

$$\begin{aligned}
 f_{\xi}(u) &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{m=1}^N \frac{1}{\tau} & (5) \\
 &\times \int_{(m-1)\tau}^{m\tau} \exp \{iuA_m \cos(\omega_0 t + \varphi_m)\} dt \\
 &+ \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{m=1}^N \frac{1}{\tau} \int_0^{\tau} iu\dot{A}_m \theta \cos(\omega_0 \theta + \varphi_m) \\
 &\times \exp \{iuA_m \cos(\omega_0 \theta + \varphi_m)\} d\theta \\
 &- \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{m=1}^N \frac{1}{\tau} \int_0^{\tau} iuA_m \dot{\varphi}_m \theta \sin(\omega_0 \theta + \varphi_m) \\
 &\times \exp \{iuA_m \cos(\omega_0 \theta + \varphi_m)\} d\theta.
 \end{aligned}$$

Here  $A_m$ ,  $\varphi_m$ ,  $\dot{A}_m$ , and  $\dot{\varphi}_m$  are the values of  $A(t)$ ,  $\varphi(t)$ , and their derivatives at the beginning of the  $m$ th interval. It is easy to see that the first of the sums in Eq. (5) gives Eq. (1); the other two sums give the correction depending on the finiteness of the rate of change of  $A$  and  $\varphi$ .

If we use the well-known expression determining the probability density of a stochastic variable in terms of its characteristic function, we obtain from Eq. (5),

$$\begin{aligned}
 w_{\xi}(\xi) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} f_{\xi}(u) e^{-iu\xi} du & (6) \\
 &= \frac{1}{\pi \sqrt{A^2 - \xi^2}} \left( 1 + \frac{\dot{\varphi}}{\omega_0} \right)
 \end{aligned}$$

(In this transformation the second sum in Eq. (5) gives zero.)

In this way Eq. (6) agrees with Eq. (2) to the accuracy of the correction term

$$\frac{\dot{\varphi}}{\omega_0 \sqrt{A^2 - \xi^2}}. \quad (7)$$

We note that although for a stationary process  $\dot{\varphi}$  must be equal to zero, the average value, Eq. (7), may differ from zero, since in increasing the passband,  $A$  and  $\dot{\varphi}$  in the general case are no longer independent random variables. It is interesting that the rate of change of the amplitude does not influence the probability density  $w_{\xi}(\xi)$  in the first approximation.

We note in closing that in the case in which  $\xi$  is obtained by use of a narrow-band filter from a quasi-monochromatic stationary stochastic

process, obeying the normal probability distribution law, the correction term in Eq. (6) disappears, and we are left with the correct formula, Eq. (2). This agrees with a well-known property of the normal distribution relating to the linear transformation of the spectral components.

<sup>1</sup>Blanc-Lapierre, Savelli and Tortrat, Ann. Télécomm. 9, 237 (1954).

<sup>2</sup>S. M. Rytov, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 702 (1955); Soviet Phys. JETP

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### Atomic Magnetic Moments of Ferromagnetic Metals and Alloys

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IN contemporary physics it is well known that on account of the difficulties associated with the many-electron problem, the relation for the quantitative calculation of the atomic magnetic moments  $m_{\varphi}$  and  $m_{\pi}$ , for the ferromagnetic and paramagnetic states of ferromagnetics, respectively, has not thus far been found. The relation proposed by us was obtained empirically.

1. *Pure metals.* It is well known that  $m$  is expressed as a fractional number of Bohr magnetons  $M_B$  (see Table I), and that this fraction is due to the exchange interaction between the  $s$ -electrons of the metal and the  $d$ - (or  $f$ -) electrons of the atoms, as a consequence of which  $m$  must somehow depend on the lattice parameter. On this basis we propose that in the equation for  $m$ , as well as in Eqs. (3)-(6), there enters the term

$$E = 0.641 [n_1(r_1 - R) + n_2(r_2 - R)] = E_0 + \Delta E, \quad (1)$$

where  $n_1$  is the number of nearest neighbors of an atom in the lattice,  $n_2$  the number of next-nearest neighbors,  $r_1$  and  $r_2$  the corresponding interatomic distances,  $R$  an empirical constant characterizing the given transition element,  $E_0$  the integral part of  $E$ , and  $\Delta E$  the fractional remainder,  $|\Delta E| < 1$ .

Let us first examine the first transition series of the periodic table of elements. For it\*,

$$R = 0.0325 Z^2 + \begin{cases} 30.6182 - 1.9175 Z & \text{for } Z \geq 26 \\ 27.2382 - 1.7875 Z & \text{for } Z \leq 26, \end{cases} \quad (2)$$

TABLE I. Atomic magnetic moments of ferromagnetic metals and alloys.

Metal	$N_{d(f)}$	$n_1(r_1, \text{\AA})$	$n_2(r_2, \text{\AA})$	$R, \text{\AA}$	Group	$E (\Delta E)$	$m, M_B$		Relative Error %
							calc.	exptl.	
Ni $\varphi$ $\pi$	2	12 (2.4868)		2.4082	2	0.6 (0,6)	0,6	0.604	< 1
		12 (2.4868)				0.6 (0,6)	1,6	1.61	< 1
Co $\varphi$ $\pi$	3	6 (2.499)	6 (2.507)	2.5382	1	-0.27 (-0,27)	1.73	1.715	1
		6 (2.499)	6 (2.507)			-0.27 (-0,27)	3.27	3.27	1
Fe $\varphi$ $\pi$	4	8 (2.4777)	6 (2.8610)	2.7332	1	-0.82 (-0,82)	2.18	2.217	1.6
		8 (2.4777)				-1.31 (-0,31)	3.31	3.37	2
Gd $\varphi$ $\pi$	1 (7)	6 (3.561)	6 (3.629)	3.566	1	0.22 (0,22)	7.22	7.12	1
		6 (3.561)				0.22 (0,22)	7.78	7.8-7,94	1
Dy $\varphi$ $\pi$	4	6 (3.506)	6 (3.585)	2.610	2	7.20 (0,20)	9.20	8.7*	3.8
		6 (3.506)				7.20 (0,20)	10.20	10.59	
Er $\varphi$ $\pi$	2	6(3.466)	6 (3.539)	2.325	2	9.06 (0,06)	9.06	>8.6	
		6(3.466)				9.06 (0,06)	10.06	9.53	5
Mn (in MnAs)	5	2 (2.84)	6 (3.71)	2.8632	1	3.25 (0,25)	3.25	3.40	4.6
Mn (in MnBi)	5	2 (3.06)	6 (4.30)	2.8632	2	5.80 (0,80)	3.20	3.52	10
Mn (in MnCu <sub>2</sub> Al)	5	12 (4.17)		2.8632	2	10.06 (0,06)	3.94	4.06	3
Cr (in CrTe)	4	2 (3.105)	6 (3.981)	3.0582	2	3.61 (0,61)	2.39	2.39	<< 1
Cr (in CrSb)	4	2 (2.726)	6 (4.127)	3.0582	1	3.7 (0,7)	2.7	2.7	< 1
Cr (in CrO <sub>2</sub> ) $\varphi$ $\pi$	4	2 (2.86)	8 (3.429)	3.0582	1	5.1 (0,1)	2.1	2.07	>> 1
			4 (4.41)		1	5.1 (0,1)	2.9	2.95	2
Fe (in Ni <sub>3</sub> Fe)	4	2.508 = $r_{ab}$	6 (3.546)	2.7332	2	3.13 (0,13)	2.87		
Ni (in Ni <sub>3</sub> Fe)	2	2.489 = $R_{ab}$	8 (2.508)	2.4082		0.51 (0,51)	$\frac{-0.49}{2.38^{***}}$ $m=1.19^{***}$	$\frac{[2,3]}{\pm 2.35 \pm 0.20}$ 1.21	max 8 2
Mn (in Ni <sub>3</sub> Mn)	5	2.543 = $r_{ab}$	6 (3.596)	2.8632	2	2.82 (0,82)	3.18		
Ni (in Ni <sub>3</sub> Mn)	2	2.522 = $R_a$	8 (2.543)	2.4082	2	0.69 (0,69)	$\frac{-0.31}{2.87^{**}}$ $m=1.03^{***}$	$\frac{[2,3]}{\pm 2.88 \pm 0.20}$ 1.01	max 8 2

\* The moment for dysprosium depends on the field. Here the moment is given for the maximal field.

\*\* The difference of the moments of the components of the alloy:  $m_{Fe} - m_{Ni}$  and  $m_{Mn} - m_{Ni}$ .

\*\*\* The average moment per atom of the alloy.

where  $Z$  is the atomic number of the element.

For the series of elements under consideration,  $R = R_s + R_d$  (to an accuracy of 1%), where  $R_s$  and  $R_d$  are the radii, respectively, of the  $s$ - and  $d$ -shells of the isolated atom, calculated in the manner of Slater. Let us assume that an analogous relation  $R = R_s + R_f$  ( $R_f$  = radius of the  $f$ -shell of the isolated atom) holds also for Gd, Dy and Er (the third transition series), by the use of which  $R$  is calculated for these metals also.

The pure ferromagnetic metals can be divided into two groups: group 1 consists of Gd, Fe and Co, for which  $r_1 < R$ ; group 2 consists of Ni, Dy and Er, for which  $r_1 > R$ . For the indicated groups

$$m_{\varphi\pi} = N_d + N_f - 2 + E_0 + \frac{i + \Delta E_{\varphi}}{1 + i \mp \Delta E_{\pi}}, \quad (3)$$

where  $N_d$  and  $N_f$  are the numbers of unpaired  $d$ - and  $f$ -electrons, respectively, in the atom;  $i = 1$  for group 1,  $i = 0$  for group 2. Here and in what follows, the upper signs in front of  $\Delta E$  or  $E$  are for group 1, the lower signs for group 2. We note that in front of the quantity  $\Delta E$ , related to the exchange interaction, the opposite sign appears in (3) for group 1. This recalls the change of the positive exchange integral to a negative one in the transition from the ferromagnetic state to the paramagnetic. The integer  $M_B$  in  $m_{\pi}$  is one unit larger than in  $m_{\varphi}$ .

2. *Alloys.* All transition metals, which are paramagnetic in pure form, when entering into ferromagnetic alloys are considered as "pure ferromagnetic" metals (the non-transition elements in alloys are not considered). For them

TABLE II. Curie points of ferromagnetic metals.

Metal	$n_1$	$N_{d(f)}$	$\Theta_{d(f)}$ °K	°K		$\frac{d\Theta}{dP} \cdot 10^3 \cdot \frac{\text{deg}}{\text{atm}}$	
				calc.	exptl.	calc.	exptl.
Ni	12	2	576*	600	631	+0.40	0.35±0.01; [4] 0.441
Co	12	3	1296*	1409	1393	-0.18	—
Fe	8	4	1024*	1045	1043	-0.73	—
Gd	6	7	252	286	289	-1.22	-1.2±0.05
Tb	6	6	216	—	200	—	[4]
Dy	6	4	144	156	155	—	—
Ho	6	3	108	—	—	—	—
Er	6	2	72	80	80	—	—
Tu	6	1	36	—	10?	—	—

\*  $A = 230 k$  (experiment:  $220 k$ ) for Ni,  $184 k$  (experiment:  $183 k$ ) for Fe, and  $946 k$  for Co.  $A$  for groups 1 and 2 corresponds to the increasing and decreasing branches of the curve  $A(r_1/R)$ .

$$m_{\varphi\pi} = N_d - 2 + \frac{i \pm \Delta E_{\varphi}}{1 \mp \Delta E_{\pi}}, \quad (4)$$

where  $i = 0$  for group 1 and  $i = 1$  for group 2.

The binary alloys are characterized by  $E_a$ ,  $E_b$  and  $E_{ab} = 0.641 n_{ab} (r_{ab} - R_{ab})$ ;  $R_{ab} = c_a R_a + c_b R_b$ , where  $c_a$  and  $c_b$  are the atomic concentrations of components  $a$  and  $b$ , respectively. For  $\text{Ni}_3\text{Fe}$  and  $\text{Ni}_3\text{Mn}$ ,  $E_{ab} = 0$  (Table I).

3. *Curie points  $\Theta$  and their variation with pressure  $P$ .* Let us suppose that

$$\Theta_{d(f)} = n_1^2 N_d^2 (N_f^1); \quad (5)$$

for Ni, Co and Fe:  $A = (1 \pm \Delta E_{\varphi}) k \Theta_{d(f)}$ ,

where  $A$  = exchange integral,  $k$  = Boltzmann constant and  $n_1$  is given in Table II. The change of the Curie point under pressure in units of  $10^3$  atm is

$$\Theta = \Theta_{d(f)} / 1 - 0.1(1 \pm \Delta E);$$

$$\frac{d\Theta}{dP} \cdot 10^3 \frac{\text{deg}}{\text{atm}} = \mp (1 \pm \Delta E). \quad (6)$$

We note that  $\Delta E$  in Eq. (6) is the same as in Eqs. (3)-(5). Ferrites, manganites and other ferromagnetics, for which  $r_1 > R$ , make up group 2; and therefore for them, as also for Ni, according to Eq. (6), it follows that one expects the positive sign of the effect; Gd, Fe, Co and invar alloys make up group 1 ( $r_1 < R$ ), and therefore for them one expects the negative sign of the effect.

In Tables I and II data are presented for the calculations with Eqs. (1)-(6); the results agree satisfactorily with experiment, in particular with the data of neutronographic analysis of the magnetic structure of alloys<sup>2,3</sup>.

\*  $R$  is computed with the same accuracy (to the fourth decimal place) as  $r$  is given experimentally (cf. Ref. 1).

<sup>1</sup> S. V. Vonsovskii, *The Contemporary Study of Magnetism*, Moscow, 1953.

<sup>2</sup> C. G. Shull and M. K. Wilkinson, *Phys. Rev.* **97**, 304 (1955).

<sup>3</sup> G. E. Bacon, *Neutron Diffraction*, Oxford, 1955.

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### On the Theory of the Hall Effect in Ferromagnetics

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It has been shown that<sup>1,2</sup> the increase of the ohmic resistivity in the disordering of the alloys  $\text{Fe}_3\text{Al}$  is accompanied by a proportional increase in the Hall constant  $R$ , i.e., the relation

$$R - R_0 = a(\rho - \rho_0). \quad (1)$$

exists. An analogous relation was also found for the change of  $R$  and  $\rho$  in the dispersion hardening. In addition, the question has been posed<sup>1,3</sup> of the relation between  $R$  and  $\rho$  as the temperature  $T$  changes. Karplus and Luttinger<sup>3</sup>, as a result of developing and making more precise the theory of Samoilovich and Kon'kov<sup>4</sup>, found that  $R$  as a function of  $T$  is proportional to  $\rho^2$ . However, an examination of the existing experimental data did not

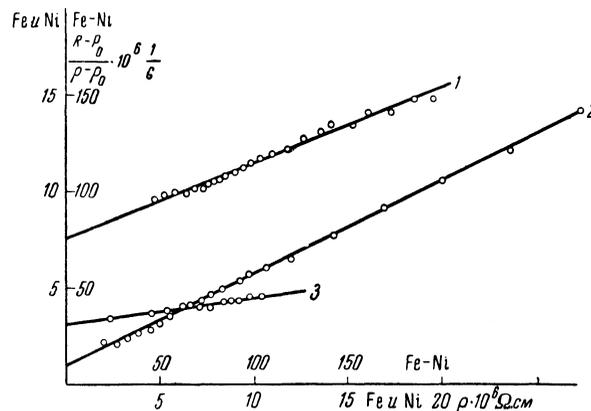


FIG. 1. 1—Ni, 2—Fe, 3—46% Ni + 53.2% Fe.

support this conclusion of the theory: The exponent of degree  $n$  proved to be less than 2 and different for different elements<sup>5</sup>. Contrary to these results, the measurements given by us for the alloys Fe-Ni gave the following dependence (see the Figure)

$$a = a_0 + a_1\rho. \quad (2)$$

In addition, the analysis of experimental data of other authors presented by us showed that the relations (1) and (2) are found to be in good agreement with experiment also for Fe and Ni in the interval from 100°K even to a temperature of the order of 0.8 $\Theta$  ( $\Theta$  = Curie point).

From Eqs. (1) and (2) it follows that, besides the Hall effect of the first kind, obeying the theory of Karplus and Luttinger, there exists a Hall effect of the second kind, for which  $a$  does not depend on  $\rho$ . Thus one can also explain the fractional values of  $n$  obtained empirically by the authors mentioned above.

A final solution of the question of the relation between  $R$  and  $\rho$  is made difficult, however, by the results on the changes of Hall effect at low temperatures, according to which there exists a region where  $dR/dT < 0$ . The quantum theory of electrical conductivity gives (taking into account the residual resistivity)

$$\rho = \rho_0 + bT^n, \quad (3)$$

where  $n = 5$ . Meanwhile, the presence of a region of the Hall effect for which  $dR/dT < 0$  requires for  $R$  the relation