"initial condition" div E = 0.

Note added in proof: After this letter had been submitted for publication the author learned of an analogous article by Case (see Ref. 5) in which, however, the case of zero rest mass is not considered.

[1] E. Schrödinger, Proc. Roy. Soc. (London) A229, 39 (1955).

[2] K. M. Case, Phys. Rev. 99, 1572 (1955).

[3] F. I. Fedorov, Dokl. Akad. Nauk SSSR 82, 37 (1952).

[4] H. J. Bhabha, Rev. Mod. Phys. 21, 451 (1946).

[5] K. M. Case, Phys. Rev. 100, 1513 (1955).

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On the Theories of Multiple Meson Production

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IN the calculations of effective cross sections for multiple production of mesons, integrations are carried out over the momenta (or over the energies) and over the angles of emergence of the product particles. Since exact calculations, taking into account all the conservation laws, are accompanied by great difficulties, various approximate methods of calculation are used. In such calculations the maximum limit for the energy of one of the particles is taken to be equal to the value $\epsilon_n = \mathcal{E} - \sum_{i=1}^{n-1} m_i$ ($\mathcal{E}$ is the total energy of colliding particles, Σm_i is the sum of the rest masses of all the product particles except the given one). The object of this article is to show that these values should be diminished as required by the laws of conservation of energy and momentum. The calculation of the maximum energy and momentum for each product particle is reduced to the problem of finding a conditional extremum.

We shall carry out the calculation in the system of the center of mass. We assume that after the collision n particles in all are formed. We denote the total energy, the momentum and the velocity of the ith product particle by ϵ_i, $\mathbf{k}_i$, v_i, and the corresponding quantities for the product particle being investigated we shall denote by the index n. Then the laws of the conservation of energy and momentum will take on the form:

$$\varepsilon_n + \sum_1^{n-1} \varepsilon_i = \mathcal{E}, \quad \mathbf{k}_n + \sum_1^{n-1} \mathbf{k}_i = 0. \tag{1}$$

It is evident that in order to make k_n a maximum it is necessary that the momenta of all the $(n-1)$ particles should be directed opposite to the momentum $\mathbf{k}_n$; then

$$k_n = -\sum_1^{n-1} k_i. \tag{2}$$

Let us find v_n:

$$v_n = k_n / \varepsilon_n = \sum_1^{n-1} k_i / (\mathcal{E} - \sum_1^{n-1} \varepsilon_i). \tag{3}$$

It is necessary to find the maximum of the function $v_n(k_1, \ldots, k_{n-1})$ which is defined by Eq. (3) under the condition

$$\left[m_n^2 + \left(\sum_1^{n-1} k_i\right)^2\right]^{1/2} + \sum_1^{n-1} (m_i^2 + k_i^2)^{1/2} = \mathcal{E}, \tag{4}$$

which follows from (1) and from the relations $\epsilon_i = m_i^2 + k_i^2$. The problem is reduced to the solution of the system of equations

$$\partial F(k_1, \ldots, k_{n-1}) / \partial k_j = 0, \quad (j = 1, \ldots, n-1), \tag{5}$$

where $F \equiv v_n + \lambda \mathcal{E}$.

In its expanded form Eq. (5) takes on the form:

$$\frac{1}{\mathcal{E} - \Sigma\varepsilon_i} + \frac{v_j \Sigma k_i}{(\mathcal{E} - \Sigma\varepsilon_i)^2} + \lambda\left[\frac{\Sigma k_i}{\mathcal{E} - \Sigma\varepsilon_i} + v_j\right] = 0. \tag{6}$$

Since λ must satisfy all the equations of the system the equations must be satisfied identically; from this it follows that $v_1 = v_2 = \ldots = v_{n-1} = v$. Then the $(n-1)$ particles may be considered as a single particle with the mass $M = \sum_{i=1}^{n-1} m_i$ and the velocity v.

From (4) we find

$$k_{n\,\max} = [(\mathcal{E}^2 - M^2 + m_n^2)^2 - 4m_n^2\,\mathcal{E}^2]^{1/2} / 2\mathcal{E}, \tag{7}$$

$$\varepsilon_{n\,\max} = {}^1/_2\,[\mathcal{E} - (M^2 - m_n^2) / \mathcal{E}],$$

$$v_{n\,\max} = [(\mathcal{E}^2 - M^2 + m_n^2)^2 - 4m_n^2\mathcal{E}^2]^{1/2} / (\mathcal{E}^2 - M^2 + m_n^2).$$

Sternheimer[1] has calculated the maximum angle of recoil of the nucleon after a collision.

$$\operatorname{tg} \theta_{\max} = (1 - v_c^2)^{1/2}\,(v_c^2 / v_{\max}^2 - 1)^{-1/2} \tag{8}$$

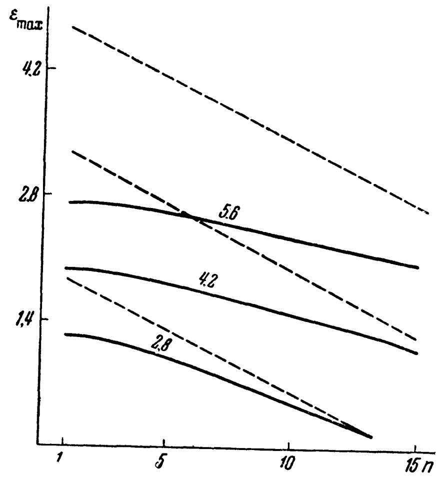

FIG. 1. Maximum energy of a π-meson formed as a result of a $\pi-N$ collision.

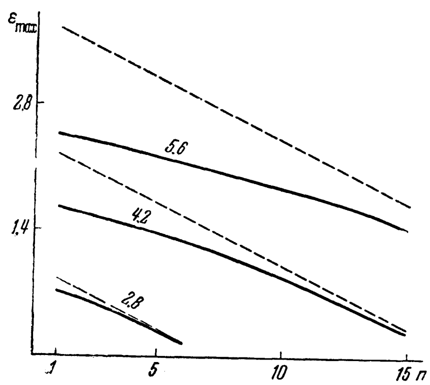

FIG. 2. Maximum energy of a π-meson formed as a result of an $N-N$ collision.

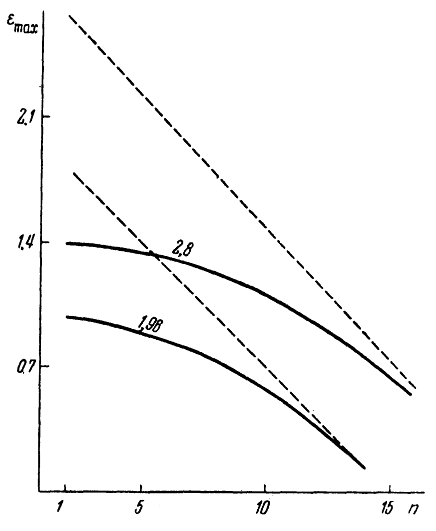

FIG. 3. Maximum energy of a π-meson formed as a result of the annihilation of a nucleon and an antinucleon.

(θ_{max} is the angle in the laboratory coordinate system, v_c the velocity of the system of the center-of-mass in the laboratory system). The author proposes to use θ_{max} as a criterion for identifying particles. Evidently the existence of a maximum angle for nucleon recoil should be taken into account during integration over the angle θ.

In Figs. 1-3 the solid curves represent maximum meson energies in certain reactions. The number of π-mesons produced is plotted along the horizontal axis; the numbers labeling the curves give the total energy of the system; the dotted curves represent results which take into account only the law of conservation of energy which, measured in the system of center-of-mass, is equal to 10^9 ev.

I express my thanks to Prof. D. Ivanenko for discussions.

[1] R. M. Sternheimer, Phys. Rev. 93, 642 (1954).

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Polarization of Neutrons Scattered by Carbon Taking Nuclear Volume into Account

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EXPERIMENTS on the double scattering of high energy nucleons by nuclei indicate a considerable polarization of the primary scattered beam[1]. A comparison of the data for light elements (H, Be, He, C) shows that the polarization is independent of the kind of nucleus and shows that the angular distribution is practically unchanged for energies in the range 100-300 mev. One may assume that the scattered particle interacts with the nucleus as a whole. In order to explain polarization accompanying scattering by nuclei with spin 0 it is useful to take into account spin-orbit forces of the form

$$U_{\mathrm{s.o.}} = (a/r)(dU_1/dr)\,\sigma \mathbf{l}. \quad (1)$$

The complete interaction potential between the nucleon and the nucleus may in such a case be written in the form