and 8* obtained with the same parameter values. Furthermore, the maximum is not found at $k_0 = \mu c^2$ as in Ref. 6, but at a considerably higher energy. The angular distribution resembles those of Refs. 6 and 8. only for very small and very large k_0, and strongly differs at other values of k_0; this is especially true of the backward scattering in the pre-resonance region and of the minimum for $\theta = \pi/2$ at resonance.

In conclusion, I wish to thank O. P. Ryibalkin, V. I. Petukhov and A. G. Trunov for carrying out the numerical computations.

Note added in proof: After completing this analysis, the author discovered that an analogous calculation was carried out independently by R. Gurzhi.

[1] Tamm, Gol'fand and Feinberg, J. Exptl. Theoret. Phys. (U.S.S.R.) **26**, 649 (1954).

[2] V. I. Ritus, J. Exptl. Theoret. Phys. (U.S.S.R.) **27**, 660 (1954).

[3] W. Pauli, Rev. Mod. Phys. **13**, 203 (1941).

[4] S. B. Batdorf and R. Thomas, Phys. Rev. **59**, 621 (1941); J. L. Powell, Phys. Rev. **75**, 32 (1949).

[5] A. M. Baldin and V. V. Mikhailikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 562 (1951); Usp. Fiz. Nauk **44**, 200 (1951).

[6] R. G. Sachs and L. L. Foldy, Phys. Rev. **80**, 824 (1950).

[7] K. M. Case, Phys. Rev. **76**, 1 (1949).

[8] V. Ia. Fainberg, J. Exptl. theoret. Phys. (U.S.S.R.) **25**, 644 (1953).

Translated by M. A. Melkanoff
223

SOVIET PHYSICS JETP VOLUME 3, NUMBER 6 JANUARY, 1957

On the S-Matrix for Particles with Arbitrary Spin

V. I. KARPMAN
Minsk State Pedagogical Institute

(Submitted to JETP editor, April 7, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1104-1111 (June, 1956)

This is a discussion of the perturbation theory for particles with arbitrary spin. The properties of the singular functions for such particles are discussed. It is shown that the elements of the *S*-matrix may be found by the Feynman rules.

THE present theory of wave fields permits us to consider fields corresponding to particles with any integer or half integer spin or with any number of integer or half integer spin states.[1-4] In spite of the fact that particles with spin larger than unity are not observed experimentally, the theory of such particles can present some interest in the interpretation of the properties of the newly discovered mesons and in the study of the isobaric states of nucleons. For instance, the theory of particles with spin 3/2 has been applied to the construction of the partly phenomenological theory of interaction of π-mesons with nucleons[4] - and the results obtained compare satisfactorily with the experimental data.

The basic problem in the study of the interaction of particles with arbitrary spin with other fields amounts to the construction of the scattering matrix *S*. It has already been assumed previously that the elements of the *S*-matrix for particles with arbitrary spin may be obtained by the Feynman rules.[5-6], in analogy to the quantum electrodynamics case.[7] However, until now no basis was given to this assumption.

A difficulty arises in the construction of the *S*-matrix for particles with arbitrary spin. The difficulty is related to the fact that for higher spins not all the components of the wave function $\psi(x)$ are dynamically independent.[1] This fact is the main impediment in the attempt to generalize, for the case of arbitrary spin, the *S*-matrix theory; the latter was derived by Dyson[7] for quantum electrodynamics (spin 1/2 in the interaction representation. (For more details on these difficulties see Refs. 5, 6).

In the present paper, it will be shown how one can obtain the elements of the *S*-matrix for particles with arbitrary spin in the Heisenberg representation. We follow the Yang-Feldman method.[8] For the sake of simplicity, we will consider the interaction with an electromagnetic field (the interaction with other fields can be treated analo-

* The constants used in this article are almost identical to those of Ref. 8.

gously).

1. The interaction of a particle with arbitrary spin* with the electromagnetic field can be written in the form [3]

$$(L_k\nabla_k + i\varkappa)\,\psi = ieA_kL_k\psi;$$

$$\Box A_k = j_k;\ j_k = e(\bar{\psi}L_k\psi)$$

$$(\hbar = c = 1;\ k = 0,\ 1,\ 2,\ 3;$$

$$\nabla_k = \partial/\partial x_k;\ \Box = \nabla_k^2), \quad (1)$$

where A_k is the quantized 4-vector potential of the electromagnetic field and L_k are matrices the properties of which are studied in Ref. 3.

It has been shown [9,10] that the matrices L_k subject to the conditions stated in the earlier footnote satsify the identity

$$(L_kp_k)^n[(L_kp_k)^2 - (\varkappa^2/\mu_1^2)p_k^2]\cdots$$

$$[(L_kp_k)^2 - (\varkappa^2/\mu_s^2)p_k^2] = 0, \quad (2)$$

where p_k is an arbitrary 4-vector, $\mu_1, \mu_2, \ldots$, μ_s is the rest mass spectrum of the particle**, and n is an integer dependent on the properties of the matrix L_0. For particles with spin ½: $n = 0$, $s = 1$; for particles with spin 0, 1: $n = 1$, $s = 1$. In the first case, (2) gives a known relation for the Dirac matrices, and in the second case, (2) gives the Deffin-Kemmer relations.

2. Let us first consider the basic singular functions satisfying equation (1).

It has been shown[9] that, in the absence of interaction the components of the wave function ψ satisfy the relation:

$$[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\pm = -R_{\rho\sigma}(x - x'), \quad (3)$$

$$R(x) = (-1)^s\left(\frac{i}{\varkappa}\right)^n(L_k\nabla_k - i\varkappa)\sum_{r=1}^{s}\Bigg\{(L_k\nabla_k)^n \times \prod_{\substack{i=1\\ i\neq r}}^{s}\left[(L_k\nabla_k)^2 - \frac{\varkappa^2}{\mu_i^2}\Box\right]\Bigg\} b_r\Delta(x,\ \mu_r), \quad (4)$$

$$b_r = \prod_{i=1}^{s}(\mu_i^2/\varkappa^2)\Big/\prod_{\substack{i=1\\ i\neq r}}^{s}(\mu_i^2 - \mu_r^2), \quad (5)$$

where n and s are defined the same way as for (2), and $\Delta(x, \mu_s)$ is the Pauli singular function corresponding to the particle with a rest mass μ_s.

$$\Delta(x,\ \mu_r) = (2\pi)^{-3}\int e^{ipx}\frac{\sin p_0x_0}{p_0}\,d^3p;$$

$$p_0 = \sqrt{p^2 + \mu_r^2}.$$

In (3) the minus sign is taken for integer spin and the plus sign for half integer spin. The vacuum expectation value (in the absence of interaction) has the form [6]

$$\langle[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\mp\rangle_0 = iR^{(1)}_{\rho\sigma}(x - x') \quad (6)$$

(the plus sign is taken for integer spin, minus for half integer spin), where $R^{(1)}(x)$ is obtained from the expression of $R(x)$ [Eq. (4)] by substituting for $\Delta(x, \mu_r)$:

$$\Delta^{(1)}(x,\ \mu_r) = (2\pi)^{-3}\int e^{ipx}\frac{\cos p_0x_0}{p_0}\,d^3p;\ p_0 = \sqrt{p^2 + \mu_r^2}.$$

The Green function of the equation $(L_k\nabla_k + ik)\psi(x) = 0$ can be presented in the form [6]:

$$S_G(x - x') = S(\nabla)\sum_{r=1}^{s}b_r\,G(x - x';\ \mu_r), \quad (7)$$

$$G(x, \mu_r) = -(2\pi)^{-4}\int e^{ipx}\frac{d^4p}{p^2 - \mu_r^2}, \quad (8)$$

where b_r may be obtained from (5) and $S(\nabla)$ is a differential operator of the form

$$S(\nabla) = -\sum_{m=0}^{n+2s-3}\left(\frac{i}{\varkappa}\right)^{m+1}(L_k\nabla_k)^m\prod_{i=1}^{s}(\Box + \mu_i^2)(\varkappa/\mu_i)^2 + \left(\frac{i}{\varkappa}\right)^n(L_k\nabla_k - i\varkappa)(L_k\nabla_k)^n \times \sum_{r=1}^{s}\prod_{\substack{i=1\\ i\neq r}}^{s}\frac{\left[(L_k\nabla_k)^2 - \frac{\varkappa^2}{\mu_i^2}\Box\right]}{\mu_r^2 - \mu_i^2}(\Box + \mu_i^2). \quad (9)$$

*We are considering only equations with finite dimensions (ψ has a finite number of components). Such equations describe particles, the spin and the rest mass of which are taking a finite number of values. Furthermore, it is assumed that the energy (in the case of integer spin) or the charge (in the half-integer case) are definite.

**the mass spectrum of a particle is determined by the relationship $\mu_i = |\varkappa/\lambda_i|$ where λ_i are [3] the non-zero eigenvalues of the matrix L_0

In particular, in the Dirac equation ($n = 0$, $s = 1$), $S(\nabla)$ takes the form $L_k \nabla_k - ik$; in the case of the Deffin–Kemmer equation ($n = 1$, $s = 1$), one obtains from (9) the known formula

$$S(\nabla) = (i/\varkappa)[(L_k\nabla_k)^2 - i\varkappa(L_k\nabla_k) - (\Box + \varkappa^2)].$$

To define completely the Green's function $S_{\sim}(x - x')$ one has furthermore to define the integration path in (8). Let us denote by $\Delta_r(x - x'; \mu_i)$ and by $\Delta_a(x - x'; \mu_i)$ the Green's function $G(x - x'; \mu_i)$ for the Klein-Fock equation $(\quad + \mu_i^2)\psi(x) = 0$, giving solutions in the form of retarded and advanced potential, respectively; by $\Delta_F(x - x'; \mu_i)$ the causal Feynman Δ–function, and by $\bar{\Delta}(x - x'; \mu_i)$ the Green's function corresponding to the principal value of the integral in (8).

The corresponding Green's function for particles with arbitrary spin will have the form

$$\bar{S}(x - x') = S(\nabla)\sum_{i=1}^{s}\bar{\Delta}(x - x'; \mu_i);$$

$$S_r(x - x') = S(\nabla)\sum_{i=1}^{s}\Delta_r(x - x'; \mu_i);$$

$$S_F(x - x') = S(\nabla)\sum_{i=1}^{s}\Delta_F(x - x'; \mu_i);$$

$$S_a(x - x') = S(\nabla)\sum_{i=1}^{s}\Delta_a(x - x'; \mu_i). \quad (10)$$

Furthermore, let us define two more functions

$$S(x - x') = S(\nabla)\sum_{i=1}^{s} b_i\Delta(x - x'; \mu_i);$$

$$S^{(1)}(x - x') = S(\nabla)\sum_{i=1}^{s} b_i\Delta^{(1)}(x - x'; \mu_i). \quad (11)$$

Using formula (9) and also the fact that $\Delta(x - x')$ and $\Delta^{(1)}(x - x')$ satisfy the Klein-Fock equation, it is easy to see that

$$S(x - x') \equiv -R(x - x');$$

$$S^{(1)}(x - x') \equiv R^{(1)}(x - x'),$$

where $R(x - x')$ and $R^{(1)}(x - x')$ are the functions defined in (3) and (6). In this fashion, Eqs. (3) and (6) can be replaced by

$$[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\pm = S(x - x'),\ \langle[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\mp\rangle_0 = -iS^{(1)}(x - x'). \quad (12)$$

From (12) one sees that, in the Green's functions as well as in the permutation relations and in the vacuum expectation values, the only operator operating on the singular Δ – function is the differential operator $S(\nabla)$; this is the same situation as in quantum electrodynamics.*** This situation will be considerably taken advantage of in our work.

It is also important to note that the relationships between the Green's functions S_r, S_a, S_F, and S are the same is in quantum electrodynamics, namely:

$$S_F(x) = \bar{S}(x) - (i/2)S^{(1)}(x), \quad (13a)$$

$$\bar{S}(x) = {}^1/_2\{S_a(x) + S_r(x)\}, \quad (13b)$$

$$S(x) = S_r(x) - S_a(x), \quad (13c)$$

$$S_r(x - x') = 0 \quad (x_0 < x_0'); \qquad S_a(x - x') = 0 \quad (x_0 > x_0'). \quad (13d)$$

However, the case of particles with spin larger than ½ differs considerably from that of quantum electrodynamics. This difference is that, in general, Eqs. (10) and (11) defining the functions $\bar{S}(x)$ and $S(x)$ do not imply the equality

$$\bar{S}(x) = {}^1/_2\varepsilon(x)S(x),$$

$$\varepsilon(x) = \begin{cases} 1 & x_0 > 0 \\ -1 & x_0 < 0, \end{cases}$$

as they do in quantum electrodynamics.

Indeed, from the definition of $S(x)$ and $\bar{S}(x)$ it follows that

$$\bar{S}(x) = S(\nabla)\{{}^1/_2\varepsilon(x)\Delta(x)\} = {}^1/_2\varepsilon(x)S(x) + {}^1/_2[S(\nabla),\ \varepsilon(x)]_-\Delta(x). \quad (14)$$

***See, for instance, Refs. 11 and 18. The proof of this statement given in Ref. 12 is erroneous, as it is based on the assumption that the matrix $(L_k p_k + K)^{-1}$ can be diagonalized. But all the relativistically invariant equations satisfying the conditions stated in the footnote *, except the Dirac and the Deffin-Kemmer equations, cannot be diagonalized (see Ref. 3, p. 726).

If the differential operator $S(\nabla)$ contains derivatives of order not higher than one, as it does only in the case of particles described by the Dirac equation, then $[S(\nabla), \epsilon(x)]\Delta(x) \equiv 0$, because $\epsilon(x)$ depends only on x_0 and

$$^1/_2 [\partial/\partial x_0, \varepsilon(x)]_- \Delta(x) = \delta(x_0)\Delta(x) \equiv 0.$$

It is easy to see that in the general case

$$^1/_2 [S(\nabla), \varepsilon(x-x')]_- \Delta(x-x') = f(L, \nabla)\delta(x-x'), \quad (15)$$

where $F(L, \Delta)$ is some polynomial in the matrix L_k and in the differential operators $\nabla_k = \partial/\partial x_k$, and $\delta(x-x')$ is a four-dimensional δ-function. Insofar as $\epsilon(x-x')$ depends only on time coordinates, the function F has a non-invariant form. To give a relativistically invariant form to all the formulas, it is necessary to change somewhat the definition of $\epsilon(x-x')$. For this purpose let us introduce the ensemble of space-like surfaces $\sigma(x)$, in such a way that one and only one of the surfaces would go through each point x of the universe. Let us denote by n_μ the unit vectors orthogonal to these surfaces. Let us introduce the notation: $x > x'$ if $\sigma(x)$ lies in the "future" with respect to $\sigma(x')$ and $x < x'$ in the opposite case. By the symbol $\epsilon(x, x')$ we will mean

$$\varepsilon(x, x') = \begin{cases} 1 & x > x' \\ -1 & x < x'. \end{cases}$$

with this definition, the relationship between $\Delta(x-x')$ and $\Delta(x-x')$ retains its form

$$\bar{\Delta}(x-x') = \varepsilon(x, x')\Delta(x-x').$$

Instead of (15) and (14) we get:

$$^1/_2 [S(\nabla), \varepsilon(x, x')]_- \Delta(x-x') = F(L_\mu n_\mu, \nabla_\nu n_\nu, \Box)\delta(x-x'), \quad (16a)$$

$$\bar{S}(x-x') = {}^1/_2 \varepsilon(x, x') S(x-x') + F(L_\mu n_\mu, \nabla_\nu n_\nu, \Box)\delta(x-x'), \quad (16b)$$

where F is some polynomial in $\Box = \nabla_\lambda \nabla_\lambda$, in $\nabla_\lambda n_\lambda$, and in the matrices $L_\mu n_\mu$. It is important to note that the quantities n_λ are involved in F in such a manner that if one puts $n_\lambda \equiv 0$, then $F = 0$.

In particular, it follows from (16b) that $\langle P'(\psi(x), \psi(x')\rangle_0$ [++] is in general not equal to $S_F(x-x')$, as it is in the case of quantum electrodynamics. Indeed, it follows from (12) that:

$$\langle P'(\psi(x), \bar{\psi}(x'))\rangle_0 = {}^1/_2 \varepsilon(x, x') S(x-x') - (i/2) S^{(1)}(x-x').$$

Using Eqs. (16b) and (13a), we get

$$\langle P'(\psi(x), \bar{\psi}(x'))\rangle_0 = S_F(x-x') - F(L_\mu n_\mu, \nabla_\nu n_\nu, \Box)\delta(x-x'). \quad (17)$$

3. Knowing the Green's function, one can solve Eq. (1) by the method of successive approximations. For this purpose it is easier to replace (1) by their equivalent integral expressions. If one uses the retarded potential, the latter will have the form

$$\psi(x) = \psi^{(i)}(x) - ie\int S_r(x-x') L_\mu A_\mu(x')\psi(x')\, d^4x',$$

$$A_\mu(x) = A_\mu^{(i)}(x) - e\int D_r(x-x') j_\mu(x')\, d^4x'. \quad (18)$$

If, on the other hand, one uses the advanced potential, one will get

$$\psi(x) = \psi^{(f)}(x) - ie\int S_a(x-x') L_\mu A_\mu(x')\psi(x')\, d^4x', \quad (19)$$

$$A_\mu(x) = A_\mu^{(f)}(x) - e\int D_a(x-x') j_\mu(x')\, d^4x',$$

where $\psi^{(i)}$, $A^{(i)}$ and $\psi^{(f)}$, $A^{(f)}$ satisy the equations and the commutation relations of the free fields. Let us assume that the interaction is adiabatically turned on at $t = x_0 = -\infty$ and turned off at $t = +\infty$. Then, according to (13d)

++ $P'(\psi(x), \bar{\psi}(x')) = \begin{cases} \psi(x)\bar{\psi}(x') & x > x' \\ \alpha\bar{\psi}(x')\psi(x) & x < x' \end{cases}$

$\alpha = \pm 1$; plus sign for integer spin, minus – for half integer.

$$\psi(x) \to \psi^{(i)}(x),$$

$$A_\mu(x) \to A_\mu^{(i)}(x) \quad \text{for} \quad t \to -\infty,$$

$$\psi(x) \to \psi^{(f)}(x),$$

$$A_\mu(x) \to A_\mu^{(f)}(x) \quad \text{for} \quad t \to +\infty. \quad (20)$$

This leads to the physical interpretation of $\psi^{(i)}$, $A^{(i)}$ and of $\psi^{(f)}$, $A^{(f)}$; $\psi^{(i)}$ and $A^{(i)}$ describe the corresponding fields when $t \to -\infty$ "before the interaction has been turned on", and $\psi^{(f)}$ and $A^{(f)}$ describes them when $t \to +\infty$, "after the interaction has been turned off." These operators have to satisfy the relations [8]

$$\psi^{(f)}(x) = S^{-1}\psi^{(i)}(x)\, S;$$

$$A_\mu^{(f)}(x) = S^{-1}A_\mu^{(i)}(x)\, S, \quad (21)$$

where S is the scattering matrix.

On the other hand, using Eqs. (18) and (19), one can express $\psi^{(f)}(x)$ and $A^{(f)}(x)$ in terms of $\psi^{(i)}(x)$ and $A^{(i)}(x)$, as a power series in e. For this purpose let us subtract (18) from (19) and use equation (13c). We get

$$\psi^{(f)}(x) = \psi^{(i)}(x) + ie\int S(x-x')\, L_\mu A_\mu(x')\, \psi(x')\, d^4x', \quad (22)$$

$$A_\mu^{(f)}(x) = A_\mu^{(i)}(x) + e\int D(x-x')\, j_\mu(x')\, d^4x'.$$

It is easy to get the desired expressions of $\psi^{(f)}$ and $A^{(f)}$ in terms of $\psi^{(i)}$ and $A^{(i)}$. First, one has to apply an iteration method to (18) to get an expression of $\psi(x)$ and $A(x)$ in terms of $\psi^{(i)}(x)$ and $A^{(i)}(x)$, and then substitute these expressions in Eqs. (22).

4. Let us now determine the elements of the S-matrix. Let us first note that $\psi^{(f)}(x)$ and $\psi^{(i)}(x)$ describe free fields, and that they can therefore be presented in the form***

$$\psi^{(f)}(x) = V^{-1/2} \times \sum_p \{c_p^{(f)}\varphi^+(p)\, e^{ipx} + c_p^{(f)*}\varphi^-(p)\, e^{-ipx}\},$$

$$\psi^{(i)}(x) = V^{-1/2} \times \sum_p \{c_p^{(i)}\varphi^+(p)\, e^{ipx} + c_p^{(f)*}\varphi^-(p)\, e^{-ipx}\}, \quad (23)$$

where $C_p^{(i)*}$ and $C_p^{(i)}$ are the creation and annihilation operators in the occupation number representation "before the interaction has been turned on", and $C_p^{(f)*}$ and $C_p^{(f)}$ – the same operators, "after the interaction has been turned off". It follows from (21) that

$$c_p^{(f)} = S^{-1}c_p^{(i)}\, S. \quad (24)$$

Analogously, one can write

$$A_\mu^{(f)}(x) = V^{-1/2} \times \sum_k \{a_k^{(f)} A_\mu(k)\, e^{ikx} + a_k^{(f)*} A_\mu^*(k)\, e^{-ikx}\},$$

$$A_\mu^{(i)}(x) = V^{-1/2} \times \sum_k \{a_k^{(i)} A_\mu(k)\, e^{ikx} + a_k^{(i)*} A_\mu^*(k)\, e^{-ikx}\}. \quad (25)$$

The physical interpretation of the operators $a_k^{(f)*}$, $a_k^{(f)}$ and $a_k^{(i)*}$, $a_k^{(i)}$ corresponds to the interpretation of $C^{(F)}$ and $C^{(i)}$. It follows from (21) that

$$a_k^{(f)} = S^{-1}a_k^{(i)}\, S. \quad (24')$$

Let us denote by ϕ_0 the vacuum wave function of the system. One can write

$$S\Phi_0 = \Phi_0. \quad (26)$$

The element of the S-matrix corresponding to the transition from the state (i) at $t \to -\infty$ (which involved the considered particles in the states $p_1, p_2, \ldots$ and photons in the states $k_1, k_2, \ldots$) to the state $(F) \sim (p_1', p_2' \ldots; k_1, k_2', \ldots\ldots)$

***In equation (23), the symbol ρ denotes the whole set of quantum numbers which define the state of the considered particle. In analogy, in Eq. (25), k denotes the set of momentum 4-vectors and the polarization of the photon; px and kx represent the scalar products of the 4-momentum and of the 4-vector of the point x.

at $t \to +\infty$ is given by the formula

$$S_{(i)}^{(f)} = \langle c_{p_1'}^{(i)} \cdots a_{k_1'}^{(i)} \cdots S c_{p_1}^{(i)*} \cdots a_{k_1}^{(i)*} \cdots \rangle_0, \quad (27)$$

where $\langle \ \rangle_0$ is the expectation value over the vacuum ϕ_0. Expressing $C_p^{(i)}$ and $a_{k'}^{(i)}$ through $C_p^{(f)}$ and $a_p^{(f)}$ with the help of (24), and taking into account (26), let us rewrite Eq. (27) in the form

$$S_{(i)}^{(f)} = \langle c_{p_1'}^{(f)} \cdots a_{k_1'}^{(f)} \cdots c_{p_1}^{(i)*} \cdots a_{k}^{(i)*} \cdots \rangle_0. \quad (28)$$

To calculate this quantity, it is necessary to express the operators $C^{(f)}$, $a^{(f)}$ through $c^{(i)}$, $a^{(i)}$ with the required accuracy with respect to e. This can be done using the expressions of $\psi^{(f)}(x)$, $A^{(f)}(x)$ in terms of $\psi^{(i)}(x)$, $A^{(i)}(x)$ [obtained with the help of Eqs. (18) and (19) by the iteration method]. After this is done, it is in principle possible to calculate the vacuum expectation value of the right hand side of (28). The procedure can be applied with relative ease to get the matrix elements to the first and second order in e. It happens that these elements are such as required by the Feynman-Dyson rules. To the vertices of the diagrams correspond the matrices L_μ [involved in (1)], and, to the solid line, the function

$$S_F(x - x') = S(\nabla) \sum_{i=1}^{s} b_i \Delta_F(x - x'; \mu_i), \quad (29)$$

where $S(\nabla)$ is given by (9),

The method given above becomes quite complicated when applied to compute the matrix elements $S_{(i)}^{(f)}$ to any order in e. It is however possible to show that even in this case, the matrix elements will have a form corresponding to the Dyson-Feynman rules.

Indeed, it follows from what was said above, that the calculation of $S_{(i)}^{(f)}$ involves only the expression of the operators $\psi^{(f)}$, $A^{(f)}$ through $\psi^{(i)}$, $A^{(i)}$, the formulas (12), the Green's functions (10) and the relations (13), (16b) between them. All of these are absolutely analogous to their equivalents in quantum electrodynamics – with only one exception: the formula (16b) has a second term, depending on the normals to the space-like surfaces, which vanishes identically in the quantum electrodynamic case. The effect of this form is such that the expression of the matrix element in the general case has the form

$$S_{(i)}^{(f)} = S_{(i)}^{\prime(f)} + S_{(i)}^{\prime\prime(f)}, \quad (30)$$

where $S_{(i)}^{\prime(f)}$ is the term which would be obtained if one set $F(L_\mu n_\mu, \nabla_\nu n_\nu, \ ) = 0$ in (16b), and $S_{(i)}^{\prime\prime(F)}$ is the term containing $F(L_\mu n_\mu, \nabla_\nu n_\nu, \ )$, $S_{(i)}^{\prime\prime(F)}$ vanishing if one sets $S = 0$. In quantum electrodynamics $S = S'$, and S' as is well known, is given by the Feynman-Dyson rules. Hence, in general, S' has also to be constructed according to these rules. Let us now show that $S'' = 0$ in the general case as well as for spin ½ particles. For this purpose, let us note that the elements of the S-matrix cannot depend on n_μ – as it can be seen, for instance, from Eq. (21). Hence S'' also does not depend on n_μ and therefore does not depend on $F(L_\mu n_\mu, \nabla_\nu n_\nu, \ )$ (because this function vanishes if $n_\mu = 0$). This is possible if, and only if $S'' = 0$.

In the matrix elements of any order in e, the terms depending on the normal to space-like surfaces have to compensate each other, and these elements are constructed according to the Feynman-Dyson rules – in the sense given above. For matrix elements of the first and second order, this was verified directly.[6]

After the completion of this work **** a paper[13] was published in which the author considers the commutation relations, the Green's function and the S-matrix for spin 3/2 particles, described by the Pauli-Fierz equation.[1] The results of this work are contained in ours, as a special case.

The author wishes to express his deep gratefulness to Prof. V. L. Ginzburg, and also to E. S. Fradkin and V. Ia. Fainberg for valuable discussion of the results of this work.

[1] M. Fierz and W. Pauli, Proc. Roy. Soc. (London) 173, 211 (1939).

[2] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) 13, 33 (1943).

[3] I. M. Gel'fand and A. M. Iaglom, J. Exptl. Theoret. Phys. (U.S.S.R.) 18, 1096 (1948); 18, 1105 (1948).

[4] Tamm, Gol'fand and Fainberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 26, 649 (1954).

[5] V. Ia. Fainberg, Thesis, Physical Inst., Acad. Sci. USSR (1953); J. Exptl. Theoret. Phys. (U.S.S.R.) 25, 636, 644 (1953).

[6] V. I. Karpman, Thesis, Physical Inst., Acad. Sci. USSR (1954).

[7] R. P. Feynman, Phys. Rev. 76, 749 (1949). F. Dyson, Phys. Rev. 75, 1736 (1949).

[8] C. N. Yang and D. Feldman, Phys. Rev. 79, 972 (1950).

****The results are given in Ref. 6.

[9]V. I. Karpman, J. Exptl. Theoret. Phys. (U.S.S.R.) 21, 1338 (1951)).

[10]F. I. Fedorov, Dokl. Akad. Nauk USSR 59, 1551 (1948).

[11]A. I. Akhizer and V. P. Berestetski, *Quantum Electrodynamics*, Moscow (1953).

[12]Y. Takahashi and H. Umezawa, Prog. Theoret. Phys. 9, 14 (1953).

[13]S. N. Gupta, Phys. Rev. 95, 1334 (1954).

Translated by E. S. Troubetskoy
228