electronic mass) in the semiconductors under consideration:

	r_{Pb}, Å	r_X, Å	V_0, eV	ΔE, eV	$\Delta E/\Delta E^{(1)}_{exp}$	$\Delta E/\Delta E^{(2)}_{exp}$	m^*_h	m^*_e
PbS	1,28	1,68	2,50	1,48	1,42	1,06	2,03	1,02
PbSe	1,25	1,82	1,95	1,13	1,36	1,08	1,54	0,88
PbTe	1,22	1,95	1,39	0,79	1,27	0,88	1,07	0,70

There are presented in the Table the values of the effective mass corresponding to the second model. The values which correspond to the first model can be derived by multiplying them by 2.25. The experimental values of the widths of the forbidden zones $\Delta E^{(1)}_{exp}$ are taken from Ref. 3, $\Delta^{(2)}_{exp}$ from Ref. 4. The value of V_0 was taken from Ref. 5. Notice that the derived value of ΔE, and also the values of the effective mass, may change somewhat, owing to some uncertainty in the values of the parameters r_{Pb}, r_X and V_0. Nevertheless, we think that the results obtained confirm the applicability of the lattice model to semiconductors and the desirability of extending its applications in this direction.

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Photoproduction of π–Meson Pairs in Hydrogen and Deuterium Near Threshold and Isotopic Invariance

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THE cross section for photoproduction of π–meson pairs in hydrogen and deuterium is calculated by considering the interaction of an electromagnetic field with a meson-nucleon system in first order perturbation theory (it is well known that this does not presuppose a small interaction between mesons and nucleons). The interaction operator consists of two parts[1] $H = S + V_3$, where S is a scalar and V_3, the third component of a vector is isotopic spin space.

The isotopic spin of the initial state in the reaction

$$\gamma + p \rightarrow n' + \pi' + \pi'' \qquad (1)$$

is equal to ½. The isotopic spin of the meson-nucleon system in the final state may be $t = ½$ and $t = 3/2$. The operator S yields transitions to states of the meson-nucleon system with isotopic spin $t = ½$, and the operator V_3 to states $t = ½$ and $t = 3/2$.

S_{Tt} V_{Tt} will be used to denote transition amplitudes to states of total isotopic spins $t = ½$ and 3/2 of the meson-nucleon system, and isotopic spin T of the two π–mesons system ($T = 0, 1, 2$). The differential cross section for photoproduction of two π–mesons then becomes

$$\sigma_1(\gamma + p \rightarrow p + \pi^0 + \pi^0) \qquad (2)$$

$$= \left|\frac{1}{\sqrt{3}} S_{0\,{}^1/_2} + \frac{1}{3} V_{0\,{}^1/_2} + \frac{2}{3}\sqrt{\frac{2}{5}}\, V_{2\,{}^3/_2}\right|^2;$$

$$\sigma_2(\gamma + p \rightarrow p + \pi^+ + \pi^-)$$

$$= \left|\frac{1}{\sqrt{3}} S_{0\,{}^1/_2} + \frac{1}{3} V_{0\,{}^1/_2} - \frac{1}{3}\sqrt{\frac{2}{5}}\, V_{2\,{}^3/_2}\right.$$

$$\left. + \frac{1}{\sqrt{6}} S_{1\,{}^1/_2} + \frac{1}{3\sqrt{2}} V_{1\,{}^1/_2} - \frac{\sqrt{2}}{3} V_{1\,{}^3/_2}\right|^2; \qquad (3)$$

$$\sigma_3(\gamma + p \rightarrow n + \pi^+ + \pi^0) \tag{4}$$

$$= \left| -\frac{1}{\sqrt{3}} S_{1\,1/2} - \frac{1}{3} V_{1\,1/2} - \frac{1}{3} V_{1\,3/2} + \frac{1}{\sqrt{5}} V_{2\,3/2} \right|^2 .$$

Near threshold, the isotopic spin T of the two π–mesons system can only be 0 or 2. Indeed, the wave functions for the states of isotopic spin $T = 0$ and 2 are symmetric with respect to charge exchange of the π–mesons coordinates (pions obey Bose statistics). The wave function for the state with isotopic spin $T = 1$ is antisymmetric with respect to charge exchange of the π–mesons and thus antisymmetric in the coordinates. Near threshold, π–mesons from reaction (1) are formed in the S state. Since the wave function for the final state must be symmetric with respect to permutations of the π–mesons, the production of two π–mesons with $T = 1$ is forbidden near threshold[1].

Substitution of S_{1t}, $V_{1t} = 0$ in Eqs. (2) – (4) yields the following inequalities among the differential cross sections for reactions near the threshold:

$$3(\sigma_1 + 2\sigma_2) \geqslant 4\sigma_3, \tag{5}$$

$$\sigma_1^{1/2} + \sigma_2^{1/2} \geqslant \sqrt{2}\,\sigma_3^{1/2} \geqslant |\sigma_1^{1/2} - \sigma_2^{1/2}|,$$

$$\sigma_1^{1/2} + \sqrt{2}\,\sigma_3^{1/2} \geqslant \sigma_2^{1/2} \geqslant |\sigma_1^{1/2} - \sqrt{2}\,\sigma_3^{1/2}|,$$

$$\sigma_2^{1/2} + \sqrt{2}\,\sigma_3^{1/2} \geqslant \sigma_1^{1/2} \geqslant |\sigma_2^{1/2} - \sqrt{2}\,\sigma_3^{1/2}|$$

and among total cross sections σ^t:

$$3(\sigma_1^t + \sigma_2^t) \geqslant 2\sigma_3^t, \quad \sigma_1^{t\,1/2} \tag{6}$$

$$+ \frac{1}{\sqrt{2}} \sigma_2^{t\,1/2} \geqslant \sigma_3^{t\,1/2} \geqslant \left| \sigma_1^{t\,1/2} - \frac{1}{\sqrt{2}} \sigma_2^{t\,1/2} \right|,$$

$$\sigma_1^{t\,1/2} + \sigma_3^{t\,1/2} \geqslant \frac{1}{\sqrt{2}} \sigma_2^{t\,1/2} \geqslant |\sigma_1^{t\,1/2} - \sigma_3^{t\,1/2}|,$$

$$\sigma_3^{t\,1/2} + \frac{1}{\sqrt{2}} \sigma_2^{t\,1/2} \geqslant \sigma_1^{t\,1/2} \geqslant \left| \sigma_3^{t\,1/2} - \frac{1}{\sqrt{2}} \sigma_2^{t\,1/2} \right|.$$

Now let us consider the photoproduction of two π–mesons in deuterium, i.e., the reaction

$$\gamma + d \rightarrow n_1 + n_2 + \pi' + \pi''. \tag{7}$$

The initial state has isotopic spin $t = 0$. The final state consists of a superposition of states with isotopic spins $t = 0$ and $t = 1$. The corresponding transition operators are S and V_3. S_{Tt_1} and V_{Tt_1} will denote transition amplitudes to states of isotopic spin T of the two π–mesons system, isotopic spin t of the two nucleons, and total spin $t = 0$ (S_{Tt_1}) and $t = 1$ (V_{Tt_1}). Various processes then give

$$\sigma_1(\gamma + d \rightarrow p + n + \pi^0 + \pi^0) \tag{8}$$

$$= \frac{1}{6} \left| S_{00} - V_{01} - \frac{2}{\sqrt{5}} V_{21} \right|^2;$$

$$\sigma_2(\gamma + d \rightarrow p + n + \pi^+ + \pi^-) \tag{9}$$

$$= \frac{1}{6} \left| -S_{00} + \frac{1}{\sqrt{2}} S_{11} + V_{01} + \sqrt{\frac{3}{2}} V_{10} - \frac{1}{\sqrt{5}} V_{21} \right|^2$$

$$\sigma_3(\gamma + d \rightarrow p + p + \pi^- + \pi^0) \tag{10}$$

$$= \left| \frac{1}{6} \right| S_{11} + \sqrt{\frac{3}{2}} V_{11} + \frac{3}{\sqrt{10}} V_{21} \left|^2 \right. .$$

If the reaction takes place without deuteron breakup, T can only be 0 or 1; therefore,

$$\sigma(\gamma + d \rightarrow d + \pi^0 + \pi^0) = 1/6\, |S_{00}|^2; \tag{11}$$

$$\sigma(\gamma + d \rightarrow d + \pi^+ + \pi^-) = 1/6\, |-S_{00} + \sqrt{3/2}\, V_{10}|^2.$$

Integration of the cross section for the reaction $\gamma + d \rightarrow d + \pi^+ + \pi^-$ eliminates interference terms and the total cross section σ^t becomes

$$2\sigma^t(\gamma + d \rightarrow d + \pi^0 + \pi^0) \leqslant \sigma^t(\gamma + d \rightarrow d + \pi^+ + \pi^-). \tag{12}$$

Near threshold, S_{1t_1} and $V_{1t_1} = 0$, and the following inequalities apply:

$$3(\sigma_1 + 2\sigma_2) \geqslant 4\sigma_3; \tag{13}$$

$$\sigma_1^{1/2} + \sigma_2^{1/2} \geqslant \sqrt{2}\,\sigma_3^{1/2} \geqslant |\sigma_1^{1/2} - \sigma_2^{1/2}|;$$

$$\sigma_1^{1/2} + \sqrt{2}\,\sigma_3^{1/2} \geqslant \sigma_2^{1/2} \geqslant |\sigma_1^{1/2} - \sqrt{2}\,\sigma_3^{1/2}|;$$

$$\sigma_2^{1/2} + \sqrt{2}\,\sigma_3^{1/2} \geqslant \sigma_1^{1/2} \geqslant |\sigma_2^{1/2} - \sqrt{2}\,\sigma_3^{1/2}|.$$

And among total cross sections σ^t:

$$3(\sigma_1^t + \sigma_2^t) \geqslant 4\sigma_3^t; \tag{14}$$

$$\sqrt{2}\,\sigma_1^{t\,1/2} + \sigma_2^{t\,1/2} \geqslant 2\sigma_3^{t\,1/2} \geqslant |\sqrt{2}\,\sigma_1^{t\,1/2} - \sigma_2^{t\,1/2}|;$$

$$\sqrt{2}\,\sigma_1^{t\,1/2} + 2\sigma_3^{t\,1/2} \geqslant \sigma_2^{t\,1/2} \geqslant |\sqrt{2}\,\sigma_1^{t\,1/2} - 2\sigma_3^{t\,1/2}|;$$

$$\sigma_2^{t\,1/2} + 2\sigma_3^{t\,1/2} \geqslant \sqrt{2}\,\sigma_1^{t\,1/2} \geqslant |\sigma_2^{t\,1/2} - 2\sigma_3^{t\,1/2}|.$$

Analyses of near threshold photoproduction of two π-mesons in deuterium without deuteron breakup, yields (here $V_{10} = 0$):

$$\sigma(\gamma + d \to d + \pi^0 + \pi^0) = \sigma(\gamma + d \to d + \pi^+ + \pi^-) \tag{15}$$

and for the total cross sections:

$$2\sigma^t(\gamma + d \to d + \pi^0 + \pi^0) = \sigma^t(\gamma + d \to d + \pi^+ + \pi^-). \tag{16}$$

The inequalities relating the cross sections of the various processes and Eqs. (15)–(16) hold away from threshold if the π-mesons produced in reactions (1) and (10) are created with equal momenta [2] $K_1 = K_2$.

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Paramagnetic Resonance and the Polarization of Nuclei in Thick Metal Foils

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ACCORDING to Overhauser[1], paramagnetic resonance induces polarization in nuclear matter. It is shown in Ref. 1 that a substantial nuclear polarization occurs in metals when the frequency

$$\Omega_0 = \mu H_0/\hbar \gtrsim kT/\hbar, \quad \text{a} \ \Omega_1 = \mu H_1/\hbar \gtrsim 1/T_{sp}$$

(H_0 and H_1 are the constant and the high frequency magnetic field intensities, μ is the magnetic moment of the electron, T_{sp} is the spin relaxation time). The later estimate, however, is correct only when the electron moves in the time T_{sp} through a homogeneous electromagnetic field. At resonance ($\omega = \Omega_0$), when $\Omega_0 \gg kT/\hbar$, this takes place only in extremely thin metallic foils, of thickness of the order of the skin depth $\delta \sim 10^{-4}$ to 10^{-6} cm. So far[2], only such specimens have been polarized by Overhauser's method.

In the present article it is shown that it is possible by means of a high frequency magnetic field of high intensity

$$H_1 \gtrsim (8\pi\delta_{\text{eff}}/c^2|Z|T_{sp})H_0$$

(Z is the surface impedance of the metal[3]), to polarize nuclei in thick foils at a considerably larger depth $\delta_{\text{eff}} \sim 10^{-2}$ to 1 cm to which electrons penetrate by diffusion in time T_{sp}.

In order to formulate a consistent theory it is necessary in this case to solve simultaneously Maxwell's equations

$$\text{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial B}{\partial t}; \ \text{rot}\,\mathbf{H}_1 = \frac{4\pi}{c}\mathbf{j}; \ \mathbf{B} = \mathbf{H}_1 + 4\pi\mathbf{M},$$

and the kinetic equation for the electron distribution operator $\hat{f}$ ($\hat{f}$ operates only on spin)

$$\frac{\partial\hat{f}}{\partial t} + \frac{\partial\hat{f}}{\partial \mathbf{r}}\mathbf{v} + \frac{\partial\hat{f}}{\partial \mathbf{p}}\left\{e\mathbf{E} + \frac{e}{c}[\mathbf{vH}]\right\} + \frac{i}{h}[\mu\mathbf{H}\hat{\sigma}, \hat{f}] + \left(\frac{\partial\hat{f}}{\partial t}\right)_{\text{col}} + \left(\frac{\partial\hat{f}}{\partial t}\right)_{\text{sp}} = 0. \tag{1}$$

$(\partial\hat{f}/\partial t)_{\text{col}}$ is here the collision integral for collisions without spin flip, $(\partial\hat{f}/\partial t)_{\text{sp}}$ is the integral for collisions with spin flip; $\hat{\sigma}$ is the spin operator; v and p are the velocity and momentum of the electron. Let us write these integrals in the form

$$(\partial\hat{f}/\partial t)_{\text{sp}} = (\hat{f} - \hat{f}_0)/T_{\text{sp}}; \ (\partial\hat{f}/\partial t)_{\text{col}} = (\hat{f} - \hat{f}^0)/t_0$$

(t_0 is the time between collisions), where