($i = 1, 2$), where $\varkappa_i$ is the absorption coefficient of the i th meson, we obtain the following expression for the effective cross section:

$$\sigma = \frac{e^2}{2\pi} |\overline{F}(\omega)|^2 \int_0^\omega \frac{E_1(\omega - E_1)}{\omega^3} \tag{10}$$

$$\times \ln \frac{E_1^2(\omega - E_1)^2 R}{\mu^2 |2E_1 - \omega|} (\sigma_{\varkappa_1} + \sigma_{\varkappa_2} - \sigma_{\varkappa_1+\varkappa_2})\, dE_1,$$

where $\sigma_\varkappa$ is the cross section for the capture, by a nucleus of radius R, of a pion with an absorption coefficient $\varkappa$.

The integration cannot be carried out in general, since the dependence of $\sigma_\varkappa$ on energy is unknown. If one takes $\varkappa$ to be independent of energy, $\varkappa_1 = \varkappa_2 = \varkappa$, then we obtain

$$\sigma = (e^2/4\pi) \ln (\omega/\mu) |\overline{F}|^2 (2\sigma_\varkappa - \sigma_{2\varkappa}). \tag{11}$$

If we introduce a cut-off in angle, then under these conditions the cross section becomes

$$\sigma = \frac{e^2}{12\pi} \left[\ln \frac{\mu^2 + g_{\max}^2}{\mu^2} - \frac{g_{\max}^2}{\mu^2 + g_{\max}^2} \right] (2\sigma_\varkappa - \sigma_{2\varkappa}). \tag{12}$$

In addition to the one considered, there are possible a series of other processes for the formation of nuclear stars by γ-quanta. However, in view of the fact that in this process an effective role is played by a region large in comparison to nuclear dimensions, one can expect that the considered mechanism is the fundamental one at high energies $\omega \gg \mu$.

The author makes use of the opportunity to express his thanks to I. Ia. Pomeranchuk for his guidance of the work.

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The Influence of the Proximity of an External Resonance on the Magnitude of the Transition Energy in a Strong Focussing Accelerator

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THE magnitude of the transition energy in a strong focussing accelerator is determined by the formula

$$E_{\mathrm{cr}} = mc^2 \alpha^{-1/2}; \quad \alpha = d \ln L / d \ln p, \tag{1}$$

where L is the length of the orbit, p is the momentum of the particle. Usually, only the direct dependence of the orbit length on the momentum is taken into account, resulting from the equation

$$\frac{d^2 r}{d\theta} - \left(\frac{l}{2\pi}\right)^2 \frac{\partial H / \partial r}{H_0 \rho} r = - \left(\frac{l}{2\pi}\right)^2 \frac{\Delta p}{p} \frac{1}{\rho}, \tag{2}$$

where $\rho = \rho(\theta)$ is the radius of curvature of the unperturbed trajectory (for $(\Delta p / p)_{\mathrm{synch}} = 0$)*, l is the length of a periodic sector, $\theta = 2\pi s / l$, s is the coordinate along the unperturbed trajectory, H is the magnetic field, r is the horizontal deviation from the equilibrium orbit. However, in the vicinity of resonances, L obviously depends on the distance from the resonances, ϵ_r, ϵ_z, and these distances depend sharply on $(\Delta p / p)_{\mathrm{synch}}$. Therefore

$$\alpha = \frac{\partial \ln L}{\partial \ln p} + \frac{\partial \ln L}{\partial \varepsilon_{0r}} \frac{d\varepsilon_{0z}}{d \ln p} + \frac{\partial \ln L}{\partial \varepsilon_{0z}} \frac{d\varepsilon_{0z}}{d \ln p} = \alpha_p + \alpha_\varepsilon, \tag{3}$$

$$\varepsilon_r = \nu_r - \nu_{\mathrm{res}} \quad \varepsilon_z = \nu_z - \nu_{\mathrm{res}} \tag{4}$$

where $\nu_{r,z}$ are the betatron quasi-frequencies of the transverse oscillations. The derivatives in Eq. (3) are taken at $(\Delta p / p)_{\mathrm{synch}} = 0$, i.e. ϵ_{0r}, ϵ_{0z} correspond to the mid-position of the synchrotron oscillations. The quantity α corresponds to a high energy $mc^2 \alpha^{-1/2}$. At such energies, the betatron oscillations about the equilibrium orbit are already sufficiently small; therefore parametric resonance, generally speaking, plays a weak role in the effect.**

Thus we have to understand L to be the length of the perturbed equilibrium periodic orbit. Clearly,

$$L \approx L_0 + \int_0^{L_0} \frac{r}{\rho_r} ds + \int_0^{L_0} \frac{z}{\rho_z} ds + \frac{1}{2}\int_0^{L_0} ds \left[\left(\frac{dr}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2\right], \quad (5)$$

$$\frac{1}{\rho_r} = \frac{H_z}{H_0 \rho} \approx \frac{1}{\rho}\left(1 + \frac{\Delta H_z}{H_0}\right), \quad \frac{1}{\rho_z} = -\frac{H_r}{H_0}\frac{1}{\rho}, \quad (6)$$

where ΔH_z and H_r are the perturbations of the magnetic field along the unperturbed equilibrium orbit. In order to calculate α_ϵ, one should substitute in Eq. (5), in place of r and z, r_ϵ and z_ϵ, which describe the perturbed equilibrium orbit in the vicinity of an external resonance for $(\Delta p/p)_{\text{synch}} = 0$. They are found from the formulas

$$r_\varepsilon \approx \frac{1}{|\varphi|_{\max}} \frac{h_r}{\varepsilon_{0r}} \operatorname{Re}[\varphi_r^* \exp(i\varepsilon_{0r}\theta - i\gamma_r)]; \quad (7)$$

$$\frac{dr_\varepsilon}{d\theta} \approx \frac{1}{|\varphi|_{\max}} \frac{h_r}{\varepsilon_{0r}} \operatorname{Re}\left[\frac{d\varphi_r^*}{d\theta} \exp(i\varepsilon_{0r}\theta - i\gamma_r)\right] \quad (8)$$

and similarly for z; $\varphi_{r,z}$ are Floquet functions,

$$\varphi(\theta) = f(\theta)\exp(i\nu e); \quad (9)$$

$$f(\theta) = f(\theta + 2\pi), \quad \varphi\varphi^{*\prime} - \varphi^*\varphi' = -iw.$$

For ΔH_z and H_r, it is necessary to substitute Eq. (5) into Eq. (6)

$$\Delta H_z \approx (\partial H_z/\partial r)\, r_\varepsilon, \quad H_r \approx (\partial H_z/\partial r)\, z_\varepsilon. \quad (10)$$

It is easy to see that only the terms quadratic in r_ϵ, z_ϵ, r'_ϵ, z'_ϵ of Eq. (5) contribute to α_ϵ.

Usually an operating point is selected such that $\nu_r \approx \nu_z$, therefore justifying the approximate equation

$$\int_0^{2\pi} \frac{\partial H/\partial r}{H_0\rho} |\varphi_r|^2 d\theta \approx -\int_0^{2\pi} \frac{\partial H/\partial r}{H_0\rho} |\varphi_z|^2 d\theta; \quad (11)$$

$$\int_0^{2\pi} |\varphi'|^2 d\theta \approx \frac{l^2}{4\pi^2 a\rho_0}\left(\int_{\text{foc}} |\varphi|^2 d\theta - \int_{\text{def}} |\varphi|^2 d\theta\right); \quad (12)$$

$$a = \frac{H_z}{\partial H_z/dr},$$

where $\int_{\text{foc}}$ and $\int_{\text{def}}$ are integrals respectively over a focussing and defocussing sector, ρ_0 is the unperturbed radius of curvature within an ordinary magnet. In addition we make use of the dependence of the frequency on the momentum

$$\Delta\nu_{r,z} = \pm\frac{1}{2\pi w}\left(\frac{l}{2\pi}\right)^2 \int_0^{2\pi} |\varphi|^2 \frac{\partial H/\partial r}{H_0\rho} d\theta \left(\frac{\Delta p}{p}\right). \quad (13)$$

The upper sign refers to ν_r, the lower to ν_z. Then we obtain

$$\alpha_\varepsilon = \frac{1}{w}\left(\frac{A_{0r}^2/a^2}{\varepsilon_{0z}} + \frac{A_{0z}^2/a^2}{\varepsilon_{0z}}\right) \quad (14)$$

$$\times\left[\frac{l}{4\pi^2\rho_0|\varphi|_{\max}}\left(\int_{\text{foc}} |\varphi|^2 d\theta - \int_{\text{def}} |\varphi|^2 d\theta\right)\right]^2,$$

where A_{0r}, A_{0z} are the amplitudes of the transverse oscillations for $(\Delta p/p)_{\text{synch}} = 0$:

$$A_{0r,z} = |h_{0r,z}/\varepsilon_{0r,z}|. \quad (15)$$

If $A_{0r} \sim A_{0z}$, $\epsilon_{0r} \sim \epsilon_{0z}$, then generally

$$\alpha_\varepsilon \sim (0.5\text{—}1)\, A_0^2/a^2\varepsilon_0 M^2, \quad 1/8M \lesssim \varepsilon_0 < 1/M, \quad (16)$$

where M is the number of periodic sectors, $L = Ml$.

It should be remarked that even in the absence of oscillations about the equilibrium orbit, the influence of parametric resonance can be told in that in place of ϵ_0 in Eqs. (15), (16), at the worst $\epsilon_0 - g$ enters, where g is the half width of the region of parametric resonance.

As can be seen from Eq. (8), α_ϵ is a strong function of the quantity ϵ_0 ($\alpha_\epsilon \sim \epsilon_0^{-3}$). From Eq. (16) it is seen that α_ϵ can easily attain a magnitude $\sim (1-5) \times 10^{-3}$. The sign of α_ϵ is determined by the sign of ϵ_0 (we note that in accelerators $\epsilon_r\, \epsilon_z > 0$ always). From this follows the important result that in accelerators with compensation for the transition energy according to the method of Vladimirsky – Tarasov[1], one should always choose the operating point in the (ν_r, ν_z) plane such that the distances from neighboring external resonances satisfy the inequalities $\epsilon_r < 0$, $\epsilon_z < 0$.

In accelerators without compensation for the transition energy the effect can introduce a large uncertainty at the moment of the phase flip. It would be possible to avoid this, if it were possible to control the deviations ϵ_{0r}, ϵ_{0z} during the course of the accelerating cycle.

However, if control of the deviations $\epsilon_{0r,z}$ is possible, then one should apply the effect toward the elimination of the transition energy [by means of a slow reduction of (ϵ_0)]. Equation (16) shows that under standard conditions this is quite realistic. It is necessary to take into account that at energies $\sim 5-10$ mev the free betatron and synchrotron oscillations are already sufficiently attenuated. This would be the cheapest way of eliminating the transition energy.

*It is necessary to remark that for the calculation of α, only that part of $\Delta p / p$ is important which corresponds to an oscillation of the momentum about some equilibrium value. We denote it by $(\Delta p/p)_{\text{synch.}}$

**By parametric resonance we mean one due to a perturbation of the gradient $\partial H_z/\partial r$; by an external resonance, one due to a perturbation of the field H_z.

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Relaxation Times T_1 and T_2 in Anthracite

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THE authors were the first to measure the electronic para-magnetic resonance in anthracite (Ref. 1). It was found that the half-width of the absorption line in anthracite is $\Delta H = 0.7$ oersted i.e., considerably smaller than in other types of stone coals. The value $\Delta H = 0.3$ oersted was obtained for anthracite in Ref. 2. Probably the half-width varies somewhat for the different kinds of anthracite. Our last measurements on the samples of Kuzbask anthracite for the frequencies 12.25 and 22 mc gave $\Delta H = 0.5$ oersted. We wanted to determine for anthracite the time of spin-lattice relaxation, T_1. For this purpose, with the above mentioned frequencies, measurements of the degree of saturation (Ref. 3) were made for different amplitudes of the oscillating magnetic field. The magnitude of the amplitude was determined with the method previously used in Ref. 4. The method was checked on $\alpha\alpha$-diphenyl $-\beta-$ picrylhydrazyl, for which $T_1 = 6.6 \times 10^{8}$ sec; moreover, the parameter of the half-width T_2 was taken equal to 6.0×10^{8} sec in correspondence with the halfwidth of the line $\Delta H = 0.95$ oersted found for the monocrystal of the above-named free radical (Ref. 5). The magnitude of T_1 is in good agreement with the researches of Refs. 3 and 6. For the Kuzbask anthracite sample the time T_1 was equal to 12×10^{-8} sec for the core $T_2 \doteq 11.4 \times 10^{-8}$ sec.

The theory of paramagnetic resonance in systems with large exchange interaction (Ref. 5) demands that $T_1 \approx T_2$; therefore, our result confirms the presence of strong exchange in anthracite, noted in Ref. 1.

In conclusion, we point out that for the temperature of liquid air, the relaxation time for anthracite is somewhat longer, since the saturation occurs for smaller amplitudes of the oscillating field. This is in agreement with the concept that the carriers of paramagnetism in anthracite are "broken bonds" between the carbon atoms.

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Concerning the Blatt, Butler, and Shafroth Paper on Superfluidity and Superconductivity Theory

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IN a series of papers, Blatt, Butler and Shafroth[1-6] concern themselves with the theory of superfluidity and superconductivity, and come forth with some far-reaching conclusions, with which it is impossible to agree. Two points stand out.[1-6] The first, associated with a consideration of the superfluidity and superconductivity of an ideal Bose gas in a vessel, has already been discussed,[7] and has only methodological significance. The second essential point – the statement concerning the finiteness of the correlation length Λ for the momenta of a pair of particles in all real systems, in contrast to an ideal Bose gas, is incorrect. The momentum correlation coefficient is introduced[3] in such a way that it is not directly