The character of the resonance can be seen in the sketch, where is shown $R(H)/R_0$ versus $\omega/\Omega = mc\,\omega/eH$ for various ratios of ω to $1/t_0$, in the simplest case of an ellipsoidal Fermi surface with l/t_0 independent of p_x.

The conclusions reached above are also valid when several zones are present. An experimental study would in principle allow one : (a) to find out, from the existence or not of the resonance, whether the surface $E(p)=E_0$ is closed, (b) to determine the degree of filling of the zones, i. e. how far the Fermi surface differed from ellipsoidal shape; (c) to establish the speed of electrons at the Fermi surface (4), by determining from H_{res} the value of $(dS/dE)_{ext}$. In the presence of several surfaces, we can determine the speed on each in turn; in equation (4) only $(dS/dE)_{ext}$ enters. Note that here we discuss only the main surfaces, not the anomalously small zones.[5]

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[*] $\delta/r \sim Hm^{1/2}h^{-1}n^{-5/6} \sim 10^{-6}\ H \ll 1$. In semiconductors, where diamagnetic resonance is observed, $\delta/r \gg cm(|e|t_0)^{-1}(nkT)^{-1/2} \gtrsim 1$ (t_0 – time of free path, n – density of electrons, $\omega t_0 >> 1$, ω – angular frequency of electromagnetic field, T – temperature).

[**] Unfortunately, diamagnetic resonance has often been called cyclotron resonance in the literature. The present nomenclature seems more appropriate.

[***] It turns out that near resonance, for a non-quadratic law of dispersion the complex tensor B_{ik} can be reduced to principal axes. For a quadratic law of dispersion $E(p)=\frac{1}{2}\mu_{ik}p_ip_k$ and $1/t_0$ independent of p_x, this is possible for all ω and H, and equation (2) is valid for $\delta << r < l$ and becomes an interpolation formula for all $H \ll v\sqrt{2\pi m_0 n} \sim 10^6$ G.

[****] The derivation of these equations, and detailed discussion of some further points, will be the subject of a separate article.

Note added in proof: Quite recently a paper has appeared[6] on a resonance in bismuth: this is to be distinguished from the resonance discussed here, since the latter (1) occurs at multiple frequencies, (2) occurs independently of the sign of the magnetic field, (3) occurs only for magnetic fields exactly parallel to the surface of the specimen (the angle ϕ must satisfy $\phi > (\delta/r)^{2/5}$). In particular, condition (3) is not fulfilled in the work referred to.

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Quantum Theory of Electrical Conduction in a Magnetic Field

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IN a previous paper we have developed a theory of galvanomagnetic phenomena in strong magnetic fields, treating the electrons classically as Fermi-particles with a general dispersion law $E=E(p)$.[1] There, however, we did not treat certain specific phenomena connected with the quantisation of the electronic energy levels (for example, the oscillations in resistance as the magnetic field changes). Such effects are observed experimentally[2], but previous theoretical investigations[3] are not entirely satisfactory. In the present paper we shall construct a consistent quantum-mechanical theory of metallic conduction in a magnetic field.

1. In quasi-classical approximation, the spacing of levels in a magnetic field in the z direction is given by[4]

$$\Delta\varepsilon_n = \varepsilon_{n+1} - \varepsilon_n = \mu^* H; \qquad (1)$$

$$\mu^* = \frac{e\hbar}{m^* c}; \quad 2\pi m^* = \frac{\partial S}{\partial \varepsilon},$$

where $S=S(E,p_z)$ is the area cut by the surface $E(p)=E$ in the plane p_z = constant. Thus the essentially quantum-mechanical effects appear when $\mu^* H \sim kT$.

The Hamiltonian $\mathcal{H}$ of an electron in a magnetic field $H_z = H$ and an electric field E may be written

$$\hat{\mathscr{H}} = \hat{\mathscr{H}}^0 + \hat{U}; \quad \hat{U} = -e\mathbf{E}\mathbf{r}, \tag{2}$$

where $\hat{H}^0$ is the kinetic energy operator of an electron in the magnetic field. This is obtained (at least in the quasi-classical approximation) from $E(p_x, p_y, p_z)$ by substituting for the momentum components of the kinetic momentum operator satisfying $[\hat{p}_x, \hat{p}_y] = eH/c$, suitably symmetrized. The exact significance of this need not be discussed in this approximation.

According to the quantum theory the electronic state is described by the density matrix $\hat{f}$. In the absence of an electric field, $\hat{f} = \hat{f}^0$ is the equilibrium Fermi distribution.

$$\hat{f}^0 = f^0(\hat{\mathscr{H}}^0); \quad f^0(z) = (1 + e^{(z-\zeta)/kT})^{-1}.$$

When the electric field is present, we put $\hat{f} = \hat{f}^0 + \hat{f}^1$. The kinetic equation for $\hat{f}^1$ is given by

$$[\hat{f}^1, \hat{\mathscr{H}}] + [\hat{f}^0, \hat{\mathscr{H}}] + \widetilde{W}\hat{f}^1 / t_0 = 0$$

or including only terms linear in E

$$[\hat{f}^1, \hat{\mathscr{H}}_0] + \widetilde{W}\hat{f}^1 / t_0 = -[\hat{f}^0, \hat{U}] \tag{3}$$

where $\widetilde{W} / t_0$ is a certain linear transformation, corresponding to the collision integral, and t_0 has the nature of a relaxation time. The operator f^1 is diagonal with respect to p_z but not with respect to n. Equation (3) when written in terms of matrix elements becomes

$$\frac{i}{\hbar}(\varepsilon_{n'} - \varepsilon_n) f^1_{n,n'} + (\widetilde{W} f^1)_{n,n'} \frac{1}{t_0}$$

$$= -e\mathbf{E}\mathbf{v}_{n,n'} \frac{f^0_{n'} - f^0_n}{\varepsilon_{n'} - \varepsilon_n},$$

where $\mathbf{v}_{n,n'}$ is the matrix element of the velocity which corresponds to the classical quantity $\mathbf{v} = dE/d\mathbf{p}$. From equation (1) we have $E_{n'} - E_n = (n' - n)\mu^* H$. Thus assuming that $f^1 = -et_0\mathbf{E}\vec{\psi}$, we get after some transformation

$$ik\vec{\Psi}_k + \gamma W_{k,k'}\vec{\Psi}_{k'} = \gamma g_k \mathbf{v}_k; \tag{4}$$

$$\gamma = H_0^* / H; \qquad H_0^* = m^* c / et_0;$$

$$\vec{\Psi}_{n,n+k} \equiv \Psi_k(\varepsilon_n); \quad \mathbf{v}_{n,n+k} \equiv \mathbf{v}_k(\varepsilon_n);$$

$$g_k = [f^0(\varepsilon_n + k\mu^* H) - f^0(\varepsilon_n)] / k\mu^* H$$

$$k \neq 0; \quad g_0 = \partial f^0 / \partial\varepsilon.$$

In the quasi-classical approximation, the matrix elements $\mathbf{v}_k$ are the Fourier components of the classical velocity vector $\mathbf{v}$ expanded in terms of $e^{ik\phi}$, where ϕ is the angle variable around the orbit in momentum space

$$\mathbf{v}(\varepsilon, p_z, \varphi) = \sum \mathbf{v}_k e^{ik\varphi};$$

$$\varphi = 2\pi \int \frac{dl}{v_\perp} \Big/ \oint \frac{dl}{v_\perp}; \quad dl = |d\mathbf{p}|,$$

$v_\perp = \sqrt{v_x^2 + v_y^2}$. In the limit $h \to 0$, $(\mu^* H \to 0)$ we have $g_k \to df^0 / dE$, and equation (4) becomes the classical equation for the Fourier components of the distribution function $\vec{\psi}$. The quantities $W_{k,k'}$ in this case become the Fourier components of the collision operator. At low temperatures where impurity scattering is predominant, $W_{k,k'}$ in (4) coincides with its classical analogue. Similar substitution is not however permissible for the quantity g_k when $\mu^* H \sim kT$.

The expressions for the current $\mathbf{j}$ and the conductivity $\sigma^{xx'}$ are given by the formulas

$$\mathbf{j} = e\,\mathrm{Sp}\,\hat{f}^1\hat{\mathbf{v}}; \quad \sigma^{x,x'} = -e^2 t_0\,\mathrm{Sp}\,\hat{v}^x\hat{\Psi}^{x'} \tag{5}$$

$$= \frac{e^2 t_0}{h^3} \sum_n \int \left\{ v_0^x \Psi_0^{x'} + 2\mathrm{Re} \sum_{k=1}^{\infty} v_{-k}^x \Psi_k^{x'} \right\} dp_z\, m^* \Delta\varepsilon_n.$$

In the limit $\hbar \to 0$ the expression (5) also tends to the corresponding classical formula.

2. The connection established above between the quantum-kinetic Eq. (4) and its classical analogue allows us to find an expression for the quantum effects by means of the solutions of the classical equations investigated earlier [1], and in particular to clarify the connection between the quantum oscillations of conductivity and their mean values, and also the oscillations in susceptibility (de Haas-van Alphen effect). Omitting all calculation and discussion (which is rather lengthy) we shall give here only the results obtained.

Let us write the expression for the classical conductivity tensor in terms of the 'mobility tensor' $q^{x,x'} = q^{x,x'}(E, p_z, H)$ (the value of $q^{x,x'}$ is obtained from the solution of the classical problem):

$$\sigma^{xx'} = h^{-3} \iint f^0 q^{x,x'} m^* \, d\varepsilon \, dp_z = N^0 \overline{q^{x,x'}}. \tag{6}$$

Here N° is the number of electrons in the conduction zone, and $q^{x,x'}$ is the mean value of the mobility, taken with the weighting factos $f.^{\circ}$ Then the expression for the oscillating part $\Delta\sigma^{x,x'}$ (for $x,x' \neq z,z$) may be written in terms of the independent variables H, ζ as

$$(\Delta\sigma^{x,x'})_{H,\zeta} = q_m^{x,x'} H (\partial \ln S / \partial\varepsilon)_m \Delta M^z, \tag{7}$$

where ΔM^z denotes the oscillatory part of the moment M^z in the de Haas-van Alphen effect and the values $q_m^{x,x'}$ and $(d \ln S / dE)_m$ are taken at the extreme cross-section of the Fermi surface $E(p) = \zeta$ by the planes $p_z = \text{const}$. An analogous expression for $\Delta\sigma^{xz}$, not given here, contains $\Delta\sigma^{zz}$ and also $d\,\Delta M^z / dH$.

As is known[4] each zone (or more precisely, each extreme section of the Fermi surface) gives a separate contribution to ΔM^z, and in fields $\sim 10^4$ G only anomalously small zones show themselves.[4] Note that in $\sigma^{x,x'}$ the major contribution is from 'normal' zones, which complicates comparison of Eqs. (6) and (7).

In comparing with experiment, we must also take account of the oscillations in the chemical potential $\zeta = \zeta(H)$, which are determined by the conditions that the total number of electrons in all zones remain constant. Taking account of the additional term $\Delta\sigma_1^{x,x'} = (\partial\sigma^{x,x'} / \partial\zeta)\Delta\zeta$, we obtain finally for the case where several conduction zones are present.

$$\Delta\sigma^{x,x'} = \sum_i \frac{\{ q_{mi}^{x,x'} - \sum_k \frac{\partial N_k^0 \overline{q_k^{x,x'}}}{\partial\zeta}}{\sum_k \frac{\partial N_k^0}{\partial\zeta} \}} H \left(\frac{\partial \ln S_i}{\partial\varepsilon}\right)_m \Delta M_i^z \tag{8}$$

(the summation to be taken over all zones).

Thus, for example, for a single zone with a small number of electrons and an isotropic dispersion law $E = p^2/2m$, $q^{xy} = ec/H(1+\gamma^2)$, and we have:

$$\frac{\Delta\sigma^{xy}}{\sigma^{xy}} = {}^4\!/_3\,\gamma^2 \frac{\partial \ln t_0}{\partial\zeta} \cdot H \frac{\Delta M^z}{N^0}; \quad t_0 = t_0(\zeta).$$

The presentation of other results and also the mathematical treatment will be the subject of a detailed communication later.

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