where n is the index of refraction in the region, ψ is the angle of refraction, and $(\gamma_z - \gamma_x)$ is a parameter, effectively independent of λ, which characterizes the properties of the layer. The values of n were determined for the given compounds.

The parameter $(\gamma_z - \gamma_x)$ was determined according to Eq. (1) from the results of very many and very accurate measurements at $\lambda = 5460$ A, and this value was used in calculations for other values of λ. The results of the calculations according to Eqs. (1) and (2) are shown by dotted lines. The measurements of δ for other wavelengths are somewhat less accurate as a result of great experimental difficulties. As can be seen, the experiment agrees sufficiently well with theory. It should be noted that measurements in the neighborhood of 0 and π are somewhat less accurate.

The existence of a transition layer on the surface seems unquestionable; this idea has been accepted in physics and physical chemistry for a relatively long time, and as far as we know has met up with no particular objections. The core of the problem, it seems to us, is in establishing a molecular mechanism for the creation of such a layer.

Measurements have been performed in which the liquid being investigated has been carefully purified by chemical methods, multiple distillation and recrystallization, and further distillation in evacuated ampoules by Martin's method. This guaranteed the absence of impurities in the liquid and on its surface or of chemical reactions on the latter. The ellipticities obtained in this way differ from those measured with the proper precautions for an uncovered surface by a negligible amount — from 5 to 10 per cent. This shows that the existence of a surface layer is not caused by impurities, but by the surface structure.

We have obtained several data establishing the connection of the ellipticity and its temperature dependence with parameters that characterize the liquid and its structure, the course of the crystallization process, etc.; all this also supports the above assumption. Details of these measurements will be published separately.

[1] D. V. Sivukhin, J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 374 (1956); Soviet Phys. JETP 3, 269 (1956).

[2] V. A. Kizel', J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 658 (1955); Soviet Phys. JETP 2, 520 (1956).

[3] V. A. Kizel' and A. F. Stepanov, Proc. Lenin Centr. Asia State U., Phys., Vol. 2, Tashkent, (1956).

[4] D. V. Sivukhin, J. Exptl. Theoret. Phys. (U.S.S.R.) 18, 976 (1948); Dokl. Akad. Nauk SSSR 36, 247 (1942); Vestn. (Herald) Mosc. State Univ. 7, 63 (1952).

Translated by E. J. Saletan
105

A Theorem on the Equality of the Cross Sections for Photoproduction of Charged π-Mesons on Nuclei with Isotopic Spin Zero

V. L. POKROVSKII
West Siberian Branch of the Academy of Sciences, USSR

(Submitted to JETP editor June 12, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 537-538 (September, 1956)

THE interaction Hamiltonian H_{int} of nucleons and π-mesons with the electromagnetic field can be represented as the sum of a scalar S and the third component of a vector V_3 with respect to the group of rotations in isotopic spin space:[1]

$$H_{int} = S + V_3; \quad S = -\frac{ie}{2}\sum_{\nu} \mathbf{v}^{\nu}\mathbf{A}(x_{\nu}); \qquad (1)$$

$$V_3 = ie\left[\sum_{\nu} T_3^{\nu}\mathbf{v}^{\nu}\mathbf{A}(x_{\nu}) + \sum_{\mu} T_3^{\mu}\mathbf{v}^{\mu}\mathbf{A}(x_{\mu})\right].$$

Here $\mathbf{v}^{\nu}$ is the velocity operator, $T_3{}^{\nu}$ is the projection operator for the isotopic spin of the νth nucleon; ν^{μ}, $T_3{}^{\mu}$ are the same quantities for the μth meson.

From (1) we obtain[1] the conservation law for the projection of the isotopic spin of the system of nucleons and π-mesons on the 3-axis (charge conservation law) and the following selection rules for the total isotopic spin T of the system

$$\Delta T = 0, \pm 1. \qquad (2)$$

We note that the matrix elements of the operators which correspond to transitions in which the number of nucleons is conserved are invariant under the group P_n of permutations only of the isotopic spin variables of the nucleons. Therefore the matrix element of an operator differs from zero when its direct (Kronecker) product with the representation of the group P_n (according to which the wave functions of the initial and final states transform) contains the unit representation.

We shall show that for photoproduction of charged mesons on nuclei with isotopic spin

zero the matrix element $(\psi_f | S | \psi_i)$ vanishes. In fact, by assumption, the wave function ψ_i of the initial state transforms according to the representation of the group P_n to which corresponds Young's diagram in the form of two horizontal lines of equal length. Since a charged π-meson appears in the final state, the isotopic spin changes and with it the charge symmetry of the nucleus. The operator S is symmetric under arbitrary permutation of the charge variables. In agreement with the above, the matrix element $(\psi_f | S | \psi_i)$ vanishes.*

From the selection rule (2) and the usual rules for combining angular momenta, it follows that the isotopic spin of the nucleus in the final state may be either 1 or 2. Making use of the well-known expressions for the matrix elements of a vector (see for instance Landau and Lifshitz[2]), it can be shown that the photoproduction cross section for charged mesons of opposite signs for both possible values of the isotropic spin of the nucleus in the final state are given by

$$d\sigma_1(\pi^+) = d\sigma_1(\pi^-), \qquad d\sigma_2(\pi^+) = d\sigma_2(\pi^-). \quad (3)$$

Summing these cross sections over the possible final states, we obtain

$$d\sigma(\pi^+) = d\sigma(\pi^-).$$

The experimental values of $r = d\sigma(\pi^-)/d\sigma(\pi^+)$ for light nuclei (D, C^{12}, N^{14}, O^{16}) are, within the limits of experimental error, equal to unity.[3]

The decrease of r as the charge increases is evidently explained by the capture of negative π-mesons by nuclei.

*The matrix element $(\psi_f|S|\psi_i)$ vanishes identically if we limit ourselves to dipole interactions. Here, however, no such limitation is made.

[1] L. A. Radicati, Phys. Rev. 87, 521, 1952.

[2] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Part I, GTTI (1948) p. 111.

[3] R. M. Littauer add D. Walker, Phys. Rev. 82. 746 (1951); 86, 838 (1952).

Translated by E. J. Saletan
113

The Magnetic Field in the Two-dimensional Motion of a Conducting Turbulent Liquid

IA. B. ZIL'DOVICH
Institute of Chemical Physics
(Submitted to JETP editor March 30, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 154-155
(July, 1956)

THE problem of magnetic fields arising spontaneously in the motion of a liquid has been considered by Batchelor.[1] He came to the conclusion that the magnetic field increases without limit for sufficient conductivity in the given field. His conclusion was based on nonrigorous considerations of the analogy between the magnetic field and a velocity vortex.

In the present work, the particular case of two-dimensional motion is considered: $v_z = 0$, v_x and v_y depend only on x and y; the liquid is incompressible, div $\mathbf{v} = 0$. In this case, we have succeeded in treating the problem rigorously. The results differ essentially from the conclusions of Batchelor: In two-dimensional motion in the absence of external fields, the initial magnetic field can increase no more than a definite number of times, and thereafter certainly dies out. In the presence of external fields on the boundaries of the region of motion, the fields in the moving liquid in the stationary state are proportional to the external fields. In the absence of mean regular flow the turbulently moving, conducting liquid behaves as a diamagnet with permeability μ inversely proportional to the intensity of the turbulent mixing.

Following Batchelor, we set up the equation in the quasi-stationary approximation, neglecting the displacement current and the density of free charges. We employ $c = 1$ and the Heaviside system (without 4π), φ = scalar potential, $\mathbf{A}$ = vector potential, div $\mathbf{A} = 0$, $\mathbf{J}$ = current, div $\mathbf{J} = 0$, the specific resistance of the liquid is r.

The equations have the form:

$$r\mathbf{j} = \mathbf{E} + \mathbf{v}\times\mathbf{H}; \quad \mathbf{H} = \operatorname{curl}\mathbf{A}; \quad \mathbf{E} = (\partial\mathbf{A}/\partial t) - \nabla\varphi; \qquad (1)$$

$$\mathbf{J} = \operatorname{curl}\nabla^2\mathbf{A}.$$

It follows from this equation that

$$(\partial\mathbf{A}/\partial t) + \mathbf{v}\times\operatorname{curl}\mathbf{A} = r\nabla^2\mathbf{A} + \nabla\varphi. \qquad (2)$$

Taking the curl of (2), we obtain an equation which reduces to Batchelor's: