Compton Effect in the Extended Electron

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IT is well known that the introduction of cutting-off form factors into the mathematical formalism of the present field theory results necessarily in the condition of the gauge invariance relations.[1] In this case the supplementary Lorentz condition $\partial A\mu/\partial x\mu = 0$ is no longer an integral of motion, and all four possible polarizations of the γ-quanta will contribute to the effective cross sections $d\sigma = d\sigma_{\perp} + d\sigma_{\parallel}$. However, if the contribution of the γ-quanta with longitudinal and scalar polarizations was small, i.e., if $d\sigma_{\parallel} << d\sigma_{\perp}$, then the theory could be considered as a first approximation of a more exact theory which would satisfy exactly the conditions for gauge invariance, and in which $d\sigma_{\parallel}$ would vanish. If, in the opposite case, the effective cross section $d\sigma_{\parallel}$ is comparable to $d\sigma_{\perp}$, then the possibility of emitting scalar and longitudinal γ-quanta is a basic result of the theory with a form factor. Let us consider these problems in the simple case of the Compton scattering.

The matrix element of the Compton effect on an extended electron will be written in the form:

$$\langle p_2 s_2;\ k_2 e_2 |S| p_1 s_1;\ k_1 e_1\rangle = \frac{ie^2}{(2\pi)^6}\frac{1}{2\sqrt{k_1 k_2}}\int d^4(x_1\ldots x_3')\,F(x_1x_2x_3)\,F(x_1'x_2'x_3')$$
$$\times\,\bar{u}(p_2)\,\gamma_\mu \bar{S}(x_3 - x_1')\,\gamma_\nu u(p_1)\,e^{-ip_2x_1 + ip_1x_3'}$$
$$\times\left\{e^{ik_1x_2' - ik_2x_2}\,e_1^\nu e_2^\mu + e^{ik_1x_2 - ik_2x_2'}\,e_1^\mu e_2^\nu\right\}.$$

Here p_1, s_1 and p_2, s_2 are the energy-momentum and spin vectors of the electron in the initial and final states: k_1, e_1 and k_2, e_2 are the energy-momentum and polarization vectors of the γ-quanta in the initial and final states. The theory of non-local interactions not being gauge invariant, one has to take into account all four possible and independent polarizations e/μ of the γ-quantum k_1, and all four possible and independent polarizations $e_{2\mu}$ of the γ-quantum k_2. If ξ_μ is the fraction of the γ-quanta with a polarization $e_{1\mu}$ in the scattered beam, then

$$d\sigma(p_2;\ k_2e_2\,|\,p_1;\ k_1) = \sum_{\mu=1}^{4}\xi_\mu d\sigma(p_2;\ k_2e_2\,|\,p_1;\ k_1e_{1\mu})\Big/\sum_{\nu=1}^{4}\xi_\nu.$$

In the particular case where all four independent polarizations have equal probabilities, i.e., when $\xi_1 = \xi_2 = \xi_3 = \xi_4$:

$$d\sigma(p_2;\ k_2\,|\,p_1;\ k_1) = \frac{r_0^2k_2^2 d\Omega}{4k_1^2}\left\{\frac{1}{k_1}\Phi_1^2\left[k_2 + m\left(1+\frac{m}{k_1}\right)\right] + \frac{1}{k_2}\Phi_2^2\left[k_1 - m\left(1-\frac{m}{k_2}\right)\right] + m\left(\frac{1}{k_1} - \frac{1}{k_2} - 2\frac{m}{k_1k_2}\right)\Phi_1\Phi_2\right\}, \tag{1}$$

$$\Phi_1 = \varphi(p_2;\ p_2 + k_2)\,\varphi(p_1 + k_1;\ p_1); \tag{2}$$
$$\Phi_2 = \varphi(p_2;\ p_2 - k_1)\,\varphi(p_2 - k_1;\ p_2);\quad \varphi(p;q) = (2\pi)^8\int F(x_1x_2x_3)\,e^{-ip(x_1-x_2)-iq(x_2-x_3)}\,d^4(x_1x_2x_3);$$

where θ is the angle between the vectors $\overline{k_1}$ and $\overline{k_2}$.

If the energy k_1 of the scattered γ-quantum and the angle θ are such that $k_1(1-\cos\theta) >> m$ then $k_1 >> k_2$ and $\phi_2 \approx 1$ for $\lambda \lesssim 10^{-12}$ cm. (It follows from the comparison with the experimental data that $\lambda < 1.2\times10^{-13}$; see below.) In this case

$$(d\sigma/d\sigma_{\mathrm{loc}})(p_2;\ k_2\,|\,p_1;\ k_1) \approx 1/2. \tag{3}$$

The right-hand side of (3) differs from ½ only in the case of small angles (for $k_1 \gtrsim 10^9$ ev; $\theta \lesssim 3$ or $5°$)

$$(d\sigma/d\sigma_{\mathrm{loc}})(p_2;\ k_2|p_1;k_1) = \Phi_1^2 \to 0;\ \vartheta \sim 0;\ k_1 \to \infty. \tag{4}$$

As the energy of the scattered γ-quantum increases, this region becomes smaller.

The effective cross section (1) accounts for the transverse, as well as for the longitudinal and scalar polarizations of the γ-quantum. It is interesting to consider also the effective cross section $d\sigma_{\perp}(p_2;\ k_2/p_1;\ k_1)$ for the case where the polarization of the γ-quanta is purely transverse.

If it happens that for $\xi_2 = \xi_4 = 0$ the difference $d\sigma_{\parallel} = d\sigma - d\sigma_{\perp} \ll d\sigma_{\perp}$, then the probability of emission of longitudinal and scalar γ-quanta, in the theory with non-local interaction, is negligibly small. We get by a standard method:

$$d\sigma_{\perp}(p_2; k_2 \mid p_1; k_1) = \frac{r_0^2 k_2^2}{2k_1^2 m} d\Omega \left\{ \frac{1}{k_1} \Phi_1^2 (mk_2 + k_1^2 \sin^2 \vartheta) + \frac{1}{k_2} \Phi_2^2 \left(mk_1 - \frac{1}{2} k_2^2 \sin^2 \vartheta \right) + \Phi_1 \Phi_2 \left(\frac{1}{2} k_2 - k_1 - m \right) \sin^2 \vartheta \right\}, \tag{5}$$

where $\phi_i = \phi_i(\theta)$ as informula (2).

The cross section (5) for Compton scattering differs considerably from the effective cross section got by Klein, Nishina and Tamm for energies $k_1 \sim \hbar\lambda/c$ and for small scattering angles. Comparing with (1), it follows that the γ-quanta with longitudinal and scalar polarizations contribute considerably to the cross section. As the energy k_1 of the scattered γ-quanta increases, the magnitude of the effective cross sections σ and $\sigma_{\perp}$ decreases rapidly.

Lawson[3] has measured the Compton scattering effective cross section for an energy $k_1 = 80$ mev of the γ-quanta. The experimental error is of 15% and the result agrees with the cross section computed by the Klein-Nishina-Tamm formula. It follows from these experiments that in any case $\lambda < 10^{-12}$ cm. More exact measurments are reported[4] for an energy $k_1 = 250$ mev; the experimental error is of 10%, and the result also agrres with the Klein-Nishina-Tamm calculation. It follows from Eqs. (1) and (5) that for $k_1 = 250$ mev and $\theta = 4°$ the magnitude of the constant λ cannot exceed 1.2×10^{-13} cm. This result is obtained for form-functions $\phi_1 = e^{-2\lambda^2 m k_1}$, $\phi_2 = e^{-2\lambda^2 m k_2}$, but it does not change appreciably when the form of these functions is varied.

To conclude, let us note that the study of the creation of electron-positron pairs by cosmic-rays electrons with energies of 0.1 to 10 bev, in photoemulsions, also yields results in agreement with the calculations performed by the known methods of the theory of local interaction.[5] For electrons with energies of 100 bev there are some indications of disagreement with theory. However, the large experimental errors prohibit the drawing of any definite conculsions.

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Relation between Neutron Scattering in Polycrystals and Specific Heat

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THE cross section for the inelastic coherent scattering of thermal neutrons in polycrystals, relative to a single nucleus and for an arbitrary frequency spectrum, is expressed in the following way, correct to terms of order $1/M$ (that is, neglecting multiple phonon processes and thermal factors):*

$$\sigma_{\pm}(E, \omega)\, d\omega = \frac{\sigma_0}{3M} \sqrt{1 \mp \hbar\omega/E}\, (2 \mp \hbar\omega/E) \times \frac{(\pm 1)}{1 - e^{\mp \hbar\omega/T}} \frac{E}{\hbar\omega} \nu(\omega)\, d\omega. \tag{1}$$

Here $\sigma_{\pm}(E, \omega)\, d\omega$ is the scattering cross section for a neutron with energy E, as a result of which a phonon with a frequency in the interval $d\omega$ is excited (or absorbed); σ_0 is the cross section for the scattering of a neutron by a single massive nucleus, M is the mass number of a nucleus of the crystal, T is the temperature (in units of energy and $\nu(\omega)$ is the frequency spectrum of the crystal, relative to a single nucleus.

The crystalline frequency spectrum entering into Eq. (1) is in turn linked in a well-defined manner with the lattice specific heat at constant volume, $C(T)$, by the relation