can all be derived as special cases ofthe relations given above. It is to be noted that the first equation of Kolmogoroff-Feller permits the derivation of the complete system of equations for the desired probability densities. Comparison of the two equations permits determining the symmetry properties of the function $f_{v_1}^{v_2}(1;2)$ with respect to an interchange of indices, from which follows the general formulation of the principle of optical reversibility.

The probability density $f_{v_1}^{v_2}(1;2)$ considered here is closely connected, of course, with the transmission and reflection functions of V. A. Ambartsumian and with the probability of emergence of a photon employed by V. V. Sobolev. The authors hope to take up these problems in detail.

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The Lagrangian Function for a System of Identically Charged Particles

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DARWIN[1] has shown that it is possible to write the Lagrangian function for a system of charged particles, correct to the second order terms in the ratio of the velocity of the particle to the velocity of light. This is possible because the radiation of light is a third order effect in v/c and does not enter inthe second order approximation.

It is of interest to point out the possibility of obtaining the Lagrangian function for a system of identically charged particles to a higher order of approximation. It is well known that in a system of identical particles (with precisely the same ratio of charge to mass) the radiation is proportional to the fifth power of v/c and not to the third power. Therefore the Lagrangian function for such a system can be written to the term v^4/c^4. It is easiest to use the method given in the book of Landau and Lifshitz[2] for its calculation.

It is not difficult to show that the third order terms in the Lagrangian function go to zero. A calculation of the fourth order terms leads to the following expression, which must be added to the second order Lagrangian function.

$$L^{(4)} = -\sum_a \frac{m_a v_a^6}{16c^4} + \frac{e^2}{8c^4}\sum_{b>a}\frac{1}{R_{ab}}\{2(\mathbf{v}_a\mathbf{v}_b)^2 \quad (1)$$

$$- v_a^2 v_b^2 + (\mathbf{n}\mathbf{v}_a)^2 v_b^2 + (\mathbf{n}\mathbf{v}_b)^2 v_a^2 - 3(\mathbf{n}\mathbf{v}_a)^2(\mathbf{n}\mathbf{v}_b)^2\}$$

$$+ \frac{e^2}{8c^4}\sum_{b>a}\{2(\mathbf{n}\mathbf{v}_a)(\mathbf{v}_a\dot{\mathbf{v}}_a) - 2(\mathbf{n}\mathbf{v}_b)(\mathbf{v}_b\dot{\mathbf{v}}_a) - v_a^2(\mathbf{n}\dot{\mathbf{v}}_b)$$

$$+ v_b^2(\mathbf{n}\dot{\mathbf{v}}_a) + (\mathbf{n}\mathbf{v}_a)^2(\mathbf{n}\dot{\mathbf{v}}_b) - (\mathbf{n}\mathbf{v}_b)^2(\mathbf{n}\dot{\mathbf{v}}_a)$$

$$- 3R_{ab}(\dot{\mathbf{v}}_a\dot{\mathbf{v}}_b) + R_{ab}(\mathbf{n}\dot{\mathbf{v}}_b)(\mathbf{n}\dot{\mathbf{v}}_a)\},$$

where $\mathbf{n}$ is a unit vector in the Direction $\mathbf{R}_{ab}$. Of course in making calculations the terms that contain the total derivative with respect totime are dropped.

The accelerations can be expressed here through the coordinates and velocities of the charges, consistent with the equations of motion, obtained by completely neglecting the retarded potentials, that is, from the Lagrangian function of zero approximation. Thus in the simplest case of two charges we have

$$\dot{\mathbf{v}}_1 = (e^2/m)\,\mathbf{n}/R^2; \qquad \dot{\mathbf{v}}_2 = -(e^2/m)\,\mathbf{n}/R^2,$$

where $\mathbf{R}_{21} = -\mathbf{R}_{12} = \mathbf{R}$ and $\mathbf{R}/R = \mathbf{n}$; after substituting in (1) we obtain

$$L^{(4)} = -\frac{mv_1^6}{16c^4} - \frac{mv_2^6}{16c^4} \quad (2)$$

$$+ \frac{e^2}{8c^4}\left\{\frac{1}{R}[2(\mathbf{v}_1\mathbf{v}_2)^2 - v_1^2v_2^2 + (\mathbf{n}\mathbf{v}_1)^2v_2^2 + (\mathbf{n}\mathbf{v}_2)^2v_1^2\right.$$

$$- 3(\mathbf{n}\mathbf{v}_1)^2(\mathbf{n}\mathbf{v}_2)^2] + \frac{3e^2}{m}[(\mathbf{n}\mathbf{v}_1)^3 + (\mathbf{n}\mathbf{v}_2)^2]$$

$$\left. - \frac{e^2}{m}(v_1^2 + v_2^2) + \frac{2e^4}{m^2R^3}\right\}.$$

The Lagrangian function of two identical charges with accuracy to the fourth order can be used for investigating the relativistic corrections inthe scattering of high speed protons, and also for generalizing the well-known formula of Breit forthe interaction of electrons (see Refs. 3,4). The calculation of the formula of Breit to fourth order was carried out by Maksimov; the results however are very lengthy, and we will not include them here.

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Charge Renormalization for an Arbitrary, Not Necessarily Small, Value of e_0

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RECENTLY Taylor[1], combining the results of Landau, Abrikosov and Khalatnikov[2] with the general theory of Gell-Mann and Low[3], attempted to prove in general that the only case in which the present electrodynamics does not lead to a contradiction is that with the renormalized charge e_c equal to zero (since otherwise the "seed" charge e_0 turns out to be imaginary and the interaction operator is non-Hermitian). In accordance with Ref. 3, Taylor writes the quantity $e_c^2 d_c(e_c^2, \xi)$ [d_c is the renormalized propagation function of the photon, and $\xi = \ln(-k^2/m^2)$] in the form of some function of only one variable*:

$$e_c^2 d_c = \Phi(\lambda_c),\ \lambda_c(\xi) = e_c^2 \Big/ \left[1 - \frac{e_c^2}{3\pi}\xi - e_c^2 f_c(e_c^2)\right], \quad (1)$$

where $f_c(e_c^2)$ is some unknown function. Comparing Eq. (1) with the results of Ref. 2, Taylor shows that $\lim_{y \to 0} y f_c(y) = 0$, and

$$\Phi(\lambda_c) \approx \lambda_c, \quad \text{if} \quad \lambda_c \to +0. \quad (2)$$

With $\xi = L$, $L = \ln(\Lambda^2/m^2)$, where Λ is the cut-off limit for momentum, Eq. (1) determines the charge renormalization

$$e_0^2 = e_0^2 d(L) = e_c^2 d_c(L) = \Phi[\lambda_c(L)]. \quad (1')$$

If in Eq. (1) e_c^2 is regarded as an arbitrary fixed quantity, then for $L \to \infty$, $\lambda_c \to -3\pi/L$. Taylor obtains the result stated above by assuming that (2) holds also for $\lambda_c \to -0$:

$$\Phi(\lambda_c) \approx \lambda_c,\ \lambda_c \to -0, \quad (2')$$

i.e., that $\lambda_c \Phi(\lambda_c)$ is a function of λ_c continuous at zero. Indeed, for $L \to \infty$, $\lambda_c \approx -3\pi/L \to -0$, it follows from (1′) and (2′) that

$$e_0^2 \approx -3\pi L, \quad (3),$$

i.e., e_0 turns out to be an imaginary quantity.

Unfortunately, it is so far quite impossible to find any basis for the assumption on the continuity of the function $\lambda_c \Phi(\lambda_c)$ at the point $\lambda_c = 0$, so that Taylor's whole proof remains without foundation. Even the reverse appears more probable: that the function $\Phi(\lambda)$ has an essential singularity at $\lambda = 0$, for example, of the type $\exp(1/\lambda)$; this would correspond to the fact that all expansions in powers of e_c^2 are apparently asymptotic series. In any case one can display many functions $\Phi(\lambda)$ for which the condition (2) is fulfilled (i.e., the relation (1) of Gell-Mann and Low goes over for $e_c^2 \to 0$ into the formula $d_c = [1 - (e_c^2 \xi/3\pi)]^{-1}$ of Landau, Abrikosov and Khalatnikov), but (2′) and consequently also (3), are invalid**.

If, however, we consider all quantities before renormalization and confine ourselves to a simpler problem than the one attacked by Taylor, the whole discussion can be carried through quite rigorously. In fact, we assume that the "seed" coupling constant e_0^2 is an arbitrary fixed quantity. Moreover, $e_c^2 = e_c^2(e_0^2, L)$. We shall show that under these conditions $e_c^2 \to 0$ for $L \to \infty$ not only for $e_0^2 \ll 1$, but for arbitrary $e_0 \gtrsim 1$.

Indeed, according to Gell-Mann and Low[3], we have***

$$e_0^2 d(e_0^2, L - \xi) = \Phi(\lambda_0),\ \lambda_0(\xi)$$
$$= e_0^2 \Big/ \left[1 + \frac{e_0^2}{3\pi}(L - \xi) - e_0^2 f_0(e_0^2)\right], \quad (4)$$

where $\Phi(\lambda)$ is the same function as in (1), and $f_0(e_0^2)$ is some unknown function. For $e_0 \to 0$ this equation goes over into the relation

$$d = \left[1 + \frac{e_0^2}{3\pi}(L - \xi)\right]^{-1} \quad (5)$$

of Ref. 2 only if

$$\lim_{y \to 0} [y f_0(y)] = 0 \quad (6)$$

and, moreover, the relation (2) must be satisfied. [The condition (6) must be fulfilled in any case: otherwise, for $e_0^2 \to 0$, Φ would depend on a quantity different from $e_0^2[1 + (e_0^2/3\pi)(L - \xi)]^{-1}$, and (5) would be in contradiction with (4)]. It must be noted that Eqs. (2) and (6) are not independent conditions; one follows from the other. In