$$t = E^{-1}\sqrt{2UM/Ze}. \quad (2)$$

Thus the time of a complete cycle with retarding fields $E_1$ and $E_2$ is

$$T = l\sqrt{2M/UZe} + 2\,(1/E_1+1/E_2)\sqrt{2UM/Ze}. \quad (3)$$

The condition $\partial T/\partial U = 0$ gives

$$1/E_1 + 1/E_2 = l/2U_0. \quad (4)$$

If the magnitudes of $U_0$, $l$, $E_1$ and $E_2$ are chosen to satisfy the relation (4) we shall achieve first-order space-time grouping (focusing) of ions of different energies. In that event, the time of a complete cycle will be

$$T_0 = 2l\sqrt{2M/U_0Ze}. \quad (5)$$

However, this will not satisfy the condition for second-order focusing

$$\partial^2 T/\partial U^2 = l\sqrt{M/2ZeU_0^5}$$

and the spread in time of a cycle for ions of identical mass but energy spread $\Delta U$ is

$$\Delta T_U \approx (l/2)\sqrt{M/2ZeU_0^5}\,(\Delta U)^2.$$

The difference in the period of a cycle for ions of mass difference $\Delta M$ but identical energy is

$$\Delta T_M \approx l\sqrt{2/ZeMU_0}\,\Delta M.$$

By equating these quantities we obtain a formula for the limit of resolution which is determined by the energy spread of the ions:

$$\Delta M/M = {}^1/_4\,(\Delta U/U_0)^2. \quad (6)$$

The precision of the measurements is determined by the duration of a pulse or its linear size. Thus the ratio of the duration of an ion pulse to its transit time must be smaller than the resolution limit. Since in the error formula $\Delta M/M = 2\Delta T/T$, the magnitude of $\Delta M/M$ is given, whereas $\Delta T$ is limited by the experimental possibilities, for the purpose of improving accuracy, it is necessary either to lengthen the tube (which is impracticable) or to cause the ions to complete the number of cycles which is required by the formula

$$N = (2M/\Delta M)\,\Delta T/T. \quad (7)$$

On the assumption that the retarding fields and the accelerating potential are supplied by the same source of power the stability of the latter is given by $\Delta V/V = \Delta M/M$. The strength of the longitudinal magnetic field is set at a value such that increase in the sensitivity of the instrument will not diminish its resolving power, that is, it will focus ions whose transverse energy component is below the limit given by

$$H \sim R^{-1}\sqrt{(2M/Ze)\,\Delta U}, \quad (8)$$

where $R$ is the radius of the tube.

Thus ions which are scattered by the residual gas will be lost in the walls at scattering angles which are greater than the permissible values. If, for example, the parameters are as follows: $U_0 = 100$ v, $\Delta U = 0.2$ v, $l = 50$ cm, $M = 100$, $\Delta T = 0.05\,\mu$sec, we obtain $T_0 = 145\,\mu$sec, $\Delta M/M = 10^{-6}$ and $N = 690$. At a pressure of $1 - 5\times10^{-6}$ mm Hg, the number of ions which reach the detector is a few percent of the initial number.

* Of course it would be possible to employ a field with a quadratic potential distribution so that the oscillation period of the ions would be absolutely independent of their energies, but there are great experimental difficulties involved in the generation of such a field.

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99

# Electron Broadening of Spectral Lines

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THERE is considerable experimental evidence of the fact that collisions with electrons play an important part in the broadening of atomic spectral lines in a plasma[1-5]. Moreover, there is every reason to believe that a number of lines of metals in stellar atmospheres are broadened due to collisions with electrons[6]. At the same time the existing theory of the broadening of spectral lines does not apply to electrons, because its basic assumption that the relative motion of the atom and of the perturbing particle is quasiclassical is correct only for heavy particles[7].

The shape of a line that is broadened due to the interaction of an atom with the particles surrounding it (in this case with electrons) is obtained from the integral

$$A = \int \prod_i \psi^*_{n''i}\psi_{n'i} d\mathbf{r}_i, \qquad \text{where}$$

$\psi_{n'i}$ and $\psi_{n''i}$ are the wave functions of the relative motion of the atom and electrons for the initial and final states of the atom, respectively.

The calculation of $A$ is greatly simplified if broadening by electrons is regarded as being of impact character[7] (in the terminology of classical theory). In this case the instants of time during which an electron is close to the atom are disregarded, since it is possible to neglect radiation during the actual collision. For integration in the intervals between collisions, when the distance to the nearest electron is large, simple asymptotic expressions can be used for the functions $\psi_{ni}$. The entire calculation can be carried through to the end in this general form. This is extremely important. A number of articles[8-10] on electron broadening have recently appeared. Their authors attempt to solve the problem without making any assumptions in advance regarding the mechanism of the broadening. Because of the difficulties of the calculation they confine themselves to the Born approximation, which is of doubtful validity in this case, as collisions with relatively slow electrons, whose energy is 0.5 to 1 ev, is of the greatest practical interest.

As will be shown below, the general equations which we derive give the results obtained in Refs. 8-10 for the Born approximation.

Our calculations showed that electron impact broadening is of dispersive character; for the line width $\gamma$ and shift $\Delta$ we obtain:

$$\gamma = 2Nv\sigma_r, \tag{1}$$

$$\sigma_r = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\{1-\cos 2(\eta_l' - \eta_l'')\};$$

$$\Delta = Nv\sigma_i, \quad \sigma_i = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin 2(\eta_l' - \eta_l''), \tag{2}$$

where $k = \mu v/\hbar$, $\hbar\sqrt{l(l+1)}$ is the angular momentum; $\eta_l'$ and $\eta_l''$ are the quantum-mechanical scattering phases for the initial and final states of the atom.

In the quasi-classical approximation Eqs. (1) and (2) go over into the corresponding formulas of classical theory. Indeed (see Ref. 11), in scattering by the field $\hbar D_n/R^n$

$$\eta_l = -\frac{\mu D_n k^{n-2}}{2\hbar l^{n-1}} \frac{\Gamma(1/2)\Gamma((n-1)/2)}{\Gamma(n/2)}.$$

After substituting $\hbar k = \mu v$ and $k\rho \approx l$ we obtain

$$2(\eta_l' - \eta_l'') = -\frac{D_n' - D_n''}{v\rho^{n-1}}\sqrt{\pi}\frac{\Gamma((n-1)/2)}{\Gamma(n/2)} = \eta(\rho), \tag{3}$$

where $\eta(\rho)$ is the phase shift of the atomic oscillator in Weisskopf's theory[7]. Using (3) and replacing the summation over $l$ by integration over $\rho$ we easily obtain

$$\sigma_r = 2\pi\int_0^{\infty}\{1-\cos\eta(\rho)\}\rho d\rho; \tag{4}$$

$$\sigma_i = 2\pi\int_0^{\infty}\sin\eta(\rho)\rho d\rho.$$

Estimates show that in some cases (3) and (4) are a quite good approximation. In the transition to the Born approximation we express the cross section $\sigma_r$ in terms of the scattering amplitudes:

$$\sigma_r = \pi\int_0^{\pi}|f'(\theta) - f''(\theta)|^2 \sin\theta\, d\theta,$$

$$f(\theta) = \frac{1}{2ik}\sum(2l+1)\{e^{2i\eta_l} - 1\}P_l(\cos\theta).$$

Considering that in the Born approximation

$$f(\theta) = \frac{\mu e^2}{2\hbar^2}\frac{(Z - F(\theta))}{\sin^2\theta/2}$$

and introducing the notation $x = 4a_0^2k^2\sin^2(\theta/2)$ and $a_0 = \hbar^2/\mu e^2$, we obtain for the broadening cross section

$$\sigma_r = \frac{2\pi}{k^2}\int_0^{4a_0^2k^2} x^{-2}(F'' - F')^2\, dx. \tag{5}$$

Here $F'$ and $F''$ are form factors for the initial and final atomic states. Equation (5) agrees with

Eqs. (22) and (23) of Ref. 8, thus establishing the relationship with the results obtained in Refs. 8-10. We note in conclusion that when inelastic collisions are taken into consideration, Eqs. (1) and (2) are replaced by

$$\sigma_r = \frac{\pi}{k^2} \sum (2l+1) \{1 - e^{2(\beta_l' + \beta_l'')} \cos 2(\eta_l' - \eta_l'')\}, \quad (6)$$

$$\sigma_i = \frac{\pi}{k^2} \sum (2l+1) e^{2(\beta_l' + \beta_l'')} \sin 2(\eta_l' - \eta_l''). \quad (7)$$

Here $\eta_l + i\beta_l$ are complex scattering phases.


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100

## Obtaining Polarized Electron Beams


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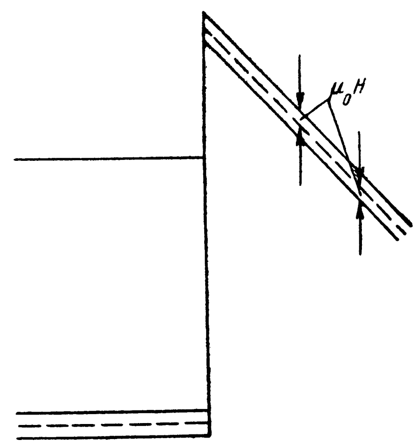
IF one can arrange for a sufficiently intense beam of polarized electrons it is possible to carry out a series of experiments with electrons as well as with circularly polarized bremsstrahlung[1]. Some methods of obtaining polarized electrons are well known (scattering of electrons by heavy nuclei[2], the photoelectric effect on aligned atoms[3], etc.). However, the values of the polarization and intensity obtained therewith are small; these methods are particularly ineffective in the high-energy region. Below is shown the principle of operation of a source of polarized electrons, which apparently has not previously been described in the literature.

We consider field emission from a cathode, to whose surface (along the normal) is applied a strong electric field $E$ and a magnetic field $H$. The behavior of the potential near the surface of the metal (see Figure), owing to the presence of the




The dotted line shows the behavior of the potential with the magnetic field absent.

magnetic field, is shifted by the value $\pm\mu_0 H$ for each of the two electron groups with opposite spin orientations. But in the equilibrium state, the Fermi energies of both groups are equal, and consequently, the penetrability of the barrier differs for conduction electrons with different spin orientations. An elementary value for the ratio $n_-/n_+$ for the emitted electrons is given by (see, for example Ref. 4)

$$n_-/n_+ \sim \exp(2\sqrt{2\varphi/mc^2}H/E),$$

where $\varphi$ is the work function. For $\varphi \sim 1$ ev (coated cathode) one needs an electric field strength of the order of $E \sim 10^6$ v/cm, and an observable polarization $\zeta = (n_+ - n_-)/(n_+ + n_-)$ is obtained, if the magnetic field is hundreds of kilogauss. Obviously, our derivation refers to the case $kT \lesssim \mu_0 H$, i.e., the cathode must be at the temperature of liquid hydrogen.

In this manner we are able to create a source of polarized electrons depending essentially on the