TABLE I

Number of Mesons	Charge distribution in $\bar{p}p$ and $n\bar{n}$ annihilation: Charge States	Charge distribution in $\bar{p}p$ and $n\bar{n}$ annihilation: Relative statistical weight	Charge distribution in $\bar{p}n$ annihilation: Charge States	Charge distribution in $\bar{p}n$ annihilation: Relative statistical weight
2	0 0	0.167	− 0	1
	+ −	0.833		
3	0 0 0	0.150	+ − −	0.700
	+ − 0	0.850	− 0 0	0.300
4	+ + − −	0.400	+ − − 0	0.800
	+ − 0 0	0.578	− 0 0 0	0.200
	0 0 0 0	0.022		
5	+ + − − 0	0.640	+ + − − −	0.286
	+ − 0 0 0	0.340	+ − − 0 0	0.629
	0 0 0 0 0	0.020	− 0 0 0 0	0.085

TABLE II

$T=1$, $T_3=1$	III	IV	$T=1$, $T_3=1$	III	IV	$T=0$	I	II
+ + − 0	3/5	1	+ + − −	4/5	0	+ + − −	8/15	1/3
+ 0 0 0	2/5	0	+ − 0 0	1/5	1	+ − 0 0	2/15	2/3
						0 0 0 0	3/15	0

TABLE III

$T=1$, $T_3=1$	V	VI	VII	VIII	$T=1$, $T_3=0$	V	VI	VII	VIII	$T=0$	IX
+ + + − −	24/35	4/10	4/10	0	+ + − − 0	8/35	8/10	2/10	1	+ + − − 0	2/3
+ + − 0 0	8/35	3/10	6/10	1	+ − 0 0 0	12/35	2/10	8/10	0	+ − 0 0 0	1/3
+ 0 0 0 0	3/35	3/10	0	0	0 0 0 0 0	15/35	0	0	0	—	—

In conclusion I wish to thank Professor S. Z. Belen'kii who suggested this problem.

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Account of Retardation in the Interaction of Neutral Atoms

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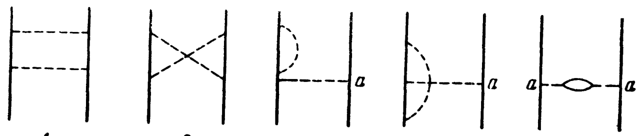
(Submitted to JETP editor March 2, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1152-1154 (June, 1956)

CASIMIR and Polder[1] considered retardation in the interaction of two neutral atoms. They showed that the energy of interaction for distances



1 2 3 4 5

large compared to the wavelengths in the spectrum of the atom decreases with the distance as R^{-7} and not as R^{-6} as follows from the theory of London for the Van der Waals forces. Subsequently, these deductions were verified in Ref. 2 for the independent macroscopic approach to the question. However, the calculations of Casimir and Polder have a number of shortcomings. In their original method of asymmetric perturbation theory there appeared divergent expressions which could not be completely and correctly removed until the appearance of the present technique of renormalization. Moreover, when integrating resonance denominators, basic to their thoery, the authors used rules of circumvention, which did not naturally follow from their method. In connection with the above, there appeared recently a paper (Ref. 3) in which the author doubts the results obtained by Casimir and Polder. Using a cumbersome nonrelativistic form of the perturbation theory this author obtained the physically scarcely detectable R^{-3} dependence for the interaction energy. We did not check the calculations in Ref. 3, and in order to solve this question we made calculations for the case of two hydrogen atoms in the ground state using the contemporary invariant technique of Feynman.

It is known that the interaction between neutral atoms is an effect of the fourth order in the charge e in perturbation theory. The S-matrix, averaged in the photon vacuum, is in the fourth approximation:

$$S^{(4)} = \frac{\pi^2}{2}\iiiint P[j_\mu(1)\, j_\mu(2)\, j_\nu(3)\, j_\nu(4)] \times D^F(12)\, D^F(34)\, dx_1\, dx_2\, dx_3\, dx_4.$$

The matrix elements, corresponding to the diagrams shown in Fig. 1 contribute to the effect which interests us. From these diagrams the divergent diagrams 3, 4 and 5 are zero after renormalization, since they will be proportional (neglecting the relativistic effects and multipole interactions), to the average value of the dipole moment of the atoms (point a). The contribution from diagrams 1 and 2 is finite and is equal to

$$S^{(4)}=2\pi^2 \iiiint P[j_\mu^{(1)}(1)\, j_\nu^{(1)}(3)]\, P[j_\mu^{(2)}(2)\, j_\nu^{(2)}(4)] \times D^F(12)\, D^F(34)\, dx_1\, dx_2\, dx_3\, dx_4.$$

The current density $j_\mu(x)$ in our case has operators with the following matrix elements:

$$\langle \mathbf{j}\rangle_{nm} = e\psi_n^* \alpha \psi_m, \quad \langle j_4\rangle_{nm} = ie\,(\psi_n^*\psi_m - \delta(\mathbf{r})\,\delta_{nm}),$$

where ψ_n is the wave function of an atom in the state n.

Let us use the known expansion of D^F in the Fourier integral and then integrate for all times. Then for the effective energy of interaction U, corresponding to the given element of the S-matrix (see for instance, Ref. 4), we get

$$U = \frac{i}{16\pi^5}\iiiint d\mathbf{r}_1\, d\mathbf{r}_2\, d\mathbf{r}_3\, d\mathbf{r}_4 \times \iint d\mathbf{p}'\, d\mathbf{p}'' \exp\{i\,\mathbf{p}'(\mathbf{r}_1-\mathbf{r}_2)+i\,\mathbf{p}''(\mathbf{r}_3-\mathbf{r}_4)\} \times \int_{-\infty}^{\infty} \frac{d\omega}{(p'^2-\omega^2)(p''^2-\omega^2)}$$
$$\sum_{n,m} \left\{\frac{j_{\mu on}^{(1)} j_{\nu no}^{(1)}}{\omega_{no}-\omega} + \frac{j_{\nu on}^{(1)} j_{\mu no}^{(1)}}{\omega_{no}+\omega}\right\}\left\{\frac{j_{\mu 0m}^{(2)} j_{\nu m0}^{(2)}}{\omega_{m0}+\omega} + \frac{j_{\nu 0m}^{(2)} j_{\mu m0}^{(2)}}{\omega_{m0}-\omega}\right\}.$$

The known rule of Feynman is used for the circumvention of the poles when integrating with respect to ω.

Neglecting relativistic effects and the effects of higher multipoles, and also taking into consideration that the ground state of hydrogen is an s-state, we obtain finally

$$U = \frac{i}{16\pi^5}\iint d\mathbf{p}'\, d\mathbf{p}''\, e^{-i(\mathbf{p}'+\mathbf{p}'')\mathbf{R}} \int_{-\infty}^{\infty} \frac{d\omega}{(p'^2-\omega^2)(p''^2-\omega^2)} \times \alpha_1(\omega)\,\alpha_2(\omega)\,[3\omega^4 - \omega^2(p'^2+p''^2) + (\mathbf{p}'\mathbf{p}'')^2].$$

Here $\alpha(\omega)$ is the real part of the polarization of atom

$$\alpha(\omega) = \sum_n 2\omega_{n0}\,|d_{0n}|^2/(\omega_{n0}^2 - \omega^2),$$

where d_{0n} is the matrix element of the dipole moment.

Divergent integrals with respect to p' and p'' must be calculated with a cutting-off multiplier of the type $e^{-\lambda pR}$, subsequently allowing λ to go to zero. Actually, in neglecting the effects of the higher multipoles, we assume that $p \ll 1/a$, where a is of the order of the size of the atom. In this sense $\lambda \to 0$ means neglecting values of the order of magnitude of a/R.

For $R \ll \lambda_0$ (λ_0 is the order of magnitude of the wavelengths in the spectrum of the atom) U is the form of London's formula. In the other limiting case for $R \gg \lambda_0$ in the integrals with respect to p and ω, the regions $p \sim \omega \sim \lambda_0/R$ are important, and the main member in U will be

$$U = \frac{i}{16\pi^5}\alpha_1(0)\ \alpha_2\ (0)$$

$$\times \lim_{\lambda\to 0}\iint d\,\mathbf{p}'\,d\,\mathbf{p}''e^{-i(\mathbf{p}'+\mathbf{p}'')\mathbf{R}-\lambda R(p'+p'')}$$

$$\times \int_{-\infty}^{\infty}\frac{d\omega}{(p'^2-\omega^2)\,(p''^2-\omega^2)}\,[3\omega^4-\omega^2(p'^2+p''^2)$$

$$+(\mathbf{p}'\mathbf{p}'')^2] = -\frac{23\alpha_1(0)\,\alpha_2(0)}{4\pi R^7},$$

which exactly coincides with the results obtained by Casimir and Polder.

I express my gratitude to L. P. Gorkov for taking part in the discussion of the above problem.

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The Energy Spectrum of γ-Quanta from Decaying π^0-Mesons

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(Submitted to JETP editor February 17, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1150-1151 (June, 1956)

THE relationship which determines the energy spectrum of the γ-radiation $W(\epsilon_\gamma, \theta_N)$ observed in any coordinate system at some angle θ_N, as a function of the energy and angular distributions of neutral mesons $F(\epsilon_\pi, \theta)$ in the same coordinate system is

$$W(\varepsilon_\gamma, \theta_N) = \int_{\varepsilon_{min}}^{\infty}\frac{d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}}\cdot\frac{1}{2\pi}\int_0^{2\pi}F(\varepsilon_\pi, \theta)\,d\varphi'. \quad (1)$$

Here ϵ_π and ϵ_γ are the total energy of the π^0-meson and the energy of the γ-quantum in units of the π^0-meson rest energy: $\epsilon_{min} = \epsilon_\gamma + 1/(4\epsilon_\gamma)$

$$\theta = \arccos(\sin\vartheta\cos\varphi'\sin\theta_N + \cos\vartheta\cos\theta_N);$$

$$\vartheta = \arccos\frac{\varepsilon_\pi - 1/2\varepsilon_\gamma}{\sqrt{\varepsilon_\pi^2-1}}.$$

For π^0-mesons whose angular distribution is proportional to $\cos^2\theta$ we obtain

$$W(\varepsilon_\gamma, \theta_N) = \frac{1}{2}(3\cos^2\theta_N - 1)\int_{\varepsilon_{min}}^{\infty}\frac{\cos^2\vartheta\,F(\varepsilon_\pi)\,d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}} \quad (2)$$

$$+\frac{1}{2}\sin^2\theta_N\int_{\varepsilon_{min}}^{\infty}\frac{F(\varepsilon_\pi)\,d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}}.$$

From this it follows that the γ-spectrum recorded at angle $\theta_N^* = \arccos(1/\sqrt{3})$ is logarithmically symmetrical with respect to the energy $\epsilon_\gamma = 1/2$ just as in the case of isotropic distribution of π^0-mesons. This means that from the γ-spectrum at a given angle and for an angular distribution of mesons of the form $a + b\cos^2\theta$, it is possible to obtain directly the energy distribution of the mesons and their mass by the method described in Ref. 1. Angles which are characterized as noted above will hereinafter be called "isotropic".

Another characteristic of an "isotropic" angle is the dependence of the total gamma flux at a given angle on the ratio of the constants a and b in the angular distribution of the π^0-mesons.

By integrating (2) with respect to the energy we obtain the angular distribution of the γ-rays produced through the decay of π^0-mesons whose angular distribution is proportional to $\cos^2\theta$, in the form

$$W_\gamma(\theta_N) = \frac{1}{2}(3\cos^2\theta_N - 1) \quad (3)$$

$$\times\int_0^\infty d\varepsilon_\gamma\int_{\varepsilon_{min}}^{\infty}\frac{\cos^2\vartheta\,F(\varepsilon_\pi)\,d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}} + \frac{1}{2}\sin^2\theta_N.$$

It follows that the gamma flux at angle $\theta_N^* = \arccos(1/\sqrt{3})$ remains unchanged in the transition from the $\cos^2\theta$ law of meson angular distribution to an isotropic distribution if only there is no change in the total number of mesons produced per unit time.

It also follows from (3) that when the angular distribution of π^0-mesons is $a + b\cos^2\theta$ the angular