then are given by

$$M(pd\to\pi^-ppp)=r_t+2n_t, \quad (33)$$
$$M(pd\to\pi^+nnp)=-r_t+n_t+n_s,$$
$$M(pd\to\pi^+npn)=-r_t+n_t-n_s,$$
$$M(pd\to\pi^+pnn)=3r_t.$$

In the energy range where n_s and n_t are negligible with respect to r_t, the $T_{\pi N} = 3/2$ state predominates in the π–nucleon interaction. The large π^+– π^- production branching ratio then follows from Eq.(33):

$$d\sigma^+:d\sigma^-=11:1. \quad (34)$$

For the neutral mesons one then obtains [from (25)]:

$$d\sigma^0=\tfrac{6}{11}d\sigma^+, \quad (34')$$

This can also be obtained from expressions of the type (34) which have not been written down here.

The case of heavier nuclei with $T = 0$ is considerably more involved. Even for He^4 it is not possible to obtain equations of type (34); the complications arise from states with $T_{NN} = 2$.

It should be mentioned that relations of the type (34) have been derived earlier[19] under the assumption that the meson is created in collisions of the incoming particle with individual nucleons of the target nucleus. In that approximation, (34) is valid for all $T = 0$ nuclei.

The strong predominance of the $T_{\pi N} = 3/2$ state in the energy region 150 to 200 mev also shows up strongly in the interaction of mesons with nuclei such asthe deuteron. Considerations of similar nature to the foregoing show that the following relation holds between the charge exchange and non—charge exchange scattering cross sections:

$$2d\sigma(\pi^- d \to \pi^-) = 5d\sigma(\pi^- d \to \pi^0). \qquad (35)$$

The probability for charge exchange scattering istherefore much smaller than the probability for straight scattering, in marked contrast to π-nucleon scattering in the same energy region.

7. It has been deduced from experimental evidence on hyperons and K-particles[20, 21, 22] that the isotopic spin of the different particles is $T_\lambda = 1/2$; $T_\lambda = 0$; $T_\vartheta = 1/2$; $T_\tau = 1/2$. With this assignment it is possible to understand many properties ofthe "strange" particles. The large value of the K^-/K^+ ratio in K-production with nuclei can be explained in this way. Furthermore, the scheme of associated production of K and Y particles in nucleon—nucleon collisions removes the difficulty of the long half—life of the particles as compared withthe rather high production probability, etc.

The reactions of creation of new particles in π-nucleon collisions have several peculiarities as compared with creation in nucleon—nucleon collisions. In particular, it is possible to create a pair of K-particles without creating a hyperon ($\pi + N \to N + K + \widetilde{K}$). This reaction will occur at a kinetic energy of the meson greater than

$$W = (2m_K - m_\pi)c^2[1 + (2m_K + m_\pi)/2M_p] \cong 1400 \text{ mev}$$

($\widetilde{K}$ denotes the antiparticle of K; for example if $T_z^{\vartheta+} = 1/2$ $T_z^{\vartheta 0} = -1/2$, then $T_z^{\widetilde{\vartheta}-} = -1/2$, and $T_z^{\widetilde{\vartheta}0} = 1/2$).

The conservation laws of charge and projection of the isotopic spin allow justthe following reactions

$$\pi^+ + p \to p + K^+ + \widetilde{K}^0, \quad \pi^- + p \to n + K^+ + K^-, \quad \pi^- + p \to p + K^0 + K^-. \qquad (36)$$

No production of $\widetilde{K}^0$ particles is possible in π^-–p collisions since in a reaction like $\pi^- + p \to p + K^- + \widetilde{K}^0$, allowed by conservation of charge, T_z is not conserved. The cross sections of the allowed reactions can be expressed in the form

$$\begin{aligned} &\sigma(\pi^+ p \to pK^+\widetilde{K}^0) = |\alpha_3|^2 = \sigma_{3/2}(\pi N \to NK\widetilde{K}),\\ &\sigma_+(\pi^- p \to nK^+K^-) = {}^1/_3\{{}^2/_3|\alpha_3 - \alpha_1|^2 + 2|\beta_1|^2\},\\ &\sigma_+(\pi^- p \to pK^0K^-) = {}^1/_9|\alpha_3 + 2\alpha_1|^2, \end{aligned} \qquad (37)$$

Here α_1 and β_1 correspond to transitions with $T = 3/2$. The cross sections $\sigma_{1/2}$ for K-pair production in the $T = 1/2$ state are given by

$$\begin{aligned} \sigma_{1/2}(\pi N \to NK\widetilde{K}) &= {}^1/_2\{3[\sigma_+(\pi^- p \to K^+K^-) + \sigma_+(\pi^- p \to K^0K^-)] - \sigma_{3/2}(\pi N \to NK\widetilde{K})\}\\ &= {}^1/_2\{3\sigma(\pi^- p \to K\widetilde{K}) - \sigma(\pi^+ p \to K\widetilde{K})\}, \end{aligned} \qquad (38)$$

which has the same form as (4).

One sees from (37) that the probabilities of K^+ and K^- production have comparable magnitude. The direct dependence of the relations between the different cross sections on the isotopic spin also follows immediately from (37). If one considers, in addition to the reactions (36), K-pair creation induced by π^0 —mesons, one can easily derive relations between the different crosssections analogous to Heitler's relations for π—meson scattering.

8. In connection with the possibility of experiments in the bev range, it is of interest to consider thecreation andthe interactions of antinucleons. One of the rather low threshold processes is the nucleon pair creation in π– N collisions, i.e., the reaction $\pi + N \to N + N' + \widetilde{N}'$. Its threshold lies at a meson kinetic energy of

$$W = 4Mc^2\left(1 - \frac{m_\pi}{2M}\right)\left(1 + \frac{m_\pi}{4M}\right) \cong 3760 \text{ mev}.$$

Nucleon pairs can be produced in the following meson—proton collisions:

$$\begin{aligned} &\pi^+ + p \to p + p + \widetilde{n};\\ &\pi^- + p \to n + p + \widetilde{p};\ \pi^- + p \to n + n + \widetilde{n};\\ &\pi^0 + p \to p + p + \widetilde{p};\quad \pi^0 + p \to p + n + \widetilde{n}. \end{aligned}$$

Introducing crosssections for $\widetilde{n}$ and $\widetilde{p}$ creation, summed over nucleons, i.e., denoting

$$\sigma(\pi^- p \to \widetilde{p}) = \sigma(\pi^- p \to np\widetilde{p}) + \sigma(\pi^- p \to pn\widetilde{p})$$

etc., one can write for the relation between the crosssections

$$\sigma(\pi^- p \to \tilde{n}) + \sigma(\pi^- p \to \tilde{p}) + \sigma(\pi^+ p \to \tilde{n}) = 2[\sigma(\pi^0 p \to \tilde{p}) + \sigma(\pi^0 p \to \tilde{n})], \quad (39)$$

This equation allows one to obtain the cross section of antinucleon creation by neutral mesons from experimentally determined cross sections of antinucleon creation by charged mesons.

Similarly we have

$$\sigma_{1/2}(\pi N \to \tilde{N}) = {}^1/_2\{3[\sigma(\pi^- p \to \tilde{p}) + \sigma(\pi^- p \to \tilde{n})] - \sigma(\pi^+ p \to \tilde{n})\} = {}^1/_2\{3\sigma(\pi^- p \to \tilde{N}) - \sigma(\pi^+ p \to \tilde{N})\}. \quad (40)$$

If the $T = 3/2$ state predominates in the π–N interaction, one obtains

$$\sigma(\pi^+ p \to \tilde{n}) : \sigma(\pi^- p \to \tilde{p}) : \sigma(\pi^- p \to \tilde{n}) = 9:2:1.$$

On the other hand, if the $T = 1/2$ is the important one, we have

$$\sigma(\pi^- p \to \tilde{n}) + \sigma(\pi^- p \to \tilde{p}) = 2[\sigma(\pi^0 p \to \tilde{n}) + \sigma(\pi^0 p \to \tilde{p})]. \quad (41)$$

From (40) and (41) it follows immediately that

$$d\sigma(\pi^- p \to d + \tilde{p}) = 2d\sigma(\pi^0 p \to d + \tilde{n}) = 2d\sigma(\pi^0 n \to d + \tilde{p}) \quad (42)$$

in processes where the final state contains a deuteron, or also in the inverse process of the annihilation of antinucleons on deuterium in the emission of a single meson.

Relations between antinucleon–nucleon annihilation cross sections in the emission of two and three π–mesons have been derived earlier by Kobzarev and Shmushkevich.[23,24] For two–meson antiproton annihilation processes on deuterium one can obtain

$$2[\sigma(\tilde{p}d \to n\pi^+\pi^-) + \sigma(\tilde{p}d \to n\pi^-\pi^+)] = 4\sigma(\tilde{p}d \to n\pi^0\pi^0) + \sigma(\tilde{p}d \to p\pi^-\pi^0) + \sigma(\tilde{p}d \to p\pi^0\pi^-), \quad (43)$$

which for the total cross sections go over into

$$2\sigma(\tilde{p}d \to \pi^+\pi^-) = 4\sigma(\tilde{p}d \to \pi^0\pi^0) + \sigma(\tilde{p}d \to \pi^0\pi^-) \quad (43')$$

which coincide with the expressions for antinucleon–nucleon annihilation obtained by Kobzarev and Shmushkevich. For three-meson antiproton annihilation on deuterium one obtains

$$\sigma_+(\tilde{p}d \to \pi^+\pi^-\pi^-) = 2\sigma(\tilde{p}d \to \pi^0\pi^0\pi^0) + \sigma_+(\tilde{p}d \to \pi^0\pi^0\pi^-), \quad (44)$$

and for the total cross sections we do get Shmushkevichs expressions for three-meson nucleon–antinucleon annihilation. For annihilation cross sections involving π^+, π^-, π^0 –mesons, which do not appear in (44), one obtains the inequality

$$\sigma_+(\tilde{p}d \to \pi^+\pi^-\pi^0) \geqslant \sigma_+(\tilde{p}d \to \pi^0\pi^0\pi^-). \quad (45)$$

The cross sections of (44) and (45) are summed over π-mesons, for example

$$\sigma_+(\pi^+\pi^-\pi^-) = \sigma(\pi^+\pi^-\pi^-) + \sigma(\pi^-\pi^+\pi^-) + \sigma(\pi^-\pi^-\pi^+).$$

The requirement of isotopic invariance yields a number of inequalities for nucleon pair creation in N–N collisions, such as

$$3\sigma_+(pp \to \tilde{n}) \geqslant \sigma(pp \to \tilde{p}), \qquad 2[\sigma(np \to \tilde{p}) + \sigma(np \to \tilde{n})] \geqslant \sigma(pp \to \tilde{p}) + \sigma(pp \to \tilde{n}), \quad (46)$$

The cross sections σ_1 and σ_0 of pair creation in states of definite isotopic spin of the N–N system are given by

$$\sigma_1(NN \to \tilde{N}) = \sigma(pp \to \tilde{p}) + \sigma(pp \to \tilde{n}), \qquad \sigma_0(NN \to \tilde{N}) = 2[\sigma(np \to \tilde{n}) + \sigma(np \to \tilde{p})] - \sigma_1(NN \to \tilde{N}) \quad (47)$$

in complete analogy with relation (12).

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