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Translated by A. Skumanich
162

Effect of the Rate of Flow of a He II Film on Its Thickness

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IT is well known that below the λ-temperature point the wall of a container of liquid helium becomes covered with a thin ($\sim 3 \times 10^{-16}$ cm), rapidly creeping film which moves (under isothermal conditions) in the direction of a lower level[1-4] (i.e., toward a lower gravitational potential). It is of interest to ascertain how the motion influences the thickness of the film. The fact that some influence is expected follows from the dependence of the thermodynamic potential of He II on the rate of relative motion of the superfluid and the normal component[5,6].

We shall consider the leaking of a film over a vertical wall. The rate of overflow of the film, i.e., the volume of fluid Q, transported by the film in unit time and across a unit perimeter, shall be considered as a known quantity. The hydrodynamic equations of motion in a constant potential field are obtained in the customary manner from the conservational laws and the requirement of the existence of a velocity potential for the superfluid motion[5,6]. The equations of motion for the superfluid component prove to be

$$\mathbf{v}_s + \nabla \{ {}^1/_2 v_s^2 + \tilde{\mu} + U \} = 0. \tag{1}$$

Here $\tilde{\mu}$ is the chemical potential of helium per unit mass in the absence of a field, U is the potential energy per unit mass of fluid. For the problem under consideration, U equals $\beta y^{-3} + gz$, where β is a constant specifying the Van der Wall's interaction with the wall. The y-axis is directed along the normal to the wall and lies in the plane that contains the horizontal surface of the helium in the container.

We shall consider the motion quasi-stationary. In view of the extremely small film thickness we can neglect the motion of the normal component in comparison to the superfluid: $v_n = 0$. If we insert $\tilde{\mu} = \mu(P, T) - (\rho_n/2\rho) v_s^2$ into Eq. (1) and integrate over the free surface of the film, we find that

$$(\rho_s / 2\rho) v_s^2 + \mu(P, T) + gz - \beta\delta^{-3} = \mu(P_0, T_0), \tag{2}$$

where δ is the thickness of the film at height z and P_0, T_0 is the pressure and temperature, respectively, at the horizontal surface. We shall consider the flow isothermal. Then

$$\mu(P, T) - \mu(P_0, T_0) = (P - P_0)/\rho. \tag{3}$$

The boundary condition at the free surface requires

$$P_\sigma + P = P_0, \tag{4}$$

where P_σ is the pressure associated with the curved surface. The change in the pressure of helium vapor with height is neglected. If we consider a z sufficiently large in comparison with the capillary constant of helium so that the film can be considered plane-parallel, then we obtain from Eq. (2) through (4)

$$(\rho_s / 2\rho) v_s^2 + gz - \beta\delta^{-3} = 0. \tag{5}$$

In this part of the film the flow rate is

$$Q \equiv \frac{\rho_s}{\rho} \int_0^\delta v_{sz}\, dy \approx v_s \delta \frac{\rho_s}{\rho}.$$

Expressing v_s in terms of Q and allowing for the fact that

$$(\rho/6\rho_s)(Q^2/\beta)(\beta/gz)^{1/3} \equiv q \ll 1,$$

we find from Eq. (5) that

$$\delta = (\beta/gz)^{1/3}(1 - q). \tag{6}$$

For observed values of Q we find that $q \sim 10^{-1}$ to 10^{-2} [1,2]. For $Q = 0$ we obtain the usual equation for thin stationary helium films[7]. We note that the form of the first term in Eq. (5) is specified by the dependence of the chemical potential on the relative rate of flow.

I use this opportunity to express my sincere gratitude to Prof. I. M. Lifshitz for detailed discussions on the problems considered in this note.

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167

Specific Heat of Solid Oxygen between 20° and 4° K

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WE have previously reported measurements of the heat capacity of solid oxygen below 4° K [1]. These were undertaken to clarify the question of the character of the magnetic anomalies in solid oxygen. The absence (in the expression for the heat capacity) of a term which depends linearly on the temperature suggested that, at liquid helium temperatures, solid oxygen is in an antiferromagnetic state. It was surmised that a transition of the oxygen to an antiferromagnetic state took place in the temperature interval between 4° and 10° K. In order to verify this surmise, measurements of the heat capacity of solid oxygen have been carried out between 4° and 20° K in the present work.

A calorimeter similar to that described in Ref. 2 was used for the measurement of the heat capacity [2]. A high vacuum was produced in the container as a result of the adsorption by activated charcoal of heat-exchanging helium or hydrogen introduced therein. The temperature scale in these experiments was established with the aid of a carbon resistance thermometer, which was calibrated before each experiment in terms of the vapor pressure of helium and hydrogen*.

TABLE

First series of measurements		Second series of measurements		Third series of measurements	
$T°$ K	C cal/mole-degree	$T°$ K	C cal/mole-degree	$T°$ K	C cal/mole-degree
4.325	0.0343	4.275	0.0361	12.50	1.06
4.387	0.0372	4.32	0.0367	13.06	1.16
4.723	0.0481	4.66	0.0463	13.15	1.21
4.89	0.0544	4.685	0.0485	13.93	1.43
4.947	0.0556	4.727	0.0481	14.02	1.36
5.235	0.0686	5.01	0.0592	14.09	1.51
5.282	0.0756	5.12	0.0575	14.17	1.42
5.47	0.0788	5.67	0.0840	14.26	1.48
5.52	0.0798	5.71	0.0883	14.73	1.61
5.76	0.0937	6.49	0.140	14.8	1.62
5.81	0.0961	6.55	0.140	14.89	1.60
6.32	0.118	7.41	0.213	14.97	1.69
6.37	0.132	7.46	0.215	15.06	1.66
6.95	0.167	8.14	0.285	15.39	1.80
7.67	0.227	8.19	0.304	15.47	1.83
7.78	0.236	8.25	0.296	15.95	2.03
8.70	0.346	8.43	0.312	16.04	2.01
8.75	0.350	8.63	0.350	16.13	1.98
9.25	0.424	8.68	0.343	16.78	2.28
9.72	0.470	8.76	0.359	16.88	2.15
—	—	8.82	0.364	16.97	2.25
—	—	8.97	0.397	17.68	2.59
—	—	10.38	0.599	17.77	2.58
—	—	—	—	18.52	2.74
—	—	—	—	18.61	2.82
—	—	—	—	19.25	3.03
—	—	—	—	19.35	3.08