And among total cross sections σ^t:

$$3(\sigma_1^t + \sigma_2^t) \geqslant 4\sigma_3^t; \tag{14}$$

$$\sqrt{2}\,\sigma_1^{t^{1/2}} + \sigma_2^{t^{1/2}} \geqslant 2\sigma_3^{t^{1/2}} \geqslant |\sqrt{2}\,\sigma_1^{t^{1/2}} - \sigma_2^{t^{1/2}}|;$$

$$\sqrt{2}\,\sigma_1^{t^{1/2}} + 2\sigma_3^{t^{1/2}} \geqslant \sigma_2^{t^{1/2}} \geqslant |\sqrt{2}\,\sigma_1^{t^{1/2}} - 2\sigma_3^{t^{1/2}}|;$$

$$\sigma_2^{t^{1/2}} + 2\sigma_3^{t^{1/2}} \geqslant \sqrt{2}\,\sigma_1^{t^{1/2}} \geqslant |\sigma_2^{t^{1/2}} - 2\sigma_3^{t^{1/2}}|.$$

Analyses of near threshold photoproduction of two π–mesons in deuterium without deuteron breakup, yields (here $V_{10} = 0$):

$$\sigma(\gamma + d \to d + \pi^0 + \pi^0) = \sigma(\gamma + d \to d + \pi^+ + \pi^-) \tag{15}$$

and for the total cross sections:

$$2\sigma^+(\gamma + d \to d + \pi^0 + \pi^0) = \sigma^t(\gamma + d \to d + \pi^+ + \pi^-). \tag{16}$$

The inequalities relating the cross sections of the various processes and Eqs. (15) – (16) hold away from threshold if the π – mesons produced in reactions (1) and (10) are created with equal momenta [2] $K_1 = K_2$.

The author wishes to thank I.M. Shmushkevich for discussion of the results and for advice given during the course of this investigation.

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Translated by M.A. Melkanoff
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Paramagnetic Resonance and the Polarization of Nuclei in Thick Metal Foils

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(Submitted to JETP editor May 18, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 357-359
(August, 1956)

ACCORDING to Overhauser[1], paramagnetic resonance induces polarization in nuclear matter. It is shown in Ref. 1 that a substantial nuclear polarization occurs in metals when the frequency

$$\Omega_0 = \mu H_0/\hbar \gg kT/\hbar, \quad \text{a } \Omega_1 = \mu H_1/\hbar \gg 1/T_{sp}$$

(H_0 and H_1 are the constant and the high frequency magnetic field intensities, μ is the magnetic moment of the electron, T_{sp} is the spin relaxation time). The later estimate, however, is correct only when the electron moves in the time T_{sp} through a homogeneous electromagnetic field. At resonance ($\omega = \Omega_0$), when $\Omega_0 >> kT/\hbar$, this takes place only in extremely thin metallic foils, of thickness of the order of the skin depth $\delta \sim 10^{-4}$ to 10^{-6} cm. So far[2], only such specimens have been polarized by Overhauser's method.

In the present article it is shown that it is possible by means of a high frequency magnetic field of high intensity

$$H_1 \gg (8\pi\delta_{eff}/c^2|Z|T_{sp})H_0$$

(Z is the surface impedance of the metal[3]), to polarize nuclei in thick foils at a considerably larger depth $\delta_{eff} \sim 10^{-2}$ to 1 cm to which electrons penetrate by diffusion in time T_{sp}.

In order to formulate a consistent theory it is necessary in this case to solve simultaneously Maxwell's equations

$$\text{rot } \mathbf{E} = -\frac{1}{c}\frac{\partial B}{\partial t}; \quad \text{rot } \mathbf{H}_1 = \frac{4\pi}{c}\mathbf{j}; \quad \mathbf{B} = \mathbf{H}_1 + 4\pi\mathbf{M},$$

and the kinetic equation for the electron distribution operator $\hat{f}$ ($\hat{f}$ operates only on spin)

$$\frac{\partial \hat{f}}{\partial t} + \frac{\partial \hat{f}}{\partial \mathbf{r}}\mathbf{v} + \frac{\partial \hat{f}}{\partial \mathbf{p}}\left\{e\mathbf{E} + \frac{e}{c}[\mathbf{vH}]\right\} + \frac{i}{h}[\mu\mathbf{H}\hat{\sigma}, \hat{f}] + \left(\frac{\partial \hat{f}}{\partial t}\right)_{col} + \left(\frac{\partial \hat{f}}{\partial t}\right)_{sp} = 0. \tag{1}$$

$(\partial\hat{f}/\partial t)_{col}$ is here the collision integral for collisions without spin flip, $(\partial\hat{f}/\partial t)_{sp}$ is the integral for collisions with spin flip; $\hat{\sigma}$ is the spin operator; v and p are the velocity and momentum of the electron. Let us write these integrals in the form

$$(\partial\hat{f}/\partial t)_{sp} = (\hat{f} - \hat{f}_0)/T_{sp}; \quad (\partial\hat{f}/\partial t)_{col} = (\hat{f} - \hat{f}^0)/t_0$$

(t_0 is the time between collisions), where

$$\hat{f}_0 = \begin{pmatrix} f_0(\varepsilon_0 + \mu H_0) & 0 \\ 0 & f_0(\varepsilon_0 - \mu H_0) \end{pmatrix};$$

$$\hat{f^0} = \int \hat{f}\,\frac{dS}{v} \Big/ \int \frac{dS}{v};\ f_0(\zeta) = [e^{(\varepsilon-\zeta)/kT} + 1]^{-1}.$$

In order to determine $\hat{f}^0$, let us compute what such an equilibrium operator would become for the given $\hat{f}$ in the absence of processes which bring about spin flip (i.e., for $\Omega_1 = 0$, $T_{sp} = \infty$). Evidently then,

$$\int \hat{f}\,dp_x\,dp_y\,dp_z = \int \hat{f}^0\,dp_x\,dp_y\,dp_z,$$
$$\mathrm{Sp}\int \hat{f}\,dp_x\,dp_y\,dp_z = \mathrm{Sp}\int \hat{f}_0\,dp_x\,dp_y\,dp_z$$

(ϵ_0 is the Fermi energy level; dS is an element of the Fermi surface).

Boundary conditions for the function f at the metal surface have the form*

$$\hat{f}\big|_{\mathbf{vn}>0} = (1-q)\,\hat{f}^0 + qf\big|_{-\mathbf{vn}} \tag{2}$$

(n is the internal surface normal, q is the coefficient for electron reflection from the surface; apparently, almost always, $q \approx 0$). Expanding $\hat{f}$ in terms of the operators $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\hat{\sigma}$, it may be shown that Eq. (1) corresponds to the following system of equations:

$$\frac{\partial f_1}{\partial t} + \frac{\partial f_1}{\partial \mathbf{r}}\mathbf{v} + \frac{\partial f_1}{\partial \mathbf{p}}\left\{\frac{e}{c}[\mathbf{vH}] + e\mathbf{E}\right\} + \frac{f_1 - f_0(\varepsilon-\varepsilon_0)}{t_0} = 0, \tag{3}$$

$$\frac{\partial \mathbf{w}}{\partial t} + \frac{\partial \mathbf{w}}{\partial \mathbf{r}}\mathbf{v} + \frac{e}{c}[\mathbf{vH}]\frac{\partial \mathbf{w}}{\partial \mathbf{p}} + \frac{\mathbf{w}}{t_0^*} + [\mathbf{w}\Omega] = \frac{\overline{\mathbf{w}}}{t_0} + [\mathbf{b}\Omega_1], \tag{4}$$

$$\mathbf{w}\big|_{\mathbf{vn}>0} = (1-q)\,\overline{\mathbf{w}} + q\mathbf{w}\big|_{-\mathbf{vn}},$$
$$f_1\big|_{\mathbf{vn}>0} = (1-q)\,f_0 + qf_1\big|_{-\mathbf{vn}} \quad \text{(see Ref. 7)},$$

$$\frac{1}{t_0^*} = \frac{1}{t_0} + \frac{1}{T_{\text{сп}}},\quad \mathbf{b} = \frac{\mathbf{H}_0}{H_0},\quad \overline{\mathbf{w}} = \int \mathbf{w}\,\frac{dS}{v} \Big/ \int \frac{dS}{v}, \tag{5}$$

where

$$\mathbf{j} = 2eh^{-3}\int f_1\,dp_x\,dp_y\,dp_z,\quad \mathbf{M} = \chi H_0(\mathbf{b} - \overline{\mathbf{w}}).$$

It is easily seen that if $T_{sp} = \infty$, $\omega = \Omega_0$, Eq. (4) has a non-trivial solution for $\Omega_1 = 0$, depending only upon ϵ. But $T_{sp} \gg t_0$; we therefore find ourselves near the eigenvalue of Eq. (4). Its solution, therefore, turns out to be near the corresponding eigenfunction (independent of r) and slowly varying as a function of the depth $\xi = \mathbf{nr}$.

At resonance (for $\omega = \Omega_0$), solving the equation, we obtain:

$$\overline{w}_z(\xi) = \frac{|\alpha|^2}{1+|\alpha|^2}\exp\{-\xi/\delta_{\text{eff}}\}, \tag{6}$$

$$\overline{w}_x + i\overline{w}_y = -\frac{\alpha e^{i\omega t}}{1+|\alpha|^2}\exp\{-\xi/\delta_{\text{eff}}\},$$

$$(t_0/T_{sp})^{1/2}(a/\delta)^{4/3} \ll 1,$$

where

$$\alpha = \frac{c(E_x + iE_y)}{2H_0 a}\sqrt{t_0 T_{sp}},\quad \delta_{\text{eff}} = a\sqrt{T_{sp}/t_0} \tag{7}$$

(the z-axis coincides with the direction of the constant field H_0, the y-axis lies in the surface of the metal), E is the amplitude of the electric field at the surface of the metal, and the quantity a is of the order of magnitude of the orbital radius of the electron $r_0 \sim mvc/eH_0$ in the case of a strong ($r_0 \ll vt_0$) magnetic field H_0 parallel to the surface, and of the order of vt_0 in all other cases.

It is easy to obtain from formulas (6) the nuclear polarization P:

$$P = I^{-1}\{(I + 1/2)\,\mathrm{cth}\,(I + 1/2)\,s - 1/2\,\mathrm{cth}\,(s/2)\},$$

$$s = \frac{|\alpha|^2}{1+|\alpha|^2}\frac{\mu H_0}{kT}e^{-\xi/\delta_{\text{eff}}}$$

(I is the magnetic moment of the nucleus).

Let us note that a slow damping of the magnetic moment M [see Eqs. (6) and (7)] brings about, according to Maxwell's equations, the appearance of a slight, though equally slow, damping of E and H_1**. Thus, in paramagnetic resonance one should observe the resonance passage of an electromagnetic wave through the film, whereby the transmitted wave becomes circularly polarized.

The transmission coefficient $K = |E_{tr}|^2/|E_{inc}|^2$ for a film of thickness d ($\delta \ll d \ll \delta_{\text{eff}}$), is of the order of magnitude

$$K = \left(\frac{cT_{sp}\chi\,|cz|^2}{2\pi d}\right)^2\left[1 + \left|\frac{c^2 z T_{sp} H_{inc.}}{2\pi d H_0}\right|^2\right]^{-2}\ (\mathbf{n} \parallel z),$$

i.e., it can be many times larger than the transmission coefficient away from resonance. It should be noted that such a selective transparency of a film occurs at any temperature.

The derivations of the results reported here will be the subject of a separate article.

The author is indebted to L. N. Rosentsveig for discussion of the results of this investigation.

* It is assumed that the electron spin is not flipped upon reflection. Let us note that Eq. (1) was obtained in Ref. 4; the operator $(\partial f/\partial t)_{sp}$, however, was not included here. Boundary conditions (13) of that reference are apparently not realizable in practice.

** The slow damping of M is linked to the fact that when $\delta \ll \delta_{eff}$ there appears an "anomalous skin-effect" for the magnetic moment: an integral relation appears between M and H_1. This does not take place out of resonance as $M \sim 10^{-6} H_1$ (while at resonance $M \sim 10^{-2} H_1$).

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Translated by M. A. Melkanoff
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"Repulsion" of Nuclear Levels

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(Submitted to JETP editor April 16, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 162-164 (July, 1956)

1. DATA on the position and parameters of nuclear levels obtained by methods of neutron spectroscopy at excitation energies of the order of the neutron binding energy[1,2] allow the investigation of the empirical regularities of level behavior with the aim of checking the predictions of existing nuclear theories and their improvement. Interesting work has been done, for example, on the systematics of neutron widths[3] and on the systematics of radiation widths[4] *. The problem of regularities in the distribution of nuclear levels and in the fluctuations of adjacent level spacing has not been discussed in the literature.

For a purely random distribution of the distance between levels ϵ about its average value D, the distribution function must be of the form:

$$W(\varepsilon)\, d\varepsilon = \exp\{-\varepsilon / D\}\, d\varepsilon / D. \qquad (1)$$

It would be more to the point to examine the data for levels in the same spin state, i.e., for target nuclei of spin 0 (even-even nuclei). However, such nuclei have few levels in the range where the resolution of contemporary methods of nuclear spectroscopy suffices. Thus it is impossible to exclude the data obtained from nuclei with odd atomic weight, and consequently with two sets of nuclear levels, corresponding to $i - 1/2$ and $i + 1/2$ (where i is the spin of the target nucleus). It should be kept in mind that the presence of two sets of levels corresponding to various spins of the intermediate nucleus makes the correlation of various levels positions less obvious.

If the distribution function (1) holds for each set of levels, then the resulting distribution function will have the same form with $D = d_1 d_2 / (d_1 + d_2)$, (where $d_{1,2}$ are the distances between levels in each set).

We made use of experimental data on the level distribution for: In^{113}, In^{115}, Cs^{133}, Tb^{159}, Ho^{165}, Tm^{169}, Hf^{177}, Hf^{179}, Ta^{181} [1], U^{235} **, U^{238} [1].

In order to eliminate mistakes in the determination of ϵ due to ignoring levels because of insufficient experimental resolution, a curve was constructed for each element of the increase of the number of discovered levels with increase of neutron energy. Levels within a suitable energy limit were used, so that this increase was approximately linear.

To increase the statistical certainty of the experimental distribution of levels for each isotope, the quantity $x_i = \epsilon_i / D$ was calculated, and the distribution of levels as a function of x_i for all the enumerated nuclei was plotted (Fig. 1). The total number of cases $N = 134$. The curve is the distribution (1) normalized to the area of the histogram.

The level distributions for U^{238} (an even-even nucleus with eleven known levels) and for U^{235} (for which D is comparable to the entire level width) are shown separately.

Comparison of the curve and the histogram allows the qualitative confirmation of the relatively small number of closely spaced levels, which may be interpreted as the result of a "repulsion" of levels.

To eliminate the possibility that the relatively