The Effect of a Transverse Magnetic Field on the Thermal Conductivity of Metals

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LET us consider a metal, in which there is a heat flow $Q = Q_x$ and a magnetic field $H = H_z$. For the calculation of the coefficient of thermal conductivity we use the model of Sommerfeld,[1] according to which the flow depending on the motion of electrons under the action of the temperature gradient is set equal to zero. Accordingly,[2]

$$I_x = -\frac{3ne}{mv^3}\int \xi f v d\varepsilon, \qquad Q_y = \frac{3n}{2v^3}\int \eta f v^3 d\varepsilon \quad (1)$$

and analogously for I_y and Q_x. Here $(-e)$ is the charge on the electron, m is the mass of the electron, v is the velocity, ξ and η are the components of the velocity along the x and y axes, ϵ is the kinetic energy of the electron. The distribution function is taken to have the form

$$f = f_0 + \xi\chi_x + \eta\chi_y, \quad (2)$$

where f_0 is the Fermi distribution function, and the functions χ_x and χ_y (found with the aid of the kinetic equation, in which the term taking into account collisions, was derived by Lorentz[3]) equal:

$$\chi_x = -l(f_1 - qf_2)/v(1+q^2), \quad (3)$$
$$\chi_y = -l(f_1 q + f_2)/v(1+q^2).$$

Here l is the length of the mean free path of the electron, and the rest of the variables are defined as follows:

$$q = \omega l/v = (eH/mc)\,l/v; \quad (4)$$
$$f_1 = \partial f_0/\partial x - eE_x \partial f_0/\partial\varepsilon;$$
$$f_2 = \partial f_0/\partial y - eE_y \partial f_0/\partial\varepsilon;$$

E_x and E_y are the components of the electric field resulting from the motion of the electrons under the action of the temperature gradient.

Calculation shows that the dependence of l on v for the present problem is immaterial, because the terms containing the derivative of l with respect to v, are small and do not enter into the expression for the coefficient for thermal conductivity $\varkappa$.

Making the usual calculation for the coefficient of thermal conductivity $\varkappa = -Q_x/\partial T/\partial x$ (in the present problem $I_x = I_y = 0$, $Q_y = 0$) with accuracy to the terms $\sim (kT/\bar{\epsilon})^3$ ($\bar{\epsilon}$ is the Fermi level), we obtain

$$\varkappa = \varkappa_0\left[1 - \frac{4\pi^2}{15}\left(\frac{kT}{mv^2}\right)^2 \frac{q^2(4+2q^2+3q^4)}{(1+q^2)^3}\right], \quad (5)$$

where $\varkappa_0 = \pi^2 nlk^2 T/3mv$ is the coefficient of thermal conductivity in the absence of a magnetic field.

Approximate calculation shows that formula (5) gives a decrease in the thermal conductivity of less than 0.01% of $\varkappa_0$ in a field of 10,000 Oersteds.

It can be shown that consideration of the effect is necessary for metals of the type of Bi which have a small number of conduction electrons.

In conclusion I must thank K. B. Tolpygo for a number of suggestions and E. I. Rashba for certain advice in the course of carrying out the work.

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[2] H. Bethe and A. Sommerfeld, Electron Theory of Metals.
[3] G. Lorents, Theory of the Electron.

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Application of the Theory of Random Processes to Radiation Transfer Phenomena

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IN this note the motion of the photon is treated as a random process under the following very general assumptions: the medium is isotropic; its properties may be functions of time and space; the photon may be scattered, absorbed by an atom and reemitted, or absorbed in a collision of the second kind; the polarization of the radiation and the motion of the atom excited by a photon are not taken into account.

We begin with the function

$$f_{v_1}^{v_2}(\mathbf{r}_1, \eta_1, \nu_1, t_1; \mathbf{r}_2, \eta_2, \nu_2, t_2)\, dV_2\, d\eta_2\, d\nu_2,$$

It represents the probability that a photon, which at time t_1 and position $\mathbf{r}_1$ possesses a frequency ν_1 and a velocity ν_1 whose direction is specified by a set of direction cosines denoted by η_1, will at time t_2 be found in the element of volume dV_2 surrounding the point $\mathbf{r}_2$, with a frequency in the interval ν_2 to $\nu_2 + d\nu_2$, and with a velocity ν_2 whose direction lies in the interval η_2 to $\eta_2 + d\eta_2$.

In introducing different values for the speed of the photon, we have in mind free photons ($v=c$, where c is the speed of light) and photons absorbed by atoms ($v = 0$). In the case of photons absorbed by atoms, η and ν' stand for the direction of their motion and the frequency before absorption. It is necessary to take these parameters into account inasmuch as in a given case the indicatrix of radiation may depend on the previous history of the photon. Such a choice of the function $f_{\nu_1}^{\nu_2}$ permits the motion of the photon to be regarded as a random process of mixed type without aftereffects. Therefore the same function $f_{\nu_1}^{\nu_2}$ must satisfy the generalized Markoff equation

$$f_{v_1}^{v_2}(1;2) = \sum_{v_3}\int f_{v_1}^{v_3}(1;3)\, f_{v_3}^{v_2}(3;2)\, dV_3 d\eta_3 d\nu_3, \quad (1)$$

where $f_{\nu_1}^{\nu_2}(1;2)$ is an abbreviation for the function introduced above.

Setting $t_3 = t_2 - \Delta t$ in (1) and letting Δt approach zero, we obtain the first integro-differential equation of Kolmogoroff-Feller for processes of mixed type

$$\begin{aligned}&\partial f_{v_1}^{v_2}(1;2)/\partial t_2 \qquad (2)\\ &= \sum_{v_3}\int f_{v_1}^{v_3}(1;\mathbf{r}_2,\eta_3,\nu_3,t_2) k_{v_3}^{v_2}(\mathbf{r}_2, t_2;\eta_3,\nu_3;\eta_2,\nu_2)\, d\eta_3 d\nu_3\\ &\quad - f_{v_1}^{v_2}(1;2)\sum_{v'}\int k_{v_2}^{v'}(\mathbf{r}_2, t_2;\eta_2,\nu_2;\eta',\nu')\, d\eta' d\nu'\\ &\quad - \sigma_{v_2}(\mathbf{r}_2, t_2, \nu_2) f_{v_1}^{v_2}(1;2) - \mathbf{v}(2)\,\mathrm{grad}\, f_{v_1}^{v_2}(1;2).\end{aligned}$$

The last term is the scalar product of the photon velocity at the point 2 with the gradient of the function $f_{\nu_1}^{\nu_2}$ at this same point.

Similarly, setting $t_3 = t_1 + \Delta t$ and letting Δt approach zero, we obtain a second equation

$$\begin{aligned}&-\partial f_{v_1}^{v_2}(1;2)/\partial t_1 \qquad (3)\\ &= \sum_{v_3}\int k_{v_1}^{v_3}(\mathbf{r}_1, t_1;\eta_1,\nu_1;\eta_3,\nu_3)\, f_{v_3}^{v_2}(\mathbf{r}_1, t_1;\eta_3,\nu_3;2)\, d\eta_3 d\nu_3\\ &\quad - f_{v_1}^{v_2}(1;2)\sum_{v'}\int k_{v_1}^{v'}(\mathbf{r}_1, t_1;\eta_1,\nu_1;\eta',\nu')\, d\eta' d\nu'\\ &\quad - \sigma_{v_1}(\mathbf{r}, t_1, \nu_1) f_{v_1}^{v_2}(1;2) + \mathbf{v}(1)\,\mathrm{grad}\, f_{v_1}^{v_2}(1;2).\end{aligned}$$

The quantity $k_{\nu_1}^{\nu_2}(\mathbf{r}, t;\eta_1,\nu_1;\eta_2,\nu_2)$ in Eqs.(2) and (3) is the probability of change of state of the photon normalized per unit time. It takes into account scattering of the photon and its re-radiation by an atom; $\sigma_{\nu_1}(\mathbf{r}, t, \nu)$ takes into account processes of photon destruction, and its form is determined only by the initial velocity of the photon.

Explicit expressions for $k_{\nu_1}^{\nu_2}$ and σ_{ν_1} are given below: $k_0^0 = 0$; $k_0^c = Apk\ (\eta_1, \nu_1;\eta_2, \nu_2)$, where A is the probability of spontaneous emission and $p_k(\eta_1, \nu_1;\eta_2, \nu_2)$ is the indicatrix of emission; $k_c^0 = k(\mathbf{r}, t, \nu_1)\, c\, \delta(\eta_1 - \eta_2)$, in which $k(\mathbf{r}, t, \nu_1)$ is the photon absorption coefficient of the atoms; $k_c^c = \varkappa(\mathbf{r}, t, \nu_1)\, cp_\varkappa(\eta_1, \nu_1;\eta_2, \nu_2)$, where $\varkappa(\mathbf{r}, t, \nu_1)$ is the coefficient and $p_\varkappa(\eta_1, \nu_1;\eta_2, \nu_2)$ the indicatrix of scattering; $\sigma_0(\mathbf{r}, t)$ is the probability of collisions of the second kind, calculated for a single excited atom; $\sigma_c(\mathbf{r}, t, \nu)$ is the coefficient of true absorption.

If we substitute the values of $k_{\nu_1}^{\nu_2}$ and σ_{ν_1} into (2) and (3), then instead of each of these expressions we obtain a system of four integro-differential equations in the functions f_0^0, f_0^c, f_c^0 and f_c^c, which can be transformed into a system of integral equations. We note that simple transformations permit producing integro-differential or integral equations that contain only one of the functions $f_{\nu_1}^{\nu_2}$.

If the distribution of the sources of radiation and of collisions of the first kind in the volume V under consideration is known, then with the aid of the function $f_{\nu_1}^{\nu_2}$ it is easy to obtain the concentration of excited atoms and the radiation intensity as functions of space and time. Thus, by applying the theory of random processes it is possible under very general assumptions about the interaction of radiation with matter to derive the complete system of equations that describe the nonstationary process of radiation transfer in an isotropic medium whose properties are functions of space and time. Naturally the equations of radiation transfer (that are well known in the literature) the equations for the volume density of radiation and for the concentration of excited atoms,

can all be derived as special cases of the relations given above. It is to be noted that the first equation of Kolmogoroff-Feller permits the derivation of the complete system of equations for the desired probability densities. Comparison of the two equations permits determining the symmetry properties of the function $f_{v_1}^{v_2}(1;2)$ with respect to an interchange of indices, from which follows the general formulation of the principle of optical reversibility.

The probability density $f_{v_1}^{v_2}(1;2)$ considered here is closely connected, of course, with the transmission and reflection functions of V. A. Ambartsumian and with the probability of emergence of a photon employed by V. V. Sobolev. The authors hope to take up these problems in detail.

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The Lagrangian Function for a System of Identically Charged Particles

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DARWIN[1] has shown that it is possible to write the Lagrangian function for a system of charged particles, correct to the second order terms in the ratio of the velocity of the particle to the velocity of light. This is possible because the radiation of light is a third order effect in v/c and does not enter in the second order approximation.

It is of interest to point out the possibility of obtaining the Lagrangian function for a system of identically charged particles to a higher order of approximation. It is well known that in a system of identical particles (with precisely the same ratio of charge to mass) the radiation is proportional to the fifth power of v/c and not to the third power. Therefore the Lagrangian function for such a system can be written to the term v^4/c^4. It is easiest to use the method given in the book of Landau and Lifshitz[2] for its calculation.

It is not difficult to show that the third order terms in the Lagrangian function go to zero. A calculation of the fourth order terms leads to the following expression, which must be added to the second order Lagrangian function.

$$L^{(4)} = -\sum_a \frac{m_a v_a^6}{16c^4} + \frac{e^2}{8c^4}\sum_{b>a}\frac{1}{R_{ab}}\{2(\mathbf{v}_a\mathbf{v}_b)^2 \quad (1)$$

$$- v_a^2 v_b^2 + (\mathbf{nv}_a)^2 v_b^2 + (\mathbf{nv}_b)^2 v_a^2 - 3(\mathbf{nv}_a)^2(\mathbf{nv}_b)^2\}$$

$$+ \frac{e^2}{8c^4}\sum_{b>a}\{2(\mathbf{nv}_a)(\mathbf{v}_a\dot{\mathbf{v}}_a) - 2(\mathbf{nv}_b)(\mathbf{v}_b\dot{\mathbf{v}}_a) - v_a^2(\mathbf{n}\dot{\mathbf{v}}_b)$$

$$+ v_b^2(\mathbf{n}\dot{\mathbf{v}}_a) + (\mathbf{nv}_a)^2(\mathbf{n}\dot{\mathbf{v}}_b) - (\mathbf{nv}_b)^2(\mathbf{n}\dot{\mathbf{v}}_a)$$

$$- 3R_{ab}(\dot{\mathbf{v}}_a\dot{\mathbf{v}}_b) + R_{ab}(\mathbf{n}\dot{\mathbf{v}}_b)(\mathbf{n}\dot{\mathbf{v}}_a)\},$$

where $\mathbf{n}$ is a unit vector in the Direction $\mathbf{R}_{ab}$. Of course in making calculations the terms that contain the total derivative with respect to time are dropped.

The accelerations can be expressed here through the coordinates and velocities of the charges, consistent with the equations of motion, obtained by completely neglecting the retarded potentials, that is, from the Lagrangian function of zero approximation. Thus in the simplest case of two charges we have

$$\dot{\mathbf{v}}_1 = (e^2/m)\,\mathbf{n}/R^2; \qquad \dot{\mathbf{v}}_2 = -(e^2/m)\,\mathbf{n}/R^2,$$

where $\mathbf{R}_{21} = -\mathbf{R}_{12} = \mathbf{R}$ and $\mathbf{R}/R = \mathbf{n}$; after substituting in (1) we obtain

$$L^{(4)} = -\frac{mv_1^6}{16c^4} - \frac{mv_2^6}{16c^4} \quad (2)$$

$$+ \frac{e^2}{8c^4}\left\{\frac{1}{R}[2(\mathbf{v}_1\mathbf{v}_2)^2 - v_1^2v_2^2 + (\mathbf{nv}_1)^2v_2^2 + (\mathbf{nv}_2)^2v_1^2\right.$$

$$- 3(\mathbf{nv}_1)^2(\mathbf{nv}_2)^2] + \frac{3e^2}{m}[(\mathbf{nv}_1)^3 + (\mathbf{nv}_2)^2]$$

$$\left. - \frac{e^2}{m}(v_1^2 + v_2^2) + \frac{2e^4}{m^2R^3}\right\}.$$

The Lagrangian function of two identical charges with accuracy to the fourth order can be used for investigating the relativistic corrections in the scattering of high speed protons, and also for generalizing the well-known formula of Breit for the interaction of electrons (see Refs. 3,4). The calculation of the formula of Breit to fourth order was carried out by Maksimov; the results however are very lengthy, and we will not include them here.

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