Letters to the Editor

Polarization of Neutrons Scattered by Lead

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IN the scattering of fast neutrons by nuclei of high Z a strong polarization should be observed at small angles due to the interaction of the magnetic moment of the neutron with the electric field of the nucleus. This interaction has the form:

$$H' = -\mu_n (eh/2M^2c^2)\,\sigma[\mathbf{E}\cdot\mathbf{p}] = \xi r^{-3}(\sigma\cdot\mathbf{l});\ \xi = +\mu_n e^2h^2Z/2M^2c^2, \tag{1}$$

where $\mu_n = -1.91$ is the magnetic moment of the neutron, σ is the neutron spin operator, p its momentum, and E the electric field of the nucleus. This effect was first shown by Schwinger[1] who evaluated the resulting polarization. In that work he considered the nucleus either as an absolutely black body or as a hard sphere.

In the present paper the polarization of neutrons was calculated more exactly. The interaction Hamiltonian of the neutron with the nucleus was expressed in the following form:

$$U = \frac{U_0(1+i\zeta)}{1+\exp[(r-r_0)/a]} + H^1 = U_1 + H^1, \tag{2}$$

where $U_0 = -40$ mev, ζ a constant describing the absorption, r_0 the nuclear radius, $2a$ the smear of the nuclear edge, and H^1 is determined by Eq. (1). The first term in the right side of Eq. (2) was written as the sum of two parts:

$$U_1 = U_2 + (U_1 - U_2),$$

where

$$U_2 = \begin{cases} U_0(1+i\zeta) & \text{for } r < r_0, \\ 0 & \text{for } r > r_0. \end{cases} \tag{3}$$

The Schrödinger equation for a neutron in the potentials U_2 was solved exactly, while the additions to the Hamiltonian, $(U_2 - U_1)$ and H^1, were taken into account by perturbation theory. In this way a neutron scattering amplitude was calculated:

$$f(\theta) = f_1(\theta) + (\sigma\mathbf{n})\,f_2(\theta). \tag{4}$$

It should be noted that for scattering angles $\theta \sim 1$–$3°$ the spin amplitude $f_2 = -i2M\,\xi/h^2\,\theta$ and thus depends only on the nuclear charge [see Eq. (1)].

Knowing the scattering amplitude, we calculated the total cross section for neutrons

$$\sigma_t = (4\pi/k)\,\mathrm{Im}\,f_1(0),$$

the differential cross section in the region $0° < \theta < 30°$; $d\sigma/do = |f_1|^2 + |f_2|^2$ (we are limited to these angles since the optical model describes scattering well only for small angles), and finally the polarization for small θ

$$P = -\mathbf{n}\,\frac{2\mathrm{Re}\,[f_1(0),\ f_2^*(\theta)]}{|f_1(0)|^2 + |f_2(\theta)|^2} = \mathbf{n}\,\frac{k\xi M\sigma_t}{\pi h^2 d\sigma(\theta)/do} = P(\theta)\,\mathbf{n}. \tag{5}$$

All the above calculations were carried out for lead, with various choices of the parameters, r_0, ζ, a and neutron energy ϵ:

$r_0 = 7; 7.5;$ and 8 (in units of 10^{-13} cm)
$\zeta = 0; 0.01; 0.02; 0.03; 0.05; 0.10$
$\epsilon = 1; 2; 3; 4; 5$ (in mev)
$a = 0; 0.4; 0.55$ (in units of 10^{-13} cm)

Comparing the magnitudes of σ_T and $d\sigma/do$ calculated for various values of the parameters it turns out that for a given neutron energy ϵ the results of the calculation depend most strongly on the nuclear radius r_0, less strongly on the value of the absorption ζ, and finally, least on the smear of the nuclear edge $2a$. The selection of best values for r_0, ζ and a must be made by a comparison of the theoretical results for σ_t and $d\sigma/do$ with the experimental values. It turns out that agreement with the experimental values occurs only for $r_0 = 8$. As illustration, Fig. 1 shows our calculations of σ_t for various parameters. Comparison of the theoretical curves of σ_t (as a function of

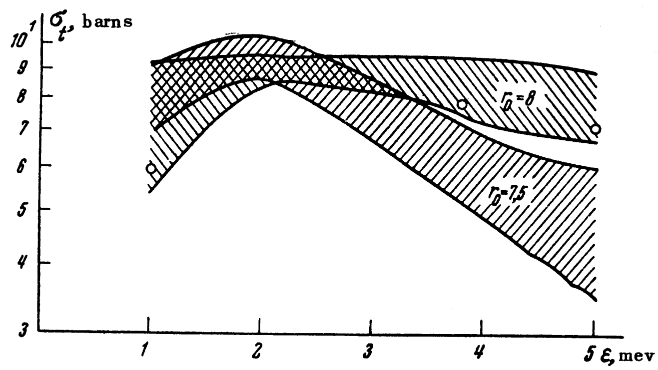

FIG. 1. Graph of total cross section. σ_t lies inside the shaded regions for $r_0 = 8$ and $r_0 = 7.5$ and various values of ζ and a. Experimental values are indicated by the circles.

neutron energy) and the curves of differential cross section $d\sigma/do$ with the experimental values[2] shows that in all cases (with the exception of the case $\epsilon = 1$ mev) the experimental points are between the curves for $r_0 = 7.5 \times 10^{-13}$ cm and $r_0 = 8 \times 10^{-13}$ cm, and usually closer to the latter. Thus the correct value of the radius must lie in the interval $7.5 < r_0 < 8$, and probably closer to $r_0 = 8$ ($r_0 = 8$ corresponds to $R_0 = 1.35$ in the radius formula $R = R_0 \times 10^{-13} A^{1/3}$ cm, and $r_0 = 7.5$ to $R_0 = 1.27$). Since all calculated quantities depend strongly on r_0 and comparitively less on ζ and a, we did not think it reasonable to determine ζ and a from the experimental data. Nevertheless some information about these quantities was obtained. The theoretical value of σ_t for $\epsilon = 1$ mev agrees with the experimental value only for $r_0 = 8$, $\zeta = 0$–0.2 and $a = 0.5$. Values $a=0$ and $\zeta > 0.02$ do not give agreement with experimental values of σ_t for any r_0.

Thus we come to the conclusion that the best agreement with experimental data is achieved for r_0 slightly smaller than 8, and an effective radius edge smear of $2a \sim 1$. These values are in good agreement with those obtained from other sources.[3] With regard to ζ it is only possible to say that for $\epsilon = 1$ mev, $\zeta \sim 0.01$, and with increase in ϵ, apparently does not grow rapidly.

The neutron polarization $P(\theta)$ was calculated from Eq. (5) for $r_0 = 8$, $a = 0.5$ and various values of ζ. It was found that $P(\theta)$ almost does not depend on ζ for neutron energies $\epsilon > 3$ mev, while at 1 mev, where this dependence is stronger, $P(1°)$ changes from 0.5 to 0.8 for variation of from 0 to 0.1. The magnitude of ζ for $\epsilon = 1$ mev is, however, known from the above, and is 0.01 so that we can calculate $P(\theta)$ for neutron energies from 1–5 mev with an accuracy of 10–20%. These curves for the angles $\theta = 1,2,3°$ are shown in Fig. 2.

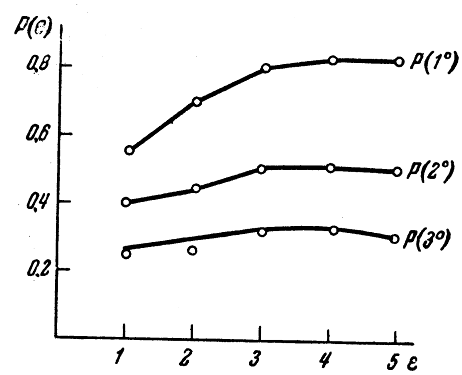

FIG. 2

Comparison of our results to those of Schwinger shows that our calculation leads to a much more rapid decrease of $P(\theta)$ with increasing θ. Schwinger[1] obtained the following values of polarization at an energy $\epsilon = 1$:

$$P(1.5°) = 1;\ P(3°) = 0.8;\ P(6°) = 0.47;\ P(9°) = 0.32.$$

Our calculation gives, at the same energy

$$P(1°) = 0.5;\ P(2°) = 0.45;\ P(3°) = 0.28.$$

It should be noted, as can be seen from Eq. (5), $P(\theta)$ depends only on σ_t and $d\sigma/do$. Since calculations made with our choice of parameters are in good agreement with experimental values of σ_t and $d\sigma/do$, our calculated polarization does not depend on a particular model, and should approach the correct value.

In conclusion, I would like to thank Ia. A. Smorodinskii for his interest in this work.

[1] J. Schwinger, Phys. Rev. 73, 407 (1948).

[2] W. J. Rhein, Phys. Rev. 98, 1300 (1955).

[3] R. D. Woods and D. S. Saxon, Phys. Rev. 95, 577 (1954).

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30

Parapositronium Annihilation Probability, with Account of the First Radiative Corrections

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IT is known that parapositronium in the ground state, as a system of even parity, cannot decay into three photons. Therefore it is interesting to calculate the first radiative corrections to the probability for two photon annihilation,[1] since in the next approximation the lifetime of parapositronium is determined just by these radiative corrections and does not depend on the four photon annihilation.

The positronium annihilation probability W for any order of radiative correction is connected with the annihilation probability W_{Free} of the free particles with zero relative velocity by the equation[2]

$$W = [(\overline{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0))\;/\;(\overline{\psi}^{\mathcal{E}\sigma}_{\text{Free}}(0)\,\psi^{\mathcal{E}\sigma}_{\text{Free}}(0))]\,W_{\text{Free}} \quad (1)$$

where $\psi^{E\sigma}(x)$ is the wave function (in relative coordinates) of the positronium in the ground state, and which satisfies a Bethe-Salpeter equation with the possible annihilation of the particles (Ref. 2; see also Ref. 3). $\psi^{\mathcal{E}\sigma}_{\text{Free}}(x)$ is the wave function of the free particles which turns out to have the same implicit set of functions as $\psi^{E\sigma}(x)$ (but with energy $\mathcal{E} = E + \epsilon$, where $\epsilon > 0$ the binding energy). The sign of σ determines the spin state. In Eq. (1), the quantity $[\overline{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)] = \mathrm{Sp}\,[\overline{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)]$ should be calculated with the same accuracy as is obtained in the calculation of W_{Free}.

If we limit ourselves to the first radiative corrections, it is not hard to calculate $[\overline{\psi}^{E\sigma}(0)\psi^{E\sigma}(0)]$ with the required accuracy to terms including the order e^2 ($\hbar = c = 1$) using the nonrelativistic approximation to the wave function $\psi^{E\sigma}(0)$. In this process we can drop the small terms of the wave functions in $(\overline{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)]$ since they are of order v^2_{rel} ($\sim e^4$). It remains to find the corrections of the order e^2 to the large components of $\psi^{E\sigma}(x)$. This can be done with the aid of ordinary perturbation theory[4] if we use the second approximation[5] to the Hamiltonian for the large components of the wave function. Simple calculations show that the first correction of the nonrelativistic value $\psi_{\text{NR}}(0)$ is of the order e^4, and therefore does not need to be taken into account. To the desired degree of approximation we thus have

$$(\overline{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)) = |\,\psi_{\text{NR}}(0)\,|^2. \quad (2)$$

The wave function $\psi^{\mathcal{E}\sigma}_{\text{Free}}(x_1, x_2)$ of the free particles used in W_{Free} is an implicit function of the full energy and momentum operators, and the full spin (since the orbital angular momentum is zero) and its projections. Since the spin operator does not include the spatial coordinates of the particle, the spatial and spin variables in $\psi^{\mathcal{E}\sigma}_{\text{Free}}$ separate, and the wave function has the form:*

$$\psi^{\mathcal{E}\sigma}_{\text{Free}}(x_1 x_2) = \Phi^{\sigma}(K)\,e^{-iK(x_1+x_2)/2}, \quad (3)$$

where the main component of the total particle momentum K is $K_0 = \mathcal{E} = 2m$, and $K \to 0$ in the center-of-mass system. Therefore, $[\overline{\psi}^{\mathcal{E}\sigma}_{\text{Free}}(0)\,\psi^{\mathcal{E}\sigma}_{\text{Free}}(0)]$ in Eq. (1) is a square of the spin function $\Phi^{\sigma}(K)$, which can always be made equal to unity.

The value of W does not change if we sum in Eq. (1) over all four spin states, since $W_{\text{Free}} = 0$ in all spin states with a total spin $S = 1$. In W_{Free} (in the summation over all spin states) let us replace the summation over the complete set of spin functions ofthe entire spin operator by a summation over the complete set of spin functions which are themselves implicit spin operators of