on this scheme shows that such nuclei must be: $_{66}Dy_{95}^{165}$ with isomeric transition of type E3[2]; $_{72}Hf_{107}^{179}$ (E3, M3)[2]; $_{74}W_{109}^{183}$ (E3, M3)[2]; $_{74}W_{111}^{185}$ (E3, M3)[2]; $_{76}Os_{115}^{191}$ (M3 + E4)[11]; as well as $_{68}Er_{99}^{167}$; $_{72}Hf_{105}^{177}$; $_{70}Yb_{101}^{171}$. In the first five nuclei isomeric states have already been observed, and their expected multipolarity (which agrees with experimental data) is given in parentheses. In an analogous manner it is possible to explain the absence of an isomeric state in the heaviest isotope $_{43}Tc_{58}^{101}$ [12], since this nucleus is more strongly deformed than the lighter isotopes Tc^{93-99} in which isomeric states are observed. Taking account of deformation makes it also possible to explain the appearance in the island $41 \leq Z, N \leq 47$ of isomeric transitions of type E3[9].

Thus, taking into account of deformation makes it possible to explain the absence of isomeric states in nuclei with $63 \leq Z \leq 75$. the small number of isomeric transitions and their anomalous multipolarity with $93 \leq N \leq 115$, as well as the absence of an isomeric state in Tc^{101} and the appearance of transitions of type E3 in the region $39 < Z, N < 49$.

In conclusion, it is a pleasant duty to express my deep gratitude to L. A. Sliv for detailed discussion of this paper.

[1] C. E. Weizsäcker, Naturwiss 24, 813 (1936).

[2] M. Goldhaber and R. D. Hill, Rev. Mod. Phys. 24, 179 (1952).

[3] L. W. Nordheim, Phys. Rev. 75, 1894 (1949).

[4] E. Feenberg and K. C. Hammack, Phys. Rev. 75, 1877 (1949).

[5] M. G. Mayer, Phys. Rev. 78, 16 (1950).

[6] M. Goldhaber and A. W. Sunyar, Phys. Rev. 83, 906 (1951).

[7] R. Katz, Phys. Rev. 91, 1487 (1953).

[8] G. Scharff-Goldhaber, Phys. Rev. 90, 587 (1953).

[9] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab. Mat. – fys. Medd., 27, 16 (1953) .

[10] S. Nillsen, Kgl. Danske Videnskab, Selskab., Mat. – fys. Medd. 29, 14 (1955).

[11] W. Michelich, Bull. Amer. Phys. Soc. 30, 46 (1955).

[12] D. R. Wiles, Phys. Rev. 93, 181 (1954).

Translated by A. V. Bushkovitch
127

Energy Spectrum of High Energy Ionizing Particles Passed Through a Thick Layer of Matter

IU. F. ORLOV

(Submitted to JETP editor December 2, 1955)
J. Exptl. Theoret. Phys.(U.S.S.R.) 30 613-614 (March, 1956)

THE energy spectrum of a fast ionizing particle after passage through a thin layer of matter for which the average energy loss $\overline{\Delta E} \ll E$ is determined by the relation

$$s \equiv \overline{\Delta E}/\varepsilon_{\max} L_i,$$

where L_i is the well-known ionization logarithm and $\epsilon_{\max}$ (E) is the maximum energy loss per ionizing collision where m is the electron rest mass,

$$L_i = 2\left(\ln \frac{2m\beta^2}{I_{\text{cp}}(1-\beta^2)} - \beta^2\right), \qquad (1)$$

$$\varepsilon_{\max} = \frac{2m(E^2 - \mu^2)}{\mu^2}, \quad \beta = v/c,$$

where m is the rest mass of an electron, μ the rest mass of the ionizing particle and I_{ave} is the ionization potential. If $W(\epsilon, E)$ is the probability per unit path length of a collision with an energy loss ϵ, then the distribution function for energy loss Δ after passage through a layer δx has the form[1,2] :

$$f(\delta x, \Delta) = (2\pi)^{-1} \qquad (2)$$

$$\times \int_{-\infty}^{\infty} dz \exp\left\{iz\Delta - \delta x \int_0^{\infty} W(\varepsilon, E)(1 - e^{-iz\varepsilon})\, d\varepsilon\right\}.$$

For $S \ll 1$ Eq. (2) gives the curve due to Landau,[1] and for $S \gg 1$ it yields a Gaussian distribution.[2]

For the case of a thick layer ($S \gg 1$) we shall obtain a more accurate distribution function than the Gaussian. If we allow first $s \gg 1$, but with $\overline{\Delta E} \ll E$, then the distribution function can be expanded in Hermite polynomials:

$$\varphi(\delta x, y) = (2\pi)^{-1/2} e^{-y^2/2}\left(1 + \sum_{n \geqslant 3} a_n H_n(y)\right); \quad (3)$$

$$H_n(y) = (-1)^n e^{y^2/2} \frac{d^n}{dy^n} e^{-y^2/2}, \quad a_n(\delta x) \quad (4)$$

$$= (n!)^{-1} \int_{-\infty}^{\infty} \varphi(\delta x, y) H_n(y)\, dy,$$

$$y=(\Delta-\overline{\Delta})(\overline{\Delta^2}-\overline{\Delta}^2)^{-1/2},\ \varphi(\delta x, y) \tag{5}$$
$$=(\overline{\Delta^2}-\overline{\Delta}^2)^{1/2} f(\delta x,\ \overline{\Delta}+y(\overline{\Delta^2}-\overline{\Delta}^2)^{1/2}),$$

$$\varphi(\delta x,\ y)=(2\pi)^{-1/2}e^{-y^2/2}\left[1+\frac{\beta_3}{6\beta_2^{3/2}}H_3(y)+\frac{\beta_4}{24\beta_2^2}H_4(y)+\frac{\beta_3^2}{72\beta_2^3}H_6(y)+\ldots\right], \tag{6}$$

$$\beta_n=\delta x\overline{\varepsilon^n}=\delta x\int_0^{\varepsilon_{\max}}\varepsilon^n W(\varepsilon,\ E)\,d\varepsilon,\ n>1. \tag{7}$$

Since $W(\epsilon, E)$ = const. $(\beta\epsilon)^{-2}$ and const. $\beta^{-2}\delta x = \overline{\Delta E}/L_i$, we have:

$$\varphi(\overline{\Delta E},\ y)=(2\pi)^{-1/2}e^{-y^2/2}\left[1+\frac{1}{12}s^{-1/2}(y^3-3y)+\frac{1}{72s}(y^4-6y^2+3)+\frac{1}{288s}(y^6-15y^4+45y^2-15)+\ldots\right]. \tag{8}$$

The series of Eq. (8) is asymptotic. For $l/s \sim 1$, $y \sim 2$ the first two terms give an accuracy of abuut 10%; for $y \lesssim 1$ one may use four terms of Eq. (8).

Assume now that $\overline{\Delta E} \sim E_0$. We shall denote the distribution function of energy after passage through a thick layer x by $\phi(x, \epsilon)$. It will evidently be given by the relation

$$\Phi(x,\ E)=\int_0^\infty \Phi(x-\delta x,\ E+\Delta) f(\delta x,\ \Delta)\,d\Delta. \tag{9}$$

Expanding ϕ as a series in powers of Δ and δx and letting δx go to zero, we obtain the differential equation

$$\frac{\partial\Phi}{\partial x}=\overline{\varepsilon}\frac{\partial\Phi}{\partial E}+\frac{1}{2!}\overline{\varepsilon^2}\frac{\partial^2 T}{\partial E^2}+\frac{1}{3!}\overline{\varepsilon^3}\frac{\partial^3\Phi}{\partial E^3}+\cdots \tag{10}$$

If we discard in Eq. (10) the terms with high order of $\overline{\epsilon^n}$, $n\geq 3$ and in the coefficients of $\overline{\epsilon}$ and $\overline{\epsilon^2}$ we replace E by $\overline{E}(x)$, we obtain a Gaussian distribution. This case corresponds to the following two assumptions.* First, $\phi(x,\epsilon)$ should be sufficiently narrow so that it is possible to neglect the dependence of the diffusion coefficient $\overline{\epsilon^2}$ on E and replace E by $\overline{E}(x)$, second, the energy of the particle should not be so large that great energy losses ϵ would take place. For highly relativistic particles these conditions are not fulfilled.

The approximation will be more accurate if instead of $\overline{\epsilon^n}$ in Eq. (10) we take the quantity

$$\int_0^\infty \Phi(x,\ E)\,dE\int_0^{\varepsilon_{\max}}\varepsilon^n W(\varepsilon,\ E)\,d\varepsilon.$$

Using the formula for a derivative of a δ-function

$$\partial^n\delta(\alpha)/\partial\alpha^n=(2\pi)^{-1}(i)^n\int_{-\infty}^{\infty}y^n\exp(i\alpha y)\,dy;$$

we obtain

For a faster computation of corrections we notice that the quantity β_n has the same additive properties according to Eq. (7) as the quantity $\beta_2 = \overline{\Delta^2} - \overline{\Delta}^2$ which enters into Eq. (5) for y. Precisely this property of β_2 was used by Pomeranchuk, who obtained the Gaussian distribution for a thick layer. Therefore we shall first obtain a distribution function for a thick layer by replacing all β_n in Eq. (6) by

$$B_n(\overline{E})=\int_{\overline{E}}^{E_0}\left|\frac{dx}{dE_1}\right|dE_1\int_0^\infty\Phi(x(E_1),\ E)\,dE\times\int_0^{\varepsilon_{\max}}\varepsilon^n W(\varepsilon,\ E)\,d\varepsilon. \tag{11}$$

In order to obtain $\beta_2 = \overline{(E-\overline{E})^2}$ to the first order of l/s, it is sufficient to take for ϕ in Eq. (11) the Gaussian curve of Pomeranchuk; to compute β_3 it is possible to let $\phi(E, E_1)$ equal to $\delta(E-E_1)$. In this approximation

$$B_2=\overline{(E-\overline{E})^2}={}^2/_3(mL_i^{-1}\mu^{-2})\times(E_0^3-\overline{E}^3+3\overline{E}\mu^2-3E_0\mu^2)+{}^4/_3(m^2L_i^{-2}\mu^{-4})\times[3E_0^4/4-E_0^3\overline{E}-\mu^2E_0^3/\overline{E}+3\mu^2E_0\overline{E}+\overline{E}^4/4-2\mu^2\overline{E}^2-3\mu^4-3\mu^4\ln(E_0/\overline{E})]; \tag{12}$$

$$B_3/6B_2^{3/2}={}^1/_6(mL_i/2\mu^2)^{1/2}[(E_0^5-\overline{E}^5)/5-2\mu^2(E_0^3-\overline{E}^3)/3+\mu^4(E_0-\overline{E})]+3\mu^4E_0/\overline{E}]\times[(E_0^3-\overline{E}^3)/3-\mu^2(E_0-\overline{E})]^{-3/2}; \tag{13}$$

$$\Phi(y)=(2\pi)^{-1/2}e^{-y^2/2}[1+(B_3/6B_2^{3/2})\times(y^3-3y)+\ldots], \tag{14}$$

$$y=-B_2^{-1/2}(E-\overline{E}). \tag{15}$$

The Eq. (14) has a simple and descriptive form for $\overline{E} \ll E_0$. For example, at the end of the passage for $E_0 \gg \mu$:

$$\Phi(y) = (2\pi)^{-1/2} e^{-y^2/2} \left[1 + \frac{1}{10} \left(\frac{3mL_iE_0}{2\mu^2} \right)^{1/2} (y^3 - 3y) \right.$$
$$\left. + \frac{1}{28} \frac{mL_iE_0}{\mu^2} (y^4 - 6y^2 + 3) + \frac{3}{400} \frac{mL_iE_0}{\mu^2} (y^6 - 15y^4 + 45y^2 - 15) + \ldots \right]; \quad (16)$$

$$y^2 \approx (E - \overline{E})^2 (3L_i\mu^2/2mE_0^3)(1 + 3mE_0/2L_i\mu^2)^{-1}; \quad (17)$$

$$B_n \approx (2m/\mu^2)^{n-1} E^{2n-1}/(n-1)(2n-1) L_i(E_0), \ \overline{E} \ll E_0. \quad (18)$$

From Eq. (16) it is evident that for sufficiently large ratio $L_i m E_0 / \mu^2$, the distribution function differs substantially from a Gaussian one even at the end of the passage. The curve has a characteristic "tail" at the low energy and a sharp cutoff on the high energy side. Its maximum is displaced in the direction of greater energies relative to its center of gravity ($y = 0$).

[1] L. D. Landau, J. Phys. USSR 8, 201 (1944).

[2] I. Ia. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) 18, 759 (1948).

Translated by M. J. Stevenson
125

The Propagation of Sound in Moving Helium II and the Effect of a Thermal Current upon the Propagation of Second Sound

I. M. KHALATNIKOV
Institute for Physical Problems, Academy of Sciences, USSR

(Submitted to JETP editor December 15, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 617-619 (March, 1956)

ROM ordinary hydrodynamics it is well known that an "entrainment" of sound occurs in a moving fluid. A similar phenomenon must take place in the hydrodynamics of a superfluid. Inasmuch as in the case of a superfluid two types of motion (normal, with a velocity $\mathbf{v}_n$, and superfluid, with a velocity $\mathbf{v}_s$) as well as two types of sound vibrations, propagated with different velocities, are possible, it is natural that the picture of sound propagation in a moving superfluid liquid should differ from the corresponding phenomenon in classical hydrodynamics.

Let sound oscillations of frequency ω be propagated in a direction characterized by the unit vector $\mathbf{n}$ (along the x-axis) through helium II in which normal and superfluid motions are taking place with the constant velocities $\mathbf{v}_n$ and $\mathbf{v}_s$. The wave vector $\mathbf{k}$ is equal to $\mathbf{n}\omega/u$, where u is the velocity of sound. We shall determine here the velocities of first and second sound in the moving Helium II under the assumption that the motion proceeds at velocities which are small by comparison with the velocity of sound. Let us write the complete set of hydrodynamic equations for Helium II:

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \mathbf{j} = 0,$$

$$\frac{\partial j_i}{\partial t} + \frac{\partial}{\partial x_k}(p\delta_{ik} + \rho_n v_{ni} v_{nk} + \rho_s v_{si} v_{sk}) = 0,$$

$$\frac{\partial \rho s}{\partial t} + \operatorname{div} \rho s v_n = 0, \quad \frac{\partial \mathbf{v}_s}{\partial t} + \nabla\left(\Phi + \frac{v_s^2}{2}\right) = 0.$$

Here ρ is the density, s is the entropy per unit mass, p is the pressure, $\mathbf{j} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$ is the mass current density, ρ_n and ρ_s are the densities of the normal and superfluid components, and $\Phi = \Phi_0(p, T) - \rho_n(\mathbf{v}_n - \mathbf{v}_s)^2/2\rho$ is the thermodynamic potential, depending upon the relative velocity $\mathbf{v}_n - \mathbf{v}_s$. If we take the pressure and temperature as independent variables, then from the thermodynamic identity for the potential

$$\rho\, d\Phi = -\rho s\, dT + dp + (\mathbf{j} - \rho \mathbf{v}_s)\, d(\mathbf{v}_n - \mathbf{v}_s)$$

it follows that the density ρ and the entropy s are functions of the relative velocity $\mathbf{v}_n - \mathbf{v}_s$:

$$s = s_0 + \frac{\partial}{\partial T}\left(\frac{\rho_n}{2\rho}\right)(\mathbf{v}_n - \mathbf{v}_s)^2, \quad s_0 = -\left(\frac{\partial \Phi_0}{\partial T}\right),$$

$$\frac{1}{\rho} = \frac{1}{\rho_0} - \frac{\partial}{\partial p}\left(\frac{\rho_n}{2\rho}\right)(\mathbf{v}_n - \mathbf{v}_s)^2,$$

$$\frac{1}{\rho_0} = \left(\frac{\partial \Phi_0}{\partial T}\right)_p$$

We shall look for increments linear in $\mathbf{v}_n$ and $\mathbf{v}_s$ to the velocity of sound for the stationary fluid. For this purpose it is necessary to rewrite the hydrodynamic equations to include terms quadratic in the velocity. We shall introduce the notation $\mathbf{v} = \mathbf{j}/\rho$ and $w = \mathbf{v}_n - \mathbf{v}_s$. The components of these vectors in the direction of the wave vector $\mathbf{k}$ we shall identify by the index k, and those in a plane perpendicular to $\mathbf{k}$ by the index $\perp$. In a traveling sound wave, all quantities depend upon