The assumption of adiabatic cessation of the interaction

$$e = e(t) = \begin{cases} e, & t > T_0 \\ e \exp \alpha t, & t < T_0 \end{cases} \quad (T_0 \to -\infty)$$

allows us to neglect the values of all the integrals taken at the lower limit. In particular, Eq. (5) depends only on the lower limit, and hence is equal to zero. Carrying out the integration with respect to $d\tau$ and $d\tau'$ in Eq. (4) in the indicated manner, and varying Eq. (4) with respect to δe^2, we take the Fourier component of the resulting relation in the momentum region $|p^2 - m^2| \ll m^2$ which is of interest to us. We find

$$\delta G(p) = \frac{i\delta e^2}{\pi} \left\{ \int [G(p) - G(p-k)] \left[\left(v^2 - \frac{(vk)^2}{k^2} \right) + \frac{(vk)^2}{k^2} d_l(k^2) \right] \frac{d^4k}{\omega^2 (k^2 + i\varepsilon)} \right\}. \quad (6)$$

The calculation is most simply carried out in the system of reference in which the velocity of the particle $v = p/m$ is equal to zero. Then

$$\delta G(p) = \frac{i\delta e^2}{\pi} \left\{ \int [G(p) - G(p-k)] \left[\left(1 - \frac{\omega^2}{k^2} \right) + \frac{\omega^2}{k^2} d_l(k^2) \right] \frac{d^4k}{\omega^2 (k^2 + i\varepsilon)} \right\}.$$

Carrying out the integration with respect to $d\omega$ according to the usual rules of contouring[3], we find that the terms sought are obtained by taking a calculation at the point $k^2 = 0$:

$$\delta G(p) = \frac{\delta e^2}{2\pi} \left\{ \int [G(p) - G(p-k)] \frac{d|k|}{|k|} \right\} [3 - d_l(0)]; \quad (\omega = +|k|). \quad (7)$$

The essential logarithmic region of interaction in this integral is $|p^2 - m^2|/m \ll k \ll m$. The region of high frequencies leads to renormalization effects, which, naturally, cannot be correctly taken into account in this technique. Carrying out the integration in (7) for low frequencies, we obtain

$$\delta G(p) = G(p) \left(\frac{\delta e^2}{2\pi} \right) [3 - d_l(0)] \ln \left(\frac{m^2}{p^2 - m^2} \right)$$

or

$$G(p) = G_0(p) \left(\frac{m^2}{p^2 - m^2} \right)^{(e^2/2\pi)\,[3 - d_l(0)]}$$

where $G_0(p)$ differs from the Green's function of a free particle by the renormalization factors. Thus, the appearance of the additional singularity (1) in the Green's function of a particle interacting with an electromagnetic field is connected only with the classical properties of the electric current being produced by the particle in its uniform motion.

In conclusion, I wish to express my deep gratitude to A. A. Abrikosov and I. M. Khalatnikov for discussions of this work.

* Designations are those used in Ref. 3.

[1] A. A. Abrikosov, Dissertation, Institute for Physical Problems, Academy of Sciences, USSR, 1955.

[2] L. P. Gor'kov, Dissertation, Institute for Physical Problems, Academy of Sciences, USSR, 1955.

[3] R. Feynman, Phys. Rev. 76, 749, 769 (1949).

Translated by M. G. Gibbons
159

Transformation of Positive Helium Ions Colliding with Inert Gas Atoms into Negative Ions

V. M. DUKEL'SKII, V. V. AFROSIMOV
AND
N. V. FEDORENKO
Leningrad Physico-technical Institute, Academy of Sciences, USSR

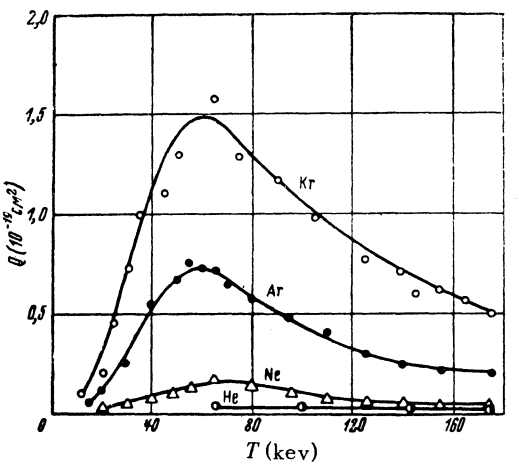
(Submitted to JETP editor January 14, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 792-793 (April, 1956)

IN studying the processes which take place during the passage of He^+ ions through rarefied gases, we noted that negative ions He^- occurred in the beam after it had passed through the gas.

The experiments were carried out in the double mass spectrometer arrangement described in Ref. 1. A beam of He^+ ions of given energy was separated by the magnetic mass-monochromator, after which it entered the gas-filled collision chamber. For a gas pressure of $\sim 3 \times 10^{-4}$ mm Hg in the chamber, we can keep the pressure in the remaining parts of the apparatus at a level $< 1 \times 10^{-5}$ mm Hg. The composition of the beam which has passed through the collision chamber is investigated by means of a magnetic mass analyzer.

If we admit a beam of He^+ ions into the collision chamber and select a suitable intensity of magnetic field in the mass analyzer, we can pass through it He^+ ions which will retain their charge and their velocity after the passage. On reversing the direction of the magnetic field in the mass

analyzer, we discovered the presence of negative ions in the beam coming from the collision chamber. These ions passed through the mass analyzer for the same value of magnetic intensity as did the He^+ ions. It is obvious that they were He^- ions which had been formed from the He^+ ions and which had kept their velocity.



Dependence of the effective cross section for the conversion process $He^+ \rightarrow He^-$ in He, Ne, Ar, Kr on the energy of the He^+ ions.

The formation of the He^- ions was established for the He^+ ion energy interval from 15 to 175 kev in four inert gases: Kr, Ar, Ne, He. We are convinced that in the case of Kr, Ar and Ne the effect is proportional to the pressure of the gas in the collision chamber right down to a pressure of 5×10^{-4} mm Hg, that is, the process $He^+ \rightarrow He^-$ occurred during *single* collisions of He^+ ions with the atoms of the gas. This shows that in interacting with the atom the He^+ ion must have simultaneously taken two electrons from it. In order to show the magnitude of the observed effect, we adduce an example. For a He^+ ion energy of 80 kev and a pressure of 2.5×10^{-4} mm Hg of argon in the collision chamber (16 cm path length of the ions in the gas), the current of secondary He^- ions was equal to 1.4×10^{-12} amp when the current of primary He^+ ions was equal to 3.3×10^{-7} amp.

On the basis of the measurements performed, we carried out an approximate determination of the effective cross sections for the process $He^+ \rightarrow He^-$. In order to do this, it was necessary to know the transmission coefficient k_- of the mass analyzer for the secondary ions He^-. We measured the coefficient k_+ for the primary ions He^+, scattered in the gas, and used its value in calculating the effective cross sections for the process $He^+ \rightarrow He^-$. In the energy interval which we used, the value of k_+ changed only slightly and was equal to 0.4-0.5. It was also verified that the angular distribution of the He^- ions was only slightly different from that of the primary He^+ ions after going through the collision chamber.

Our results are given in the figure in the form of curves showing the dependence of the effective cross section Q for the conversion process $He^+ \rightarrow He^-$ on the energy T of the He^+ ions. The curves for neon, argon and krypton have a maximum for an energy of 60-70 kev. As the atomic number of the gas atoms decreases, the curves become lower, keeping the same general shape. For krypton the magnitude of Q corresponding to the maximum of the curve is 1.5×10^{-19} cm^2. In the case of helium the conversion effect is very small (Q of the order of 10^{-21} cm^2), and hence we were not able to determine the form of the $Q(T)$ curve.

The formation of negative ions on the collision of positive ions with gas molecules was first observed for protons[2,3], and later for oxygen ions[4]. The $He^+ \rightarrow He^-$ process which we have observed is of special interest in view of the fact that the very possibility of the formation of the He^- ions is unexpected. As was shown in Ref. 5, the energy of an He^- ion in its normal state $(1s^2 2s)^2 s$ (similar to the ground state of the lithium atom) should exceed the energy of the normal state of a helium atom; hence, such an ion should not exist. In a recently published work[6] it was pointed out that a highly excited state of the negative ion He^- was possible, which, if it existed, would have to be metastable (with respect to the process of auto-ionization) and possess an average lifetime τ of the order of 10^{-13} sec. This theoretical result coincides with the results of Ref. 7, in which an analysis of the composition of canal rays showed a weak He^- line.

On the basis of our experiments we cannot judge whether the He^- ions which we have observed are stable or metastable. The He^- ions travel a path of 70 cm in our apparatus; for an energy of 60 kev the time of flight is 4×10^{-7} sec. If the He^- ions which we observed were metastable, with $\tau = 10^{-3}$ sec, then the fraction 4×10^{-4} of them should disintegrate by spontaneous emission of an electron while going along the path from the point of collision to the receiver. Such an insignificant decrease in the number of He^- ions because of their disintegration in the apparatus could not be detected in the results of our experiments, in particular, in the form of the $Q(T)$ curves which we obtained.

[1] D. M. Kaminker and N. V. Fedorenko, J. Tech. Phys. (U.S.S.R.) 25, 1843 (1955).

[2] A. C. Whittier, Can. J. Phys. 32, 275 (1954).

[3] Ia. M. Fogel', L. I. Krupnik and B. G. Safronov, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 589 (1955); Soviet Phys. JETP 1, 415 (1955).

[4] Ia. M. Fogel' and L. I. Krupnik, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 209 (1955); Soviet Phys. JETP 2, 252 (1956).

[5] Ta-You Wu, Phil. Mag. 22, 837 (1936).

[6] E. Holøien and J. Midtal, Proc. Phys. Soc. (London) A68, 815 (1955).

[7] J. W. Hiby, Ann. Physik 34, 473 (1939).

Translated by M. G. Gibbons
160

Concerning a Certain Generalization of a Renormalization Group

A. A. LOGUNOV
Moscow State University

(Submitted to JETP editor January 24, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 793-795
(April, 1956)

THE basic goal of the present note is the generalization of the Lie equations for the Green functions for the case of an arbitrary longitudinal coupling of photons.

1. We shall work with finite Green functions and shall assume that all divergences which exist in the theory have been eliminated by means of the subtraction formalism[2]. However, there still remain, even after the elimination of the infinites, final ambiguities which are associated with the finite counter-terms introduced into the Lagrangian of the same operator which occurs in the elimination of the divergences[2]. In the case where the æroth coupling of the photon contains an admixture of an arbitrary longitudinal coupling,

$$\mathfrak{G}^0_{mn}\, d(k^2) = \frac{i}{k^2}\left(g^{mn} - \frac{k_m k_n}{k^2}\right) + \frac{i}{k^2}\frac{k_m k_n}{k^2}\,\omega^2 d_l\,(k^2),$$

the calculation of the introduced counter-terms introduces a multiplicative renormalization factor for the transverse part of the photon coupling but does not change its longitudinal part. Insofar as the introduced finite counter-terms (for the transverse photon coupling) led to a renormalization of the Green function and of the electron charge, then, in the present problem, because of the circumstance shown above regarding the "incomplete renormalization" of the zeroth coupling, we shall work with the following transformation group:

$$G_1 \to G_2 = Z_2 G_1;\quad \mathfrak{G}_1 \to \mathfrak{G}_2 = Z_3\mathfrak{G}_{,1};\quad \Gamma_1 \to \Gamma_2 = Z_1^{-1}\Gamma_1;$$

$$e_1^2 \to e_2^2 = Z_3^{-1}e_1^2;\quad \omega_1^2 \to \omega_2^2 = Z_3\omega_1^2;\quad Z_1 = Z_2. \quad (1)$$

This transformation group is a generalization of the transformation group[1] for the case of an arbitrary longitudinal photon coupling. The significance of these transformations is that the set of quantities $(G_1, \mathfrak{G}_1, \Gamma_1, e_1, \omega_1)$ and $(G_2, \mathfrak{G}_2, \Gamma_2, e_2, \omega_2)$ can be used in the same form for the specification of particles with their mass and charge equal to the experimental values.

2. The transformation group obtained above permits one to derive the equations for the Green functions. If we represent the Green functions in the form

$$G(k) = i\,\frac{\hat{k}a(k^2) + mb(k^2)}{k^2 - m^2}, \quad (2)$$

$$\mathfrak{G}_{mn}(k) = \frac{i}{k^2}\left(g^{mn} - \frac{k_m k_n}{k^2}\right) d(k^2) + \frac{i}{k^2}\,\frac{k_m k_n}{k^2}\,\omega^2 d_l\,(k^2)$$

and if we reason as in Ref. 1, we obtain with no difficulty the following functional equations,

$$d(x, y, e^2) = d(-t, y, e^2)\, d(x\ \ t, y/t, e^2 d(-t, y, e^2)),$$

$$t > 0, \quad (3)$$

$$s'(x, y, \omega^2, e^2) = \frac{s'(x/t,\ y/t,\ \omega^2 d^{-1},\ e^2 d(-t, y, e^2))}{s'(-1/t,\ y/t,\ \omega^2 d^{-1},\ e^2 d(-t, y, e^2))},$$

where

$$k^2\ \lambda_2^2 = x,\quad m^2/\lambda_2^2 = y,$$

$$s'(x, y, \omega^2, e^2) = s(x, y, \omega^2, e^2)\, s^{-1}(-1, y, \omega^2, e^2).$$

The first of these equations agrees with the analogous equation for d in Ref. 2, insofar as the transverse part of the zeroth coupling.

Differentiating each of the above equations with respect to x and setting $t = -x$, we obtain,

$$\frac{\partial}{\partial x}\ln d(x, y, e^2) \quad (4)$$

$$= -\frac{1}{x}\left[\frac{\partial}{\partial \xi}\, d\left(\xi - \frac{y}{x},\ e^2 d(x, y, e^2)\right)\right]_{\xi=-1},\quad x < 0,$$

$$\frac{\partial}{\partial x}\ln s'(x, y, \omega^2 e^2)$$

$$= -\frac{1}{x}\left[\frac{\partial}{\partial \xi}\, s'\left(\xi, -\frac{y}{x},\ \omega^2 d^{-1},\ e^2 d(x, y, e^2)\right)\right]_{\xi=-1}.$$