We note that ΔE in Eq. (6) is the same as in Eqs. (3)-(5). Ferrites, manganites and other ferromagnetics, for which $r_1 > R$, make up group 2; and therefore for them, as also for Ni, according to Eq. (6), it follows that one expects the positive sign of the effect; Gd, Fe, Co and invar alloys make up group 1 ($r_1 < R$), and therefore for them one expects the negative sign of the effect.

In Tables I and II data are presented for the calculations with Eqs. (1)-(6); the results agree satisfactorily with experiment, in particular with the data of neutronographic analysis of the magnetic structure of alloys[2,3].

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* R is computed with the same accuracy (to the fourth decimal place) as r is given experimentally (cf. Ref. 1).

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On the Theory of the Hall Effect in Ferromagnetics

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It has been shown that[1,2] the increase of the ohmic resistivity in the disordering of the alloys Fe_3Al is accompanied by a proportional increase in the Hall constant R, i.e., the relation

$$R - R_0 = a(\rho - \rho_0). \tag{1}$$

exists. An analogous relation was also found for the change of R and ρ in the dispersion hardening. In addition, the question has been posed[1,3] of the relation between R and ρ as the temperature T changes. Karplus and Luttinger[3], as a result of developing and making more precise the theory of Samoilovich and Kon'kov[4], found that R as a function of T is proportional to ρ^2. However, an examination of the existing experimental data did not support this conclusion of the theory: The exponent of degree n proved to be less than 2 and different for different elements[5]. Contrary to these results, the measurements given by us for the alloys Fe-Ni gave the following dependence (see the Figure)

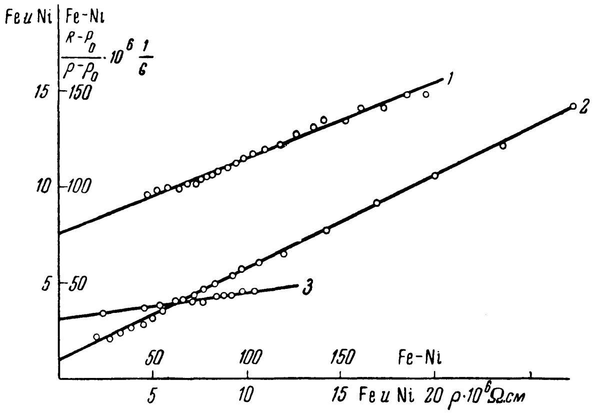

FIG. 1. *1*–Ni, *2*–Fe, *3*–46% Ni + 53.2% Fe.

$$a = a_0 + a_1\rho. \tag{2}$$

In addition, the analysis of experimental data of other authors presented by us showed that the relations (1) and (2) are found to be in good agreement with experiment also for Fe and Ni in the interval from 100° K even to a temperature of the order of 0.8Θ (Θ = Curie point).

From Eqs. (1) and (2) it follows that, besides the Hall effect of the first kind, obeying the theory of Karplus and Luttinger, there exists a Hall effect of the second kind, for which a does not depend on ρ. Thus one can also explain the fractional values of n obtained empirically by the authors mentioned above.

A final solution of the question of the relation between R and ρ is made difficult, however, by the results on the changes of Hall effect at low temperatures, according to which there exists a region where $dR/dT < 0$. The quantum theory of electrical conductivity gives (taking into account the residual resistivity)

$$\rho = \rho_0 + bT^n, \tag{3}$$

where $n = 5$. Meanwhile, the presence of a region of the Hall effect for which $dR/dT < 0$ requires for R the relation

$$R = R_0 + b_1 T^{n_1} + b_2 T^{n_2}, \tag{4}$$

where $b_1 < 0$.

The presence of a negative term in Eq. (4), if it is supported, creates serious difficulties for the present quantum-mechanical theory of the Hall effect and for the establishment of a relation between ρ and R. In particular, it contradicts Eqs. (1) and (2); therefore, further careful investigations are necessary in the region of low temperatures for the resolution of the question of the reality of the existence of the anomalous Hall effect found in Ref. 6.

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The Effect of Ordering on the Energy Spectrum of Phonons

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A METHOD was set forth in Ref. 1 for the study of the vibration spectrum of solid solutions of isotopes. The method devoloped there can be applied to the determination of the spectral density of such solutions in the presence of long-range order. In the case cónsidered, the lattice is divided into two sublattices, for which the probabilities P_i of finding the ith isotope are different and are given by the values

$$P_1^{\mathrm{I}} = 1 - c + \frac{s}{2}; \quad P_1^{\mathrm{II}} = 1 - c - \frac{s}{2}; \tag{1}$$
$$P_2^{\mathrm{I}} = c - \frac{s}{2}; \quad P_2^{\mathrm{II}} = c + \frac{s}{2}.$$

The indices I and II denote the types of sites, c = concentration of the second isotope and the quantity s characterizes the degree of order. The value $s = 1$ ($c = ½$) corresponds to complete ordering of the crystal, the value $s = 0$ to complete disorder.

To make clear the peculiarities of the spectrum, we consider the case of a small difference in masses of the two isotopes. According to the results obtained[1], the spectral density of a system with random distribution of masses can be represented in the form

$$\nu(z) = \nu_0(z) - d\xi/dz, \tag{2}$$

where, for small mass differences,

$$\xi(z) = \frac{1}{N}\Big\{ z\nu_0(z) \sum_{\mathbf{r}} \overline{\varepsilon}_{\mathbf{r}} + z^2 \int\limits_{\omega^2(\mathbf{k})=z} \frac{d\Omega}{|\nabla\omega^2|} ⨍ \frac{d\mathbf{k}'}{z - \omega^2(\mathbf{k}')} \times \sum_{\mathbf{r},\mathbf{r}'} \overline{\varepsilon_{\mathbf{r}}\varepsilon_{\mathbf{r}'}} \exp\{2\pi i(\mathbf{k} - \mathbf{k}')(\mathbf{r} - \mathbf{r}')\} + \ldots \Big\}, \tag{3}$$

$$\varepsilon_{\mathbf{r}} = (\overline{m}_{\mathbf{r}} - m_{\mathbf{r}})/\overline{m}_{\mathbf{r}}. \tag{4}$$

Here z is the square of the frequency, $m_{\mathbf{r}}$ is the value of the mass at site $\mathbf{r}$ ($\mathbf{r} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$, n_i = an integer, $\mathbf{a}_i$ = lattice constants), the bar denotes averaging, N = total number of atoms, $d\Omega$ is an element of area on the surface $\omega^2(\mathbf{k}) = z$ over which integration is to be carried out, $\nabla\omega^2$ is the gradient in the space of the wave vector $\mathbf{k}$, $⨍$ is the principal value of the integral, $\nu_0(z)$ is the unperturbed spectral density; the spectral density is normalized to unity. For a crystal with the distribution of Eq. (1), it is natural to use (for the unperturbed case) the spectral density $\nu_{0\,\mathrm{ord}}$ of a lattice with alternating masses

$$\overline{m}_{\mathrm{I}} = \overline{m}(1 - x), \quad \overline{m}_{\mathrm{II}} = \overline{m}(1 + x), \tag{5}$$

which represent the mean masses at sites of type I and II, respectively. Here $\overline{m} = (1 - c)m_1 + cm_2$, $x = s(m_2 - m_1)/2\overline{m} = s\epsilon/2$. Then

$$\overline{\varepsilon}_{\mathrm{I}} = \overline{\varepsilon}_{\mathrm{II}} = 0, \quad \overline{\varepsilon_{\mathrm{I}}^2} = \left(c - \frac{s}{2}\right)\left(1 - c + \frac{s}{2}\right)\varepsilon^2,$$

$$\overline{\varepsilon_{\mathrm{II}}^2} = \left(c + \frac{s}{2}\right)\left(1 - c - \frac{s}{2}\right)\varepsilon^2$$

and, in accord with (2) and (3), we have