For $E = 2.54$ mev:

$$\delta_0 = 90°;\ \delta_1 = 33°;\ \delta_2 = 27°;\ \delta_3 = 5.7°.$$

For $E = 3.03$ mev:

$$\delta_0 = 90°;\ \delta_1 = 48°;\ \delta_2 = 18.5°;\ \delta_3 = 14°$$

or

$$\delta_0 = 80;]\ \delta_1 = 43°;\ \delta_2 = 25°;\ \delta_3 = 10°.$$

For $E = 3.50$ mev:

$$\delta_0 = 90°;\ \delta_1 = 52°;\ \delta_2 = 16°;\ \delta_3 = 21°.$$

Experiment gives three parameters, so that the choice of phases is not unique. But the unusually small constant term in the angular distribution strongly reduces the arbitrariness in the selection of phases. Thus the phase δ_0, which accounts for potential scattering, cannot be chosen to be smaller than 70°.

Thus the scheme for the interaction of a proton with H^3 which was proposed by Baz' and Smorodinskii[2] is not in contradiction with the experimental results for the elastic scattering of protons by tritium.

In conclusion I wish to thank Professor Ia. A. Smorodinskii for suggesting the subject and for his guidance.

Note in the proof: Frank and Gammel[3] have carried through a phase analysis of $p - H^3$ scattering assuming LS coupling.

[1] Claassen, Brown, Freier and Stratton, Phys. Rev. 82, 589 (1951).

[2] A. I. Baz' and Ia. A. Smorodinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 382 (1954).

[3] R. M. Frank and J. L. Gammel, Phys. Rev. 99, 1406 (1955).

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Absorption Curve Moments for Solid Solutions

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MEASUREMENTS of resonance absorption in solid solutions enable us to make conclusions about the character of the interaction forces between the particles. The first calculation of the moments of these absorption curves was carried out by Kittel.[1] However, he took into account only the dipole interactions. Below are presented the results of a calculation of the moments of absorption curves when not only dipole, but also exchange, interaction forces of paramagnetic ions are present.

A crystal of solid solution of two salts grows in a bath of their liquid solution. The filling of any particular lattice point of the growing crystal by one or the other ion appears to be a random process. The probability of occupation of a lattice point by a paramagnetic ion will be considered to be independent of the manner in which the remaining crystal lattice points are filled. This is the case when there is a small difference between the binding energies of the ions of both metals in the crystal, between their masses, etc. We shall calculate the moments of the paramagnetic resonance absorption curve in solid solutions at high frequencies.

1. The zero order moment is

$$\nu_0 = \mathrm{Sp}\sum_j S_{x_j}^2 = \overline{N}\frac{\lambda}{2}(2S+1)^{\overline{N}}$$

$$= N_0 f\frac{\lambda}{3}(2S+1)^{N_0 f},$$

where N is the average number of paramagnetic ions, equal to $N_0 f$ (N_0 is the number of lattice points in the crystal), $\lambda = S(S+1)$ (S is the spin quantum number).

2. The second order moment is

$$\Delta\overline{\nu_2} = h^{-2}\frac{\lambda}{3}\frac{\Sigma B_{jk}^2\overline{\delta_j\delta_k}}{\overline{N}} = h^{-2}\frac{\lambda}{3}\sum_k B_{jk}^2 f,$$

where $\delta_j(\delta_k)$ are equal to unity if the lattice point $j(k)$ is occupied by a magnetic ion and to zero in the opposite case, and $\delta_j\delta_k = f$.[2]

3. The fourth order moment is

$$\begin{aligned}\Delta\overline{\nu_4} = h^{-4}\Big\{ & f\Big[\sum_k A_{jk}^2 B_{jk}^2\left(\frac{2}{5}\lambda^2 - \frac{3}{10}\lambda\right) \\ & + A_{jk}B_{jk}^3\left(\frac{4}{15}\lambda^2 - \frac{2}{10}\lambda\right) + B_{jk}^4\left(\frac{\lambda^2}{5} - \frac{\lambda}{15}\right)\Big] \\ & + f^2\Big[\sum_{k,l} 3B_{jk}^2 B_{jl}^2 + 2A_{jk}^2(B_{jl} - B_{kl})^2 \\ & + A_{jk}A_{kl}(B_{jk} - B_{jl})(B_{kl} - B_{jl}) \\ & + 2A_{jk}B_{jk}(B_{jl} - B_{kl})^2\Big]\frac{\lambda^2}{9}\Big\}\end{aligned}$$

where the remaining symbols are taken from an article of Van Vleck.[2]

The numerical values of the moments are not given in the present work, but they could be easily evaluated. It is not difficult to see that the quantity

$$X = \Delta\overline{\nu_4}/(\Delta\overline{\nu_2})^2$$

increases with a decreasing concentration of paramagnetic ions, and the moments decrease. It is possible to expect the half width of the curve to decrease as well. However, the measurements of Kashaev[3] show that the halfwidth increases at first with dilution, reaches a maximum at some value of f and then decreases with further dilution. The increase of the halfwidth at small dilutions may not even contradict an increase in the quantity X at the same time.

Indeed, such case was observed by Kumagi et al[4] who studied the change of the halfwidth of the curve with the change in separation d between two neighborhood paramagnetic ions. In particular, with a decrease in d (as long as the volume forces are small) the halfwidth increases with an increase in the quantity X though the latter is very small. The halfwidth reaches a maximum and then, as the volume forces begin to increase rapidly, it begins to decrease.

A direct comparison of the theoretical moments with the experimental data cannot be carried out not only because the available curves are not measured on sides (only halfwidths were measured) but also because the insufficient sensitivity of the present experimental techniques makes a careful investigation of the absorption curve (particularly its sides) of solid solutions almost impossible since the absorption decreases rapidly with dilution.

In conclusion the author expresses a deep gratitude for the valuable advice and discussions to C. A. Al'tshuler and B. M. Kozyrev.

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[2] J. H. Van Vleck, Phys. Rev. 74, 1168 (1948).

[3] Kh. Kashaev, Dissertation, KFAN, 1954.

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Nonlinear Equations in Quantum Field Theory

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THE purpose of the present note is to show that the equations gotten by Low[1] follow from general relations for the Green's function which were found by Lehman, Symanzik and Zimmerman[2], starting from general requirements of relativistic invariance, causality and boundary conditions. For simplicity, the proof is carried through for the example of the equation for the Green's function of the meson-nucleon system, which is simply related to the matrix element of the S-matrix for the scattering of mesons by nucleons.

The Green's function of the meson-nucleon system is defined in the following manner:

$$\tau(xx'; yy') = (\Psi_0, T\{\varphi(x)\varphi(y)\psi(x')\overline{\psi}(y')\}\Psi_0), \tag{1}$$

where Ψ_0 is the state vector of the physical vacuum in the Heisenberg representation and $\psi(x)$ and $\varphi(x)$ are the Heisenberg operators corresponding to the nucleon and meson fields. For simplicity the meson is taken to be neutral. Instead of $\tau(x\,x'; yy')$ it is convenient to consider the quantity

$$\langle p'|x, y|p\rangle = \int\limits_{x_0\to+\infty,\, y_0\to-\infty} dx'\, dy\, \overline{u}_{p'}(x')\,\gamma_4\,\tau(xx', yy')\,\gamma_4\, u_p(y'). = (\Psi_p, T\{\varphi(x)\varphi(y)\}\Psi_p). \tag{2}$$

where $u_p(x)$ is a solution of the free Dirac equation for particles with momentum p and energy $p_0 = \sqrt{p^2 + M^2}$, and Ψ_p is the state vector of one nucleon in the Heisenberg representation.

If causality and the boundary conditions are employed then, analogously to what is done in Ref. 2, it is possible to obtain relations of the following form for the Green's function:

$$\begin{aligned}\langle p'|x, y|p\rangle = \sum_{n=0}^{\infty}\sum_{\lambda}(-i)^n \Big\{\eta(x-y) \\ \times\int K_{\xi_1}\ldots K_{\xi_n}\langle p'|x, \xi_1\ldots\xi_n|\lambda\rangle \\ \times\Delta^{(+)}(\xi_1-\eta_1)\ldots\Delta^{(+)}(\xi_n-\eta_n)K_{\eta_1}\ldots K_{\eta_n} \\ \langle p|y, \eta_1\ldots\eta_\eta|\lambda\rangle^* d\xi_1\ldots d\xi_n d\eta_1\ldots d\eta_n \\ +\eta(y-x)\int K_{\xi_1}\ldots K_{\xi_n}\langle p'|y, \xi_1\ldots\xi_n|\lambda \\ \rangle\Delta^{(+)}(\xi_1-\eta_1)\ldots\Delta^{(+)}(\xi_n-\eta_n) \\ \times K_{\eta_1}\ldots K_{\eta_n}\langle p|x, \eta_1\ldots\eta_n|\lambda\rangle^* d\xi_1\ldots d\xi_n d\eta_1\ldots d\eta_n\Big\}.\end{aligned} \tag{3}$$

Here $K_x = \square_x - \mu^2$ where μ is the renormalized meson mass; * designates the Hermitian conjugate;

$$\Delta^+(x) = -i(2\pi)^{-3}\int dq\,\delta(q^2+\mu^2)\,e^{iqx}\,\eta(q)$$

is a commutator function;