finite. The transition to the point particle is accomplished after the renormalization so that infinite expressions are absent not only from the final equations (28,21) and (45,37), (46,37), but also from every stage of the calculation. The attempt to quantize the renormalized equation is a fascinating, but obviously not simple, problem.

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SOVIET PHYSICS JETP VOLUME 4, NUMBER 1 FEBRUARY, 1957

Temperature Dependence of the Viscosity of Liquid Nitrogen At Constant Density

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The results of measurements on the temperature dependence of the viscosity of liquid nitrogen at constant density are presented. These measurements were conducted over the density interval from $\rho = 0.38$ to $\rho = 0.86$ gm/cm^3 for temperatures ranging from the condensation temperature at each density to 300° K. The gas-like character of the variation of viscosity with temperature in liquid nitrogen at low densities is revealed for the first time.

IN 1950 Verkin and Rudenko[1] first investigated the temperature dependence of the viscosity of liquefied nitrogen and argon for constant values of the density. These experiments, in which it proved possible to distinguish quite fully the effects of the two factors of temperature and variation in molar volume upon the coefficient of viscosity, were conducted over a narrow density range.

The purpose of the present experiment has been to extend such an investigation to cover a wide range of densities--from a value of the density near that at the triple point to the value at the critical point. Liquid nitrogen was selected as the subject of the experiment, inasmuch as it belongs to the class of simple liquids; i.e., nonpolar, nonassociative liquids consisting of particles having spherical or nearly spherical symmetry. The full and systematic study of the properties of such substances is of great interest in connection with the problem of constructing a theory of the liquid state.

APPARATUS AND METHOD OF MEASUREMENT

In investigating the viscosity of this liquid use was made of the viscometer constructed by Verkin and Rudenko[1] and loaned for the present experiment by the Low-Temperature Laboratory of the Physico-Technical Institute, Academy of Sciences, Ukrainian SSR. The viscosity was determined from the time required for a cylindrical weight to fall along a tube of slightly greater diameter filled with the liquid to be investigated. Since pressures as high as 3000-4000 kg/cm^2 are developed as the temperature is increased with the density held constant, the tube and its contents were enclosed in a thick-walled bomb. Tube, weight and bomb were all made from the same material--beryllium bronze--and were placed within a metal Dewar flask. A core of iron-nickel magnetic alloy was pressed into the weight. Two pairs of induction coils were slipped onto the bomb, and their leads were connected in series either to a galvanometer or to an amplifier and oscillograph, with which it was possible to follow the motion of the weight down the tube through the liquid being studied, and to record the time of fall of the weight between two predetermined sections of the coils.

The experimental arrangement was such that the weight moved between the induction coils at a constant velocity, while no effect on the time of fall due to the ends of the bomb could be detected within the limits of accuracy of the measurement.

The measurement of the coefficient of viscosity of liquid nitrogen was made by means of a relative method. The value of the viscosity coefficient at the experimental temperature was determined from the formula

$$\eta_t = \eta_0 \tau_t (\rho - \rho_t) / \tau_0 (\rho - \rho_0), \qquad (1)$$

where η_0 and τ_0 are, respectively, the viscosity of nitrogen and the time of fall at $T = 18°$ C and $P = 1$ atm, τ_t is the time of fall of the weight in the liquid being studied, ρ_t is the density of the liquid, ρ_0 is the density of nitrogen at $T = 18°$ C and $P = 1$ atm, and ρ is the density of the weight. Since $\rho_0 << \rho$, we have

$$\eta_t = \eta_0 \tau_t (\rho - \rho_t) / \tau_0 \rho.$$

The maximum error in the determination of η_t did not exceed 6%.

Before being condensed into the viscometer, the liquid nitrogen was subjected to repeated distillation to eliminate solidified contaminants. The density of the liquid was determined from tabular data on the dependence of ρ upon T under equilibrium vapor pressure as it was condensed into the apparatus and sealed into the bomb.

EXPERIMENTAL RESULTS

The results of the measurements on the viscosity of liquid nitrogen are represented in Fig. 1; the fluidity φ is measured in CGS units along the y-axis, and the temperature is measured along the x-axis. Curves *1, 2, 3* and *4* are taken from the work of Verkin and Rudenko; the points along curve *4*, which were obtained in the present experiment, agree well with the values found previously[1] The remaining curves (*5, 6, 7, 8, 9, 10, 11*) have been determined here for the first time*.

On the basis of the character of their curves and their work on the viscosity of compressed nitrogen[2,3], Verkin and Rudenko suggested that "the temperature coefficient of fluidity (viscosity) for A and N_2 changes sign as the density region near the critical density is approached." This prediction has received experimental verification in the present work; the change in the character of the temperature dependence of the viscosity of liquid nitrogen, however, does not take place in the vicinity of the critical density, but rather at a density having approximately twice the value at the critical point, as may be seen from the graph above.

From consideration of Fig. 1 it is evident that there are two regions in liquid nitrogen, in which the temperature dependence of the viscosity (fluidity) has different characteristics.

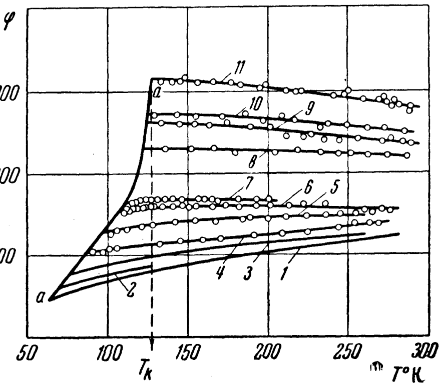

FIG. 1. Temperature dependence of viscosity in liquid nitrogen: *a* – under equilibrium vapor pressure; ρ equal to: *1* – 0.861, *2* – 0.832, *3* – 0.798, *4* – 0.746, *5* – 0.69, *6* – 0.63, *7* – 0.60, *8* – 0.50, *9* – 0.47, *10* – 0.43, *11* – 0.38 gm/cm³.

For liquid nitrogen of density $\rho > 2\rho_c$, where ρ_c is the density at the critical point, the viscosity decreases with increasing temperature, as is usual for liquids. For densities $\rho < 2\rho_c$ the viscosity increases with temperature, as is characteristic of gases.

In Fig. 1, curve *7* is distinguished by the fact that for the corresponding liquid nitrogen density $\rho \sim 2\rho_c$ the fluidity is independent of the temperature. From this same Figure it can be seen that the fluidity of liquid nitrogen under equilibrium vapor pressure does not yield a straight line in the coordinates $\varphi - T$, as was suggested by Rudenko[4]. For $\rho < 0.6$ gm/cm³, the experimental points diverge from a straight line, the fluidity

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* A. Voronel' took part in the measurements for curves *4, 5, 6* and *7*.

varying with temperature more rapidly than would be the case for a linear dependence.

The family of viscosity isotherms is plotted in Fig. 2. Values of the density ρ are given along the x-axis, and those of η, in CGS units, along the y-axis (these isotherms were obtained by rearrangement of the data of Fig. 1). It can be seen from the Figure that all of the isotherms intersect approximately in a point which for liquid nitrogen corresponds to a value for the density $\rho \approx 2\rho_c$.

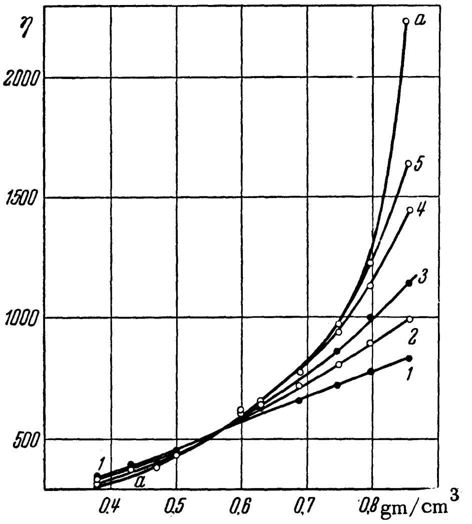

FIG. 2. Isotherms of nitrogen viscosity against density. Density in gm/cm^3 along the x-axis, viscosity, multiplied by 10^6, in CGS units along the y-axis. T equals: *1* – 273°, *2* – 200°, *3* – 150°, *4* – 100°, *5* – 85°K; *a* – *a* under equilibrium vapor pressure.

Theoretical investigation of the viscous properties of liquids in the neighborhood of the triple point[5] leads to the dependence

$$\eta = Ae^{U/kT}, \qquad (2)$$

where U, the activation energy, is a characteristic of the interaction between the liquid particles. U is usually taken to be independent of the temperature. It appeared desirable to investigate the applicability of Eq. (2) as a description of the experimental data for liquid nitrogen in the range $2\rho_c < \rho < \rho_{tr}$; i.e., in the region over which

$$\partial\varphi/\partial T\,|_{\rho=\mathrm{const}} > 0,$$

this being regarded as the fundamental feature of the viscous properties of liquids. From the given data it is possible to compute the magnitude of the activation energy U for various values of the liquid density. The values for U can be determined from the relation $\ln\eta = A + U/kT$, which, generally speaking, is correct only for values of the viscosity determined under conditions of constant molar volume.

In Fig. 3, $\ln\eta$ is plotted along the y-axis against $1/T$ along the x-axis. It is evident from the Figure that within the region for which $\partial\varphi/\partial T\,|_{\rho=\mathrm{const}}$ a linear dependence of $\ln\eta$ upon $1/T$ holds over the entire temperature range in which nitrogen exists in the liquid state. As they pass through the critical temperature, these straight lines merge smoothly into curves bending towards the x-axis. It is impossible to determine precisely the temperature at which the curvature begins, but it is clearly close to T_c. Thus, the temperature dependence of the viscosity of liquid nitrogen for constant values of the density $\rho > 0.6$ gm/cm$^3 \simeq 2\rho_c$ can be described by use of Eq. (2).

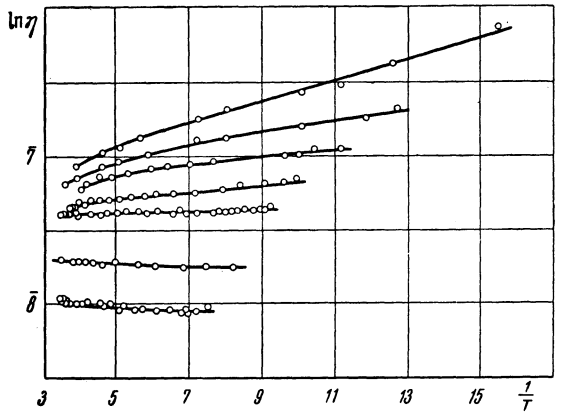

FIG. 3. Dependence of $\ln\eta$ on $1/T$ for liquid nitrogen at constant density.

The values obtained for the activation energy U show that it decreases with increasing molar volume for $U = 140$ cal/mol at $V = 32.6$ cm^3/mol to $U = 60$ cal/mol at $V = 40.6$ cm^3/mol.

The results cited above for the measurement of the temperature dependence of the viscosity of liquid nitrogen for constant values of the density permit the following conclusions to be drawn:

1. The viscosity (fluidity) of liquid nitrogen for constant values of the density depends very strongly upon the temperature. For $\rho > 2\rho_c$ the temperature dependence of the viscosity can be described by an exponential law, up to temperatures in the vicinity of T_c.

2. For liquid nitrogen

$$\text{for } \rho > 2\rho_{\text{к}} \quad (\partial\varphi/\partial T)_\rho > 0,$$

$$\text{for } \rho < 2\rho_{\text{к}} \quad (\partial\varphi/\partial T)_\rho < 0.$$

It may be suggested that the change in the character of the temperature dependence of the viscosity of nitrogen at a density near $2\rho_c$ is connected with a change in the mechanism of the viscosity. For $\rho > 2\rho_c$ the mechanism (liquid-like) prevails, while a different (gas-like) mechanism prevails for $\rho < 2\rho_c$.

In conclusion, I regard it as a pleasant duty to thank B. I. Verkin and I. S. Rudenko for directing this work.

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Production of Positive Pions in (p– p) Collisions at 660 Mev

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Using nuclear emulsions the energy spectra of positive pions produced in the reaction $p + p \rightarrow \pi^+ + \left\{ \begin{array}{l} d \\ p+n \end{array} \right.$ at an energy of 657 ± 8 mev have been measured at angles, θ_π equal to 60°, 75°, 90°, 105° and 120° in the laboratory system. The angular dependence of the cross section $d\sigma/d\omega\,(\theta^*)$ in the center-of-mass system of the colliding nucleons and the total cross section for the reactions: $p + p \rightarrow \pi^+ + \left\{ \begin{array}{l} d \\ p+n \end{array} \right.$ and $p + p \rightarrow p + n + \pi^+$ has been determined. The results are consistent with the assumption that the mesons are produced chiefly in p-states. In addition to transitions to the S-state of the final ($n - p$) system, it is found that an important contribution is made by transitions in which the final state is a P-state.

1. INTRODUCTION

IN ($p-p$) collisions at an incident energy of 660 mev there is a strong probability for meson production in addition to elastic scattering. Experiments in which elastic proton-proton scattering has been studied [1] and measurements of the total cross section for ($p - p$) interactions [2] indicate that in this energy region the probability for meson production is only slightly smaller than that for elastic scattering. Meson production can occur via the three following reactions:

$$p + p \rightarrow d + \pi^+; \qquad (1)$$

$$p + p \rightarrow + p + n + \pi^+; \qquad (2)$$

$$p + p \rightarrow p + p + \pi^0. \qquad (3)$$

Experimental studies in which (1) has been investigated[1] yield an angular dependence* of the cross section in the form $A + B\cos^2\theta^*$ which remains substantially constant over the energy range $E_p = 460$-660 mev[3,4]. Possible transitions to the $^3S_1$ state of the final ($n-p$) system are

$$^1S_0 \rightarrow {}^3S_1p_0; \quad {}^1D_2 \rightarrow {}^3S_1p_2; \quad {}^3P_1 \rightarrow {}^3S_1s_1.$$

(Here we are using the nomenclature given by Rosenfeld[5] in designating the final state of the system and it is assumed that the mesons are emitted only in s-states and p-states.) Analysis

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* The asterisk denotes quantities measured in the center-of-mass system (c.m.) of the colliding nucleons.