interactions and its inclusion is necessary. Besides this in the evaluation of the matrix elements M, the authors replaced the inverse operator $K_{\mu\nu}$ by a "nonrelativistic" approximation obtained by neglecting quantities of the type k/M_1 in comparison to 1. This is permissible in those cases if $\lambda/M_1 \ll 1$ but for satisfactory agreement between theory and experiment one must select $\lambda/M_1 \simeq 1$. Consequently, the "nonrelativistic" approximation as used by Kanazawa and Sugawara[6] is inapplicable.

In conclusion, I use this opportunity to express my gratitude to I. E. Tamm for suggesting this problem and for his continuing aid, and to Iu. A. Gol'fand, V. Ia. Fainberg and V. P. Silin for valuable discussions relating to this problem.

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Theory of Isothermal Galvanomagnetic and Thermomagnetic Effects in Semiconductors

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Isothermal galvanomagnetic and thermomagnetic effects in isotropic semiconductors are treated theoretically in the case of intermediate and strong magnetic fields.

ONE of the most effective methods of investigating the properties and parameters of semiconductors is the study of galvanomagnetic and thermomagnetic effects. A theory of these effects has been developed by a number of authors[1-7]. Most of the authors start with a quadratic dependence of the energy on the momentum, and with weakness of the magnetic field. Meanwhile, experiment has revealed many cases in which it is not legitimate to consider the effective magnetic field* φ small.

Even at room temperature, it is often necessary to deal with intermediate effective magnetic fields ($\varphi^2 \sim 1$), and sometimes even with strong ones ($\varphi^2 \gg 1$). Thus, for example, at $T = 300°$ K and $H = 10^4$ oe, $\varphi^2 \approx 1.5$ for HgSe, and $\varphi^2 = 36$ for InSb. At low temperatures we quite often deal with intermediate and strong effective magnetic fields. Davydov and Shmushkevich[3] obtained formulas for the Hall effect and for the change of electrical conductivity in the case $\varphi \gg 1$, and Madelung[6] considered the same phenomena in the case $\varphi \gtrsim 1$, but only for semiconductors with an atomic lattice.

The present work concerns the extension of the theory to the region of intermediate and strong magnetic fields, for various types of interaction of the carriers with the crystal lattice. We also determine which features of the galvanomagnetic and thermomagnetic effects depend on the statistics and on the scattering law. We consider only isothermal effects; for, as Tolpygo[5] showed, the adiabatic effects differ little in magnitude from the isothermal.

* By "effective magnetic field" we shall understand the dimensionless quantity $\varphi = uH/c$, which essentially determines the effect of the magnetic field H on the carriers of current in a semiconductor. Here u is the mobility of the carriers, and c is the speed of light.

1. SEMICONDUCTORS WITH CARRIERS OF A SINGLE TYPE

Transport Equations

The kinetic equation for the distribution function $f(\mathbf{r}, \mathbf{p})$ of the carriers, in momentum ($\mathbf{p}$) and coordinate ($\mathbf{r}$) space, has in the stationary case the well-known form

$$\mathbf{v}\nabla_{\mathbf{r}} f + \mathbf{F}\nabla_{\mathbf{p}} f = -(f - f_0)/\tau. \quad (1)$$

Here $\mathbf{v}$ is the velocity of a carrier, $\mathbf{F}$ is the external force acting on it, $f_0\left(\frac{\epsilon-\mu}{kT}\right)$ is the equilibrium distribution function, $\epsilon = \epsilon(p)$ is the energy as a function of the momentum, τ is the relaxation time, and μ is the chemical potential. In an isotropic semiconductor, the energy and the relaxation time are functions only of the magnitude p of the momentum.

A carrier in a magnetic field $\mathbf{H}$ and an electric field $\mathbf{E}$ is acted upon by the Lorentz force $\mathbf{F} = e(\mathbf{E} + [\mathbf{p}, \mathbf{H}]/mc)$, where e is the elementary charge, and m is the effective mass of the carrier. We shall, as usual, seek a solution of Eq. (1) in the form

$$f = f_0 + \mathbf{p}\chi(p, \mathbf{r}, T)/m. \qquad (2)$$

Upon substituting $\mathbf{F}$ and f in (1), we obtain, after a few transformations, the following equation for χ:

$$\chi + \frac{e\tau}{mc}[\mathbf{H}, \chi] \qquad (3)$$
$$= -\tau\left\{e\left(E - \frac{T}{e}\nabla_{\mathbf{r}}\frac{\mu}{T}\right) - \varepsilon\nabla\ln T\right\}\frac{df_0}{d\varepsilon} = \chi_0.$$

If we introduce the notation $\alpha = \mathbf{H}/H$, $\beta(p) = eH\tau(p)/mc$, the solution of (3) can be written in the form

$$\chi = \{\chi_0 + \beta[\chi_0, \alpha] + \beta^2\alpha(\chi_0, \alpha)\}/(1 + \beta^2). \quad (4)$$

The electric current density is

$$\mathbf{j} = e\int\frac{\mathbf{p}}{m^2}(\chi\mathbf{p})\frac{df_0}{d\varepsilon}d\mathbf{p} = \frac{4\pi}{3}e\int_0^\infty\frac{p^4}{m^2}\chi\frac{df_0}{d\varepsilon}dp. \quad (5a)$$

The density of the heat current transported by the electrons is

$$Q = \int\varepsilon\frac{\mathbf{p}}{m^2}\frac{df_0}{d\varepsilon}(\chi, p)\,d\mathbf{p} = \frac{4\pi}{3}\int_0^\infty\frac{p^4\varepsilon}{m^2}\frac{df_0}{d\varepsilon}\chi\,dp. \quad (5b)$$

Upon substituting (4) in (5a) and (5b), we get

$$\mathbf{j} = e^2J_{10}\widetilde{\mathbf{E}} - eJ_{11}\nabla\ln T + [e^2J_{20}\widetilde{\mathbf{E}} - eJ_{21}\nabla\ln T, \alpha] + \alpha(e^2J_{30}\widetilde{\mathbf{E}} - eJ_{31}\nabla\ln T, \alpha); \quad (6a)$$

$$\mathbf{Q} = eJ_{11}\widetilde{\mathbf{E}} - J_{12}\nabla\ln T + [eJ_{21}\widetilde{\mathbf{E}} - J_{22}\nabla\ln T, \alpha] + \alpha(eJ_{31}\widetilde{\mathbf{E}} - J_{32}\nabla\ln T, \alpha). \quad (6b)$$

Here

$$\widetilde{\mathbf{E}} = \mathbf{E} - (T/e)\nabla_{\mathbf{r}}(\mu/T),$$

$$J_{qr} = -\frac{4\pi}{3}\left(\frac{eH}{c}\right)^{q-1}\int_0^\infty\frac{p^4\tau^q}{m^{1+q}}\varepsilon^r\frac{df_0}{d\varepsilon}\frac{dp}{1+\beta^2}$$
$$= -\frac{4\pi}{3}\left(\frac{eH}{c}\right)^{q-1}\int_0^\infty\frac{p^3\tau^q}{m^q}\varepsilon^r\frac{df_0}{dp}\frac{dp}{1+\beta^2}$$

($q = 1, 2, 3$). The expansion of the integrals J_{qr} in powers of φ and $1/\varphi$ has the form:

$$J_{qr} = \frac{uN}{e}\sum_{l=0}^{i}(-1)^{l+1}a_{q+2l,r}\varphi^{q+2l-1}$$

for $\varphi^2 \ll 1$,

$$J_{qr} = \frac{uN}{e}\sum_{l=1}^{i}(-1)^{l-1}a_{q-2l,r}\varphi^{q-2l-1}$$

for $\varphi^2 \gg 1$.

$$a_{q\pm2l,r} = (J'_{q\pm2l,r}/J'_{00})(J'_{00}/J'_{10})^{q\pm2l};$$
$$J'_{q\pm2l,r} = -\frac{4\pi}{3}\times\int_0^\infty p^3\left(\frac{\tau}{m}\right)^{q\pm2l}\varepsilon^r\frac{df_0}{dp}dp;\quad u = eJ'_{10}/J'_{00}$$

(N is the carrier concentration). It can be shown that the following relations hold:

$$J'_{00} = -\frac{4\pi}{3}\int_0^\infty p^3\frac{df_0}{dp}dp$$
$$= 4\pi\int_0^\infty p^2f_0dp = N,\quad J_{1r} + J_{3r} = J'_{1r}.$$

The expansion of the integrals J_{qr} is, in general, of asymptotic type. More explicitly, from some term onward the coefficients in the expansion increase without limit; therefore, it is necessary to break off the expansion at a term preceding the smallest term of the series.

In practice, we limit ourselves to the terms proportional to φ (for $\varphi^2 \ll 1$) and to φ^{-1} (for $\varphi^2 \gg 1$). This means that for $\varphi = 1/5$ or $\varphi = 5$, for example, the error in our formulas will not exceed 4%. Actually, the accuracy of the calculation is even greater, since in the expansion of J_{qr} in each case we neglected one term in the denominator.

Galvanomagnetic Effects

Under the isothermal condition ($\nabla T = 0$)(6a) takes the form

$$\mathbf{j} = e^2 J_{10}\,\mathbf{E} + e^2 J_{20}\,[\mathbf{E},\alpha] + e^2 J_{30}\,(\mathbf{E}\alpha)\,\alpha. \tag{7}$$

We consider a specimen in the shape of a rectangular parallelepiped. Let the primary electric field be directed along the x axis; let the magnetic field lie in the xz plane. Then we may set $\alpha_x = \cos\theta$, $\alpha_y = 0$, $\alpha_z = \sin\theta$, where θ is the angle between the magnetic and the primary electric fields.

The conditions that there be no electric current in the y and z directions give two equations to determine the components E_y and E_z of the electric field:

$$E_y = \frac{E_x J'_{10} J_{20}\sin\theta}{J_{10}^2 + J_{20}^2\cos^2\theta + J_{10}J_{30}\sin^2\theta}, \tag{8}$$

$$E_z = \frac{E_x\,(J_{20}^2 - J_{10}J_{30})\sin 2\theta}{2(J_{10}^2 + J_{20}^2\cos^2\theta + J_{10}J_{30}\sin^2\theta)}.$$

The relative change of the electrical conductivity $\sigma = j_x/E_x$ in the magnetic field can be found by substituting (8) in the expression for the x component of current:

$$\Delta\sigma/\sigma_0 = (J_{10}J_{30} - J_{20}^2)\sin^2\theta/(J_{10}^2 + J_{20}^2\cos^2\theta + J_{10}J_{30}\sin^2\theta), \tag{9}$$

where $\Delta\sigma = \sigma_0 - \sigma$; σ_0 is the electrical conductivity of the semiconductor in the absence of a magnetic field. From (8) and (9) we get

$$E_z = -(\Delta\sigma/\sigma_0)E_x\,\mathrm{ctg}\,\theta. \tag{10}$$

Thus in the general case of arbitrary statistics, scattering law and dependence of the relaxation time on the momentum, the (Hall) field E_y changes its sign upon change of sign of the magnetic field ($\theta \to \theta + \pi$). If the magnetic field coincides in direction with the primary electric field, then the Hall field reduces to zero. The (longitudinal-transverse) field E_z does not change sign upon change of direction of the magnetic field. E_z reduces to zero in two cases: when H coincides in direction with E_x. The longitudinal-transverse effect was first treated by the authors.[8]

Since E_z and $\Delta\sigma/\sigma_0$ are mutually dependent, we shall henceforth give expressions only for $\Delta\sigma/\sigma_0$. From formulas (8) and (9) it is clear that all three of the effects considered have some anisotropy with respect to the direction of the magnetic field. On the basis of Schwarz's inequality it can be concluded that $J_{20}^2 - J_{10}J_{30} \leq 0$; consequently, $\sigma \leq \sigma_0$.

The integrals that occur in formulas (8) and (9) do not in general reduce to known functions; consequently, it does not seem possible to give a general description of the dependence of E_y and $\Delta\sigma/\sigma_0$ on the effective magnetic field, the carrier concentration, and the temperature. We therefore restrict ourselves to a consideration of limiting cases and of some special cases. To find the temperature dependence of the effects, it is necessary to assign a dependence of the distribution function, the relaxation time, and the energy upon the momentum and the temperature. We set*

$$\varepsilon(p) = p^2/2m;\quad f_0(p, T) = N\,(2\pi mkT)^{-3/2}e^{-p^2/2mkT};\quad \tau(p, T) = \Phi(T)\,p^{n-1}. \tag{11}$$

Upon substituting the expansions of the integrals J_{10}, J_{20} and J_{30} in (8) and (9), we get for $\varphi \ll 1$

$$E_y = a_n\varphi E_x\sin\theta,\quad \Delta\sigma/\sigma_0 = (b_n - a_n^2)\,\varphi^2\sin^2\theta. \tag{12}$$

When $\theta = \pi/2$, these formulas agree with the formulas derived by Tolpygo[5]. For $\varphi \gg 1$,

$$E_y = \frac{\sin\theta}{\cos^2\theta + a'_n\sin^2\theta}\,\varphi E_x; \tag{13}$$

$$\frac{\Delta\sigma}{\sigma_0} = \frac{(a'_n - 1)\sin^2\theta}{\cos^2\theta + a'_n\sin^2\theta};$$

$$a_n = 3\sqrt{\pi}/4\Gamma(n + 3/2)/4\Gamma^2(n/2 + 2);$$

$$b_n = 9\pi\Gamma(3n/2 + 1)/16\Gamma^3(n/2 + 2),$$

$$a_n = \frac{16}{9\pi}\Gamma\left(3 - \frac{n}{2}\right)\Gamma\left(\frac{n}{2} + 2\right),$$

$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx.$$

* The right members of formulas (11) may be regarded as the first terms of expansions in series of appropriate functions; therefore, all the results obtained may be considered approximations for more general cases.

The mobility is

$$u = (4e/3\sqrt{\pi}m)\,\Phi(T)\,(2kT/m)^{(n-1)/2}\,\Gamma(n/2+2),$$

where the form of $\Phi(T)$ is determined by the nature of the interaction between the carriers and the crystal lattice.

From formulas (12) and (13) it is clear that the Hall field, in both limiting cases, varies linearly with the effective magnetic field; the relative change of electrical conductivity increases in proportion to φ in weak magnetic fields but approaches saturation in strong fields. In the case of strong fields, the peculiar anisotropy of both effects, shown by formulas (14), should be noted.

Even with the assumptions (11), it still does not seem possible to reduce the general expressions for the galvanomagnetic and thermomagnetic effects to known functions. This calculation can, however, be carried out for a number of special cases: (1) $n = 0$ (this corresponds to a semiconductor with an atomic lattice); (2) $n = 1$ (this corresponds to an ionic lattice at temperatures below the characteristic temperature); (3) $n = 2$ (this corresponds, according to Bloch's scheme, to a semiconductor with an ionic lattice at temperatures above the characteristic temperature).

We give the exact formulas for E_y and $\Delta\sigma/\sigma_0$ in these special cases:

1) $n = 0$ (14)

$$E_y = \frac{J_2 \sin\theta}{(J_1^2 + J_2^2)\cos^2\theta + J_1 \sin^2\theta}\,E_x,$$

$$\frac{\Delta\sigma}{\sigma_0} = \frac{(J_1 - J_1^2 - J_2^2)\sin^2\theta}{(J_1^2 + J_2^2)\cos^2\theta + J_1 \sin^2\theta};$$

2) $n = 1$ (15)

$$E_y = \varphi E_x \sin\theta, \quad \Delta\sigma/\sigma_0 = 0;$$

3) $n = 2$ (16)

$$E_y = \frac{2\left(\frac{3\sqrt{\pi}}{4}t - t^2 J_2\right)\sin\theta}{[t^4 J_1^2 + (3\sqrt{\pi}\,t/4 - t^2 J_2)^2]\cos^2\theta + 2t^2 J_1 \sin^2\theta}\,E_x,$$

$$\frac{\Delta\sigma}{\sigma_0} = \frac{[2t^2 J_1 - t^4 J_1^2 - (3\sqrt{\pi}\,t/4 - t^2 J_2)^2]\sin^2\theta}{[t^4 J_1^2 + (3\sqrt{\pi}\,t/4 - t^2 J_2)^2]\cos^2\theta + 2t^2 J_1 \sin^2\theta}.$$

In (14)-(16) and below, the following notation is used:

$$J_1 = 1 - t^2 - t^4 e^{t^2}\mathrm{Ei}(-t^2); \quad J_2 = t\,[{}^1\!/_2 - t^2 + \sqrt{\pi}\,t^3 e^{t^2} F(t)];$$

$$F(t) = 1 - \mathrm{erf}(t); \quad \mathrm{erf}(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-x^2}dx; \quad \mathrm{Ei}(-t) = \int_t^\infty \frac{e^{-x}}{x}dx, \quad 0 < t < \infty.$$

For $n = 0$, $t = 3\sqrt{\pi}\,\varphi/4$; for $n = 2$, $t = 8/3\sqrt{\pi}\,\varphi$.

Thermomagnetic Effects

Let a temperature gradient exist along the x axis; let the magnetic field lie in the xz plane; and let the electric current in the specimen vanish. Then electric fields E'_y and E'_z appear in the directions of the y and z axes, and the thermoelectric field undergoes a certain increase E'_x. We will compute E'_x, E'_y and E'_z. From (6a) under the condition $\mathbf{j} = 0$ we get a system of equations for determination of the fields E'_x, E'_y and E'_z; the desired field E'_x is connected with the quantity $\tilde{E}'_x$ by the relation

$$E_x = \tilde{E}'_x - (T/e)\,\nabla_r(\mu/T) + (J'_{11}/eJ'_{10})\,\nabla \ln T.$$

By solving this system, we find

$$E'_x = -\frac{J_{10}(J_{10}J_{31} - J_{11}J_{30}) + J_{20}(J_{20}J_{11} - J_{21}J_{10}) + J_{20}(J_{31}J_{20} - J_{30}J_{21})}{eJ'_{10}(J_{10}^2 + J_{20}^2)}\sin^2\theta\,\frac{d\ln T}{dx},$$

$$E'_y = \frac{J_{11}J_{20} - J_{10}J_{21}}{e(J_{10}^2 + J_{20}^2)}\sin\theta\,\frac{d\ln T}{dx}, \quad E'_z = -E'_x \operatorname{ctg}\theta. \tag{17}$$

Thus the field E_y', which determines the transverse Nernst-Ettingshausen effect, changes sign with change of the sign of the magnetic field. The fields E_x' and E_z', which determine, respectively, the longitudinal Nernst-Ettingshausen effect and the longitudinal-transverse thermomagnetic effect, do not change sign with change of the sign of the magnetic field.

As the calculation shows, the anisotropy inherent in the galvanomagnetic effects does not appear in the thermomagnetic effects; in all further calculations, therefore, we will set $\theta = \pi/2$.

We consider the limiting cases of weak and strong effective magnetic fields. For $\varphi \ll 1$,

$$E_x' = (1-n)\left(b_n - \frac{a_n^2}{2}\right)\frac{k}{e}\varphi^2\frac{dT}{dx}, \tag{18}$$

$$E_y' = \frac{1-n}{2}a_n\frac{k}{e}\varphi\frac{dT}{dx}.$$

These formulas agree with those obtained in Ref. 5. For $\varphi \gg 1$,

$$E_x' = \frac{1-n}{2}\frac{k}{e}\frac{dT}{dx}, \quad E_y' = \frac{1-n}{2}a_n'\frac{k}{e}\frac{1}{\varphi}\frac{dT}{dx}. \tag{19}$$

Thus E_x' varies quadratically with the effective field φ in weak fields and approaches saturation in strong fields. E_y' increases linearly with φ in weak fields and is proportional to $1/\varphi$ in strong fields. Therefore, in the intermediate field range, i.e., for $\varphi \sim 1$, the function $E_y'(\varphi)$ must have at least one maximum.

We give the exact formulas for special cases:

1) $n = 0$

$$E_x' = -\frac{k}{e}\left\{t^2 - \frac{2J_1 + 3\sqrt{\pi}tJ_2/4}{J_1^2 + J_2^2}\right\}\frac{dT}{dx}, \tag{20}$$

$$E_y' = \frac{k}{e}\frac{2J_2 - 3\sqrt{\pi}tJ_1/4}{J_1^2 + J_2^2}\frac{dT}{dx};$$

2) $n = 1$

$$E_x' = E_y' = 0; \tag{21}$$

3) $n = 2$

$$E_x' = -\frac{k}{e}\left\{t^2 - \frac{2t^4J_1 + (15\sqrt{\pi}t/8)(3\sqrt{\pi}t/4 - t^2J_2)}{t^4J_1^2 + (3\sqrt{\pi}t/4 - t^2J_2)^2}\right\}\frac{dT}{dx}, \tag{22}$$

$$E_y' = \frac{k}{e}\frac{2t^2(3\sqrt{\pi}t/4 - t^2J_2) - 15\sqrt{\pi}t^3J_1/8}{t^4J_1^2 + (3\sqrt{\pi}t/4 - t^2J_2)^2}\frac{dT}{dx}.$$

Formulas (20)-(22) do in fact imply the presence of a maximum in $E_y'(\varphi)$ and of a region of saturation in $E_x'(\varphi)$. Graphs of the functions $E_x'(\varphi)$ and $E_y'(\varphi)$ are given in Figs. 1 and 2.

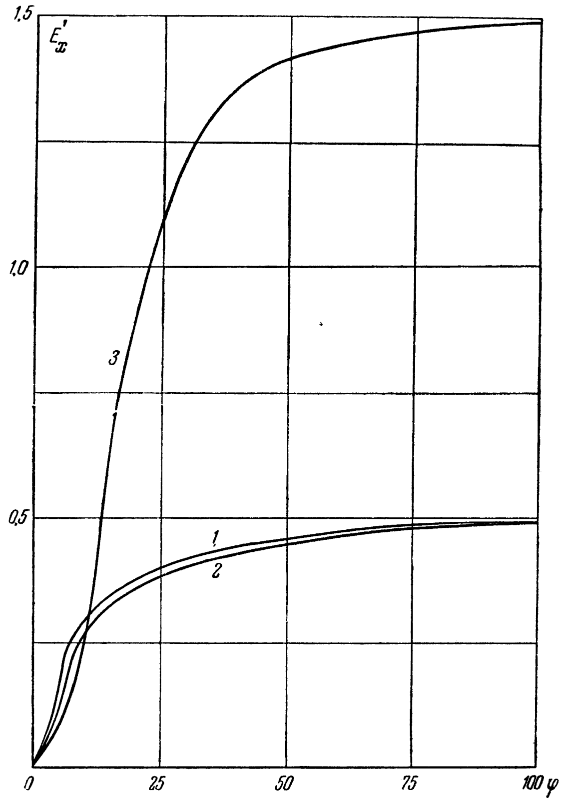

FIG. 1. Dependence of the dimensionless field of the longitudinal Nernst-Ettingshausen effect on the effective magnetic field. Curves *1*, *2*, *3* correspond to $n = 0, 2, 4$.

We consider the effect of a magnetic field on the electronic part of the heat conductivity. We set $E_y' = E_z' = 0$. Under isothermal conditions $\partial T/\partial y = \partial T/\partial z = 0$. Then from (6a) and (6b) we have

$$j_x = e^2J_{10}\tilde{E}_x' - eJ_{11}\frac{d\ln T}{dx}, \tag{23}$$

$$Q_x = eJ_{11}\tilde{E}_x' - J_{21}\frac{d\ln T}{dx}.$$

From the condition $j_x = 0$ we find $E_x' = (J_{11}/eJ_{10})d \times \ln T/dx$. The coefficient of heat conductivity due to the current carriers is

$$\lambda = -\frac{Q_x}{dT/dx} = \frac{J_{10}J_{12} - J_{11}^2}{TJ_{10}}. \tag{24}$$

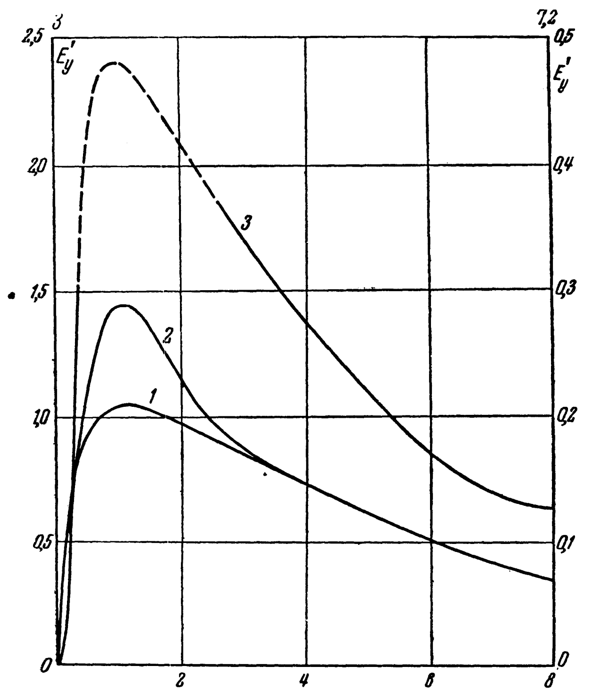

FIG. 2. Dependence of the dimensionless field of the transverse Nernst-Ettingshausen effect on the effective magnetic field. Curves *1, 2, 3* correspond to $n = 0, 2, 4$.

In the limiting cases of large and small φ we get for λ the following formulas: for $\varphi \ll 1$,

$$\lambda = (k^2T/e)\,uN\,\{2 + n/2 - b_n(n^2 - n/2 + 2)\,\varphi^2\}; \tag{25}$$

for $\varphi \gg 1$,

$$\lambda = (k^2T/e)\,uNa_n'\,(3 - n/2)\,\varphi^{-2}. \tag{26}$$

Formula (25) agrees with the corresponding formula of Ref. 5*.

From (25) and (26) it follows that the relative change of the coefficient of heat conductivity, $(\lambda_0 - \lambda)/\lambda_0$, is a quadratic function of φ in weak magnetic fields and approaches unity in strong fields.

In special cases the coefficient of the electronic part of the heat conductivity is determined by the expressions:

1) $n = 0$ (27)

$$\lambda = (k^2\,T/e)\,uN\,[(2t^2 + 6)\,J_1 - 4]\,/\,J_1,$$

2) $n = 1$ (28)

$$\lambda = (5k^2T\,/\,2e)\,uN\,/\,(1 + \varphi^2);$$

3) $n = 2$ (29)

$$\lambda = (k^2T\,/\,e)\,uN\,[(6 + 2t^2)\,J_1 - 4]\,/\,2J_1.$$

A graph of the relative change of heat conductivity as a function of the effective magnetic field is given in Fig. 3.

From formulas (18) and (19), E_x' and $E_y' > 0$ for $n < 1$; $E_x' = E_y' = 0$ for $n = 1$; and E_x' and $E_y' < 0$ for $n > 1$. It is the absolute values of E_x' and E_y' that are plotted in the graphs 1 and 2. It is evident from Fig. 2 that the maximum value of E_y' increases with increase of n. The graphs of the function $\Delta\lambda/\lambda_0(\varphi)$ for different n's differ from one another only in the region of intermediate effective fields (Fig. 3). The graphs for the case $n = 4$, which corresponds to scattering by impurity ions and to polar conductivity at high temperatures, were drawn on the basis of the asymptotic formulas.

2. SEMICONDUCTORS WITH MIXED CONDUCTION

Galvanomagnetic and thermomagnetic phenomena in semiconductors with mixed conduction have a number of peculiarities as compared with the same phenomena in semiconductors with current carriers of a single sign. The researches of one of us[10] have shown, for example, that the appearance of a few percent of minority carriers can have an appreciable influence on the character of the thermomagnetic effects.

In the case of mixed conduction, the electric or heat current is defined as the sum of the electron current and the hole current:

$$\mathrm{j} = \mathrm{j}_+ + \mathrm{j}_-, \quad \mathrm{Q} = \mathrm{Q}_+ + \mathrm{Q}_-. \tag{30}$$

Hereafter the plus sign will denote quantities pertaining to holes, the minus sign to quantities pertaining to electrons.

The calculation will be carried out on the assumptions (11)*, except that the following expression will be assumed for f_0:

* We remark that in Ref. 5 an error crept into formulas (34) and (34′) for the heat current: in the numerator of the fraction, $n^2 - n + 2$ should be $n^2 - n/2 + 2$. The same mistake was made by Avak'iants in Ref. 9.

* Original: (12).

$$f_0(p, T) = N(2\pi mkT)^{-3/2} \exp\left\{-\frac{1}{2kT}\left(w + \frac{p^2}{2m}\right)\right\},$$

where w is the width of the forbidden energy gap.

$m_{\mp}$ and $N_{\mp}$ are to be interpreted as the effective mass and concentration of the electrons or holes. It is also assumed that the scattering mechanism is the same for electrons and for holes, i.e., that $n_- = n_+ = n$.

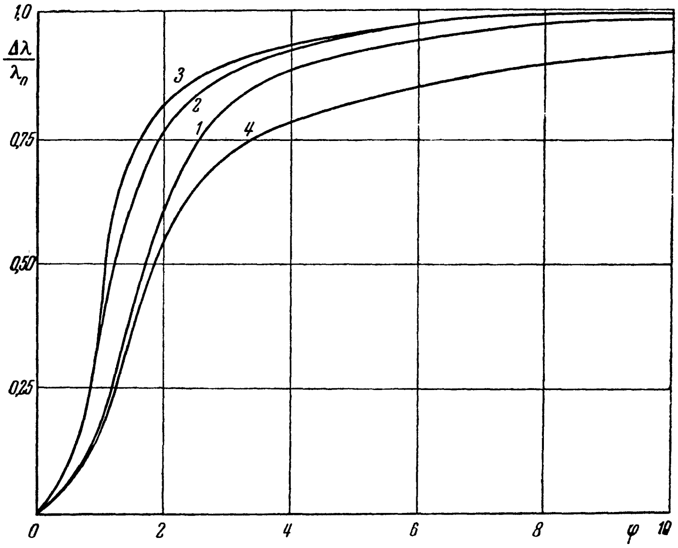

FIG. 3. Dependence of the relative change of the electronic part of the heat conductivity on the effective magnetic field. Curves *1, 2, 3, 4* correspond to $n = 0$, 1, 2, 4.

Galvanomagnetic Effects

The expression for the total electron and hole current retains the form (7), but the coefficient J_{qr} must be redefined as follows:

$$J_{10} = J_{10}^{+} + J_{10}^{-},\ J_{20} = J_{20}^{+} - J_{20}^{-},\ J_{30} = J_{30}^{+} + J_{30}^{-}. \qquad (31)$$

With the interpretation (31), formulas (8) and (10) may be carried over to the case of semiconductors with mixed conduction.

We will obtain expressions for the Hall field and for the relative change of electrical conductivity in the limiting cases of weak and strong effective magnetic fields. For $\varphi_{\pm} \ll 1$,

$$E_y = a_n \frac{N_+u_+\varphi_+ - N_-u_-\varphi_-}{N_+u_+ + N_-u_-} E_x \sin\theta, \qquad (32)$$

$$\frac{\sigma_0 - \sigma}{\sigma_0} = \frac{b_n(N_+u_+ + N_-u_-)(N_+u_+\varphi_+^2 + N_-u_-\varphi_-^2) - a_n^2(N_+u_+\varphi_+ - N_-u_-\varphi_-)^2}{(N_+u_+ + N_-u_-)^2} \sin^2\theta. \qquad (33)$$

For weak effective fields the Hall field, as in the case of semiconductors with carriers of one sign, depends linearly on $\varphi_{\pm}$; but it may change sign in its variation with the concentration and mobility ratios of the electrons and holes. The relative change of electrical conductivity is proportional to $\varphi_{\pm}^2$, as in the case of semiconductors with carriers of one sign; but for $n = 1$ it does not reduce to zero.

For $\varphi_{\pm} \gg 1$ we consider two cases:

$$\text{(a) } N_+ \neq N_-:\ E_y = \frac{(N_+ - N_-)(N_+\varphi_+ + N_-\varphi_-)}{(N_+ - N_-)^2\cos^2\theta + a_n'(N_+/u_+ + N_-/u_-)(N_+u_+ + N_-u_-)\sin^2\theta} E_x \sin\theta, \qquad (34)$$

$$\frac{\sigma_0-\sigma}{\sigma_0}=\frac{a_n'(N_+/u_+ + N_-/u_-)(N_+n_+ + N_-u_-)-(N_+-N_-)^2}{(N_+-N_-)^2\cos^2\theta + a_n'(N_+/u_+ + N_-/u_-)\sin^2\theta\,(N_+u_+ + N_-u_-)}\sin^2\theta. \tag{35}$$

In the case of strong effective fields, the expression for the electrical conductivity is of interest:

$$\sigma=\sigma_0\frac{(N_+-N_-)^2}{(N_+-N_-)^2\cos^2\theta + a_n'(N_+/u_+ + N_-/u_-)(N_+u_+ + N_-u_-)\sin^2\theta}. \tag{36}$$

For $\theta=\pi/2$ and $n=0$, 1 or 2, formulas (36) reduces to formulas obtained by Davydov and Shmushkevich[3].

(b) $N_+=N_-=N$: $$E_y=\frac{c_n}{a_n'}\frac{\varphi_+-\varphi_-}{\varphi_+\varphi_-\sin\theta}E_x, \tag{37}$$

$$\frac{\sigma_0-\sigma}{\sigma_0}=1,\quad \sigma=a_n'\frac{eN(u_++u_-)}{\varphi_+\varphi_-\sin^2\theta},$$

$$c_n=(4/3\sqrt{\pi})^3\,\Gamma(^7/_2-n)\,\Gamma^2(2+n/2).$$

From the formulas for strong effective fields it follows that the functions $E_y(\varphi_\pm)$ and $\sigma(\varphi_\pm)$ are different for different ratios between the concentrations of the electrons and of the holes. In the case $N_+\neq N_-$, $E_y(\varphi_\pm)$ increases with increase of the effective field; $\sigma(\varphi_+)$ approaches saturation. For $N_+=N_-$, $E_y(\varphi_\pm)$ and $\sigma(\varphi_\pm)$ decrease with increase of the effective field. Formulas (37) for E_y and σ are correct if θ differs considerably from zero. If θ is small, it is necessary in deriving the formulas to take account of terms of fourth order in $1/\varphi_\pm$. Then the functions $E_y(\varphi_\pm)$ and $\sigma(\varphi_\pm)$ will have the same character as in the case $N_+\neq N_-$. Formulas (37) imply a very unusual type of dedendence of the effects on the angle between the magnetic field and the primary electric field: upon decrease of the angle θ (provided θ differs considerably from zero), the Hall field and the electrical conductivity increase. The sign of the Hall effect in case (a) depends mainly on the concentration ratio of the electrons and holes; in case (b), on their mobility ratio.

In a conductor with mixed conduction, if the concentrations and the mobilities of the carriers are simultaneously equal, then $E_y=0$, and the electrical conductivity is $\sigma=a_n'\sigma_0/\varphi^2\sin^2\theta$ in the case of strong effective fields and $\sigma=\sigma_0(1-b_n\varphi^2\times\sin^2\theta)$ in the case of weak fields.

Thermomagnetic Effects

If in formulas (17) for E_x' and E_y' we make the substitutions

$$J_{11}={}^1/_2(J_{10}^+-J_{10}^-)w+J_{11}^+-J_{11}^-,$$
$$J_{21}={}^1/_2(J_{20}^++J_{20}^-)w+J_{21}^++J_{21}^-,$$
$$J_{31}={}^1/_2(J_{30}^+-J_{30}^-)w+J_{31}^+-J_{31}^-,$$

and also use (31), we get the general expressions for the longitudinal and transverse Nernst-Ettingshausen effects in semiconductors with mixed conduction. The relation between E_z' and E_x' stays the same even for semiconductors with mixed conduction. Therefore, only the expressions for E_x' and E_y' will be considered below. We consider the limiting cases of weak and strong effective fields, on the supposition that $\theta=\pi/2$:

For $\varphi_\pm\ll 1$,

$$\begin{aligned}E_x'=\frac{k}{e}\frac{1}{(N_+u_++N_-u_-)^3}\Bigg\{\Bigg\{(1-n)\Bigg\{\Big(b_n-\frac{a_n^2}{2}\Big)(N_+^3u_+^3\varphi_+^2-N_-^3u_-^3\varphi_-^2)\\+N_+N_-u_+u_-\Big[b_n(N_+u_+\varphi_+^2-N_-u_-\varphi_-^2)-\frac{a_n^2}{2}(N_-u_--N_+u_+)\varphi_+\varphi_-\Big]\Bigg\}\\-N_+N_-u_+u_-\Bigg\{\Big[(4+2n)b_n-\frac{3n+8}{2}a_n^2\Big](N_+u_+\varphi_+^2-N_-u_-\varphi_-^2)+(4+2n)b_n(N_+u_+\varphi_+^2\\-N_-u_-\varphi_-^2)-\frac{3n+8}{2}a_n^2(N_-u_--N_+u_+)\varphi_+\varphi\Bigg\}\frac{w}{kT}\Bigg\}\Bigg\}\frac{dT}{dx};\end{aligned} \tag{38}$$

$$E'_y = a_n \frac{k}{2e} \frac{1}{(N_+u_+ + N_-u_-)^2} \Big\{(1-n)(N_+^2u_+^2\varphi_+ + N_-^2u_-^2\varphi_-) - (3n+7)N_+N_-u_+u_-(\varphi_+ + \varphi_-)\left(1 + \frac{2}{3n+7}\frac{w}{kT}\right)\Big\}\frac{dT}{dx}; \quad (39)$$

For $\varphi_\pm \gg 1$: (a) $N_+ \neq N_-$:

$$E'_x = \frac{k}{2e} \frac{(1-n)(N_+^2u_+ + N_-^2u_-) + (n+9)N_+N_-(u_+ + u_-)\left(1 + \frac{2}{n+9}\frac{w}{kT}\right)}{(N_+ - N_-)(N_+u_+ + N_-u_-)}\frac{dT}{dx},$$

$$E'_y = a'_n \frac{k}{2e} \frac{1}{(N_+ - N_-)^2\varphi_+\varphi_-}\Big\{(1-n)(N_+^2\varphi_- + N_-^2\varphi_+) - (11-n)N_+N_-(\varphi_+ + \varphi_-)\left(1 + \frac{2}{11-n}\frac{w}{kT}\right)\Big\}\frac{dT}{dx}; \quad (40)$$

(b) $N_+ = N_- = N$:

$$E'_x = \frac{5k}{e}\frac{(c_n - a_n'^2)(u_+ - u_-)}{u_+ + u_-}\left(1 + \frac{w}{5kT}\right)\frac{dT}{dx}, \quad (41)$$

$$E'_y = -\frac{5k}{e}\frac{\varphi_+\varphi_-}{a'_n(\varphi_+ + \varphi_-)}\left(1 + \frac{w}{5kT}\right)\frac{dT}{dx}.$$

The formulas for the thermomagnetic effects show that for $\varphi_\pm \ll 1$, the longitudinal and transverse Nernst-Ettingshausen fields depend on the effective field in the same way as in semiconductors with carriers of one sign. In the case of strong effective fields the dependence of E'_x and E'_y on $\varphi_\pm$ is different for different ratios between the concentrations of the electrons and of the holes. When $N_+ \neq N_-$, the dependence remains of the same type as in semiconductors with carriers of one sign. If $N_+ = N_-$, however, the transverse Nernst-Ettingshausen field does not decrease, but increases, with increase of the effective field; the longitudinal Nernst-Ettingshausen field approaches saturation but the saturation value is different from the saturation value for $N_+ \neq N_-$.

Estimation indicates that formulas (37) and (41) are applicable only when the electron and hole concentrations are quite close to each other in magnitude. Thus if we put $\varphi = 5 \gg 1$ (by φ must be understood the larger of the quantities φ_+ and φ_-), the indicated formulas may be used when the concentration ratio is not less than 0.96.

Finally, we give formulas for the electronic part of the heat conductivity of semiconductors in the limiting cases of weak and strong effective fields. In view of the extreme unwieldiness of the general formulas for the case $\varphi_+ \ll 1$, we give a formula obtained on the supposition that $w/kT \gg 1$, by neglecting terms not containing w/kT or proportional to it, and keeping only terms proportional to $(w/kT)^2$:

$$\lambda \approx \frac{w^2}{eT}\frac{N_+N_-u_+u_-}{N_+u_+ + N_-u_-}\left\{1 - a_n\frac{N_+u_+\varphi_-^2 + N_-u_-\varphi_+^2}{N_+u_+ + N_-u_-}\right\}, \quad (42)$$

$$\lambda_0 = \frac{k^+T}{e}\Big\{\left(2 + \frac{n}{2}\right)(N_+u_+ + N_-u_-) + (4+n)^2\frac{N_+N_-u_+u_-}{N_+k_+ + \Lambda_-u_-}\left(1 + \frac{1}{4+n}\frac{w}{kT}\right)^2\Big\}, \quad (43)$$

λ_0 is the electronic part of the heat conductivity in the absence of a magnetic field. For $\varphi_+ \gg 1$,

$$\lambda = a'_n\frac{k^2T}{e}\Big\{\left(3 - \frac{n}{2}\right)\left(\frac{N_+u_+}{\varphi_+^2} + \frac{N_-u_-}{\varphi_-^2}\right) + \frac{N_+N_-u_+u_-(6-n)^2}{N_+u_+\varphi_+^2 + N_-u_-\varphi_-^2}\left(1 + \frac{1}{6-n}\frac{w}{kT}\right)^2\Big\}. \quad (44)$$

Thus different ratios between the electron and hole concentrations, in the case of strong fields, do not influence in an essential way the dependence of the electronic part of the heat conductivity on the effective field, as was true of the Nernst-Ettingshausen fields.

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Note added in proof: Recently one of the authors (F. G. Bass) has succeeded in showing that on the assumption of an arbitrary isotropic scattering law, an arbitrary form of the collision integral, and any statistics, the dependence of properties on magnetic field in the galvanomagnetic and thermomagnetic effects, in the limiting cases $\varphi^2 \gg 1$ and $\varphi^2 \ll 1$, is the same as that obtained in the present article under the assumptions of formulas (11).

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