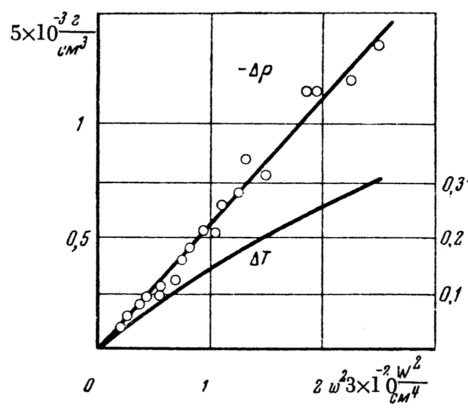

FIG. 2. The dependence of the density jump (upper curve, left scale) at the boundary between He I and He II, and the temperature (lower curve, right scale) on the square of the heat flow density.

with a jump in density and temperature. To estimate the jump in temperature more precisely, the dependence of the density change of He^4 on temperature was taken at a pressure of 1 atm. (fig. 3). The density was determined from the passage of interference bands in the same apparatus. On the X-axis is shown, instead of temperature, the helium vapor movement in the bath, because in this presentation

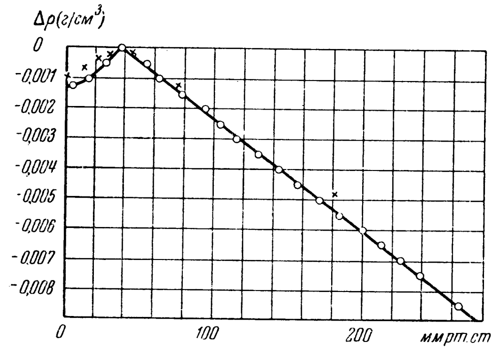

FIG. 3. The change in the density of liquid helium under 1 atm. pressure as a function of temperature, expressed in helium vapor pressures in the bath. o — present measurement data, × — data by Keesom [Physica 1 128(1933)]

the density of He I changes linearly in a large interval. Inasmuch as, for transitions of the second kind, the equality at the boundary line of temperatures and pressures, and also entropies and densities is characteristic, whereas in a transition of the first kind, the temperatures and pressures are equal and jumps occur in the entropy and the density, the transition of He I to He II and back, in presence of a heat flow, should be called either a special transition of the first kind, or a transition of zero order, since here at the boundary line not only the values of entropies and volumes, but also those of temperatures and probably pressures suffer a break. The presence of a temperature jump between two phases of liquid helium does not have direct analogy and it can be explained by a change in the mechanism of heat transfer at the boundary. If in He I one can speak of a thermal movement of strongly bonded but still separate helium atoms, the heat flow in He II is effected by a quantum movement of thermal excitations of photons and rotons. It is obvious that the interaction between these types of heat transfer is accomplished with difficulty, which causes the jump in temperature.

The experiments to study the described phenomena are continuing.

In conclusion I express my gratitude to N. I. Kondrat'ev, who built the apparatus, and also to A. I. Filimonov and I. A. Uriutov, who helped with the measurements.

Translated by S. Pakswer
108

Relativistically Invariant Formulation of Electrodynamics without Longitudinal and Scalar Fields

E. M. LIPMANOV
Novozybkov State Pedagogical Institute
(Submitted to JETP editor, February 17, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 583-584 (March, 1956)

IT was shown [1] that the generalized Coulomb field of charges can already be excluded in a relativistically invariant way in classical electrodynamics, a fact which facilitates the transition to the quantum theory. However, the problem of formulation of quantum electrodynamics in the Heisenberg representation and the transition to the interaction representation has only been superficially raised and needs a more exact treatment. In the present note, it will be shown that the formulation of quantum electrodynamics in the Heisenberg representation can be obtained from the variational principle

$$\delta \int L d\omega = 0, \qquad (1)$$

where $d\omega = dxdydzcdt$, and the Langrangian function is of the following form;*

$$L = -\frac{1}{2}\left(\frac{\partial a_\mu(x)}{\partial x_\nu}\right)^2 \qquad (2)$$

$$-\frac{\hbar c}{2}\bar{\psi}(x)\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{ie}{\hbar c}a_\mu(x)\right) + \varkappa_0\right]\psi(x)$$

$$-\frac{\hbar c}{2}\bar{\psi}'(x)\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu} + \frac{ie}{\hbar c}a_\mu(x)\right) + \varkappa_0\right]\psi'(x)$$

$$+\frac{1}{2c}n_\mu j_\mu(x)\varphi(x) - \left(\frac{\partial a_\mu(x)}{\partial x_\nu} - \frac{\partial a_\nu(x)}{\partial x_\mu}\right)n_\mu\frac{\partial\varphi(x)}{\partial x_\nu}.$$

Here $\psi(x)$ and $\psi'(x)$ are the spinor and the charge-conjugate spinor of the Dirac quantized field, $a_\mu(x)$ is the operator of the photon field, and the operator $\varphi(x)$ is assumed to be expressed in terms of the 4-current density in the following way:

$$\varphi(x) = \frac{1}{c}\int\limits_\sigma n_\nu \frac{\partial \mathscr{D}(x - x')}{\partial x_\nu} j_\lambda(x')\,d\sigma'_\lambda. \qquad (3)$$

where the space-like surface σ is passing through the point x and is chosen to be plane. The photon field is subjected to two additional conditions;

$$n_\mu a_\mu'(x) = 0; \quad \partial a_\mu(x)/\partial x_\mu = 0. \qquad (4)$$

The commutation relations between the field operators on the space-like surface σ are identical with the well-known covariant commutation relations for the "free" photon and electron-positron fields, satisfying the linear equations of motion.[2] From (1) and (2) we obtain the following equations of motion for the field operators:

$$\Box^2 a_\mu(x) = -\frac{1}{c}j_\mu'(x) \qquad (5)$$

$$+\frac{\partial}{\partial x_\mu}n_\nu\frac{\partial\varphi(x)}{\partial x_\nu} - n_\mu\Box^2\varphi(x),$$

$$\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu} - \frac{ie}{\hbar c}a_\mu(x)\right) + \varkappa_0\right]\psi(x) \qquad (6)$$

$$-\frac{ie}{2\hbar c}\gamma_\mu n_\mu\{\psi(x), \varphi(x)\} = 0,$$

$$\left[\gamma_\mu\left(\frac{\partial}{\partial x_\mu} + \frac{ie}{\hbar c}a_\mu(x)\right) + \varkappa_0\right]\psi'(x) \qquad (7)$$

$$+\frac{ie}{2\hbar c}\gamma_\mu n_\mu\{\psi'(x), \varphi(x)\} = 0,$$

where { } denotes an anti-commutator. The nonlinear terms in the equations for the operators of the electron-positron field arise from the Coulomb charge interaction. The Coulomb field itself does not enter the theory. In connection with this, the theory is no longer gauge-invariant.[1]

The transition to interaction representation is effected by the canonical transformation

$$\Psi[\sigma] = U[\sigma]\Phi, \qquad (8)$$

where Φ is the constant vector of the state of the system in the Heisenberg representation, and $\psi[\sigma]$ is the vector of state in the interaction representation. The unitary operator $U[\sigma]$ satisfies the equation **

$$U[\sigma] = U[\sigma_0] - \frac{i}{\hbar c}\int\limits_{\sigma_0}^{\sigma}\widetilde{H}(x')U[\sigma']\,d\omega', \qquad (9)$$

where $\widetilde{H}(x)$ is the energy density of the interaction of the fields***

$$\widetilde{H}(x) = -\frac{1}{c}\widetilde{j}_\nu(x)\widetilde{a}_\nu(x) - \frac{1}{2c}n_\nu\widetilde{j}_\nu(x)\widetilde{\varphi}(x). \qquad (10)$$

The field operators are transformed in the following way:

$$\widetilde{F}(x) = U[\sigma]F(x)U^{-1}[\sigma]. \qquad (11)$$

In order to find the equations of motion in the interaction representation, ir is necessary to write down the transformation formulas of the derivatives of the operators. The Schwinger transformation formulas[2] cannot be applied in this case, since it is assumed in their deduction that the transformation operator $F(x)$ commutes with the interaction Hamiltonian in various points of the space-like surface σ, and in the present case when, for instance, $F(x) = \psi(x)$, the said assumption is not fulfilled.

Making use of the relation

$$\frac{\partial}{\partial x_\nu}\int\limits_{\sigma_0}^{\sigma}F(x')\,d\omega' = \int\limits_\sigma F(x')\,d\sigma'_\nu, \qquad (12)$$

we obtain, on the basis of (11) and (9), the transformation formulas for the derivatives

$$\frac{\partial\widetilde{F}(x)}{\partial x_\nu} = \frac{\partial}{\partial x_\nu}(U[\sigma]F(x)U^{-1}[\sigma]) \qquad (13)$$

$$= U[\sigma]\frac{\partial F(x)}{\partial x_\nu}U^{-1}[\sigma] - \frac{i}{\hbar c}\int\limits_\sigma[\widetilde{H}(x'), \widetilde{F}(x)]\,d\sigma'_\nu,$$

which are applicable also in the case when the operators $F(x)$ and $H(x)$ do not commute in various points on σ.

By means of the formulas (13), we find the desired equations of motion in the interaction representation

$$(\gamma_\nu \partial/\partial x_\nu + \varkappa_0)\tilde{\psi}(x) = 0, \qquad \Box^2 \tilde{a}_\mu(x) = 0 \quad (14)$$

in accordance with Ref. 1.

---

*Notation of Refs. 1 and 2 is used throughout.

**The integral equations for $U[\sigma]$ and the vector of state $\Psi[\sigma]$. are, in contrast to the differential equations, applicable without reservation in the case of plane σ.

***Operators in interaction representation are labelled by ~.

[1] E. M. Lipmanov, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 135 (1954).

[2] J. Schwinger, Phys. Rev. 74, 1439 (1948).

Translated by H. Kasha
109

On the Theory of Asymmetric Fission of Heavy Nuclei

S. G. RYZHANOV
Kishinev State University

(Submitted to JETP editor December 2, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 599-601 (March, 1956)

IN the present communication we shall show that the dependenc of the yield of fragments on their charges and mass numbers, in the case of spontaneous fission as well as in the case of fission by thermal and fast neutrons, can be obtained by a single method, if one adopts the hypothesis suggested by Frenkel[1]. This hypothesis consists in regarding both spontaneous and induced fussion as a tunnel effect originating, respectively, from the ground or excited state of the fissioning nucleus. The potential barrier for fission is assumed to be due, on one hand, to the specific nuclear potential "well", and on the other, by the electric (Coulomb) repulsion of the fragments. From this point of view, α-decay is regarded as the most sharply asymmetric fission. The well-known quantum-mechanical formula for α-decay[2] can be easily extended to the asymmetric fission of a nucleus with charge Z_0 into fragments of charge Z_1 and Z_2.

Let E be the sum of the kinetic energies of the fragments; $v = \sqrt{2E/M}$ their relative velocity (M is the reduced mass of the fragments); ρ the radius of the specific nuclear potential well; $\sigma = Z_1 Z_2 e^2/\rho E$. It is easy to obtain the following formula for the probability of fission w:

$$w = w_0 \times \exp\left\{-\frac{4Z_1Z_2e^2}{\hbar v}(\arccos \sigma^{-1/2} - \sigma^{-1}(\sigma-1)^{1/2})\right\}, \quad (1)$$

where the coefficient w_0 in front of the exponential is of the order of magnitude of the fundamental frequency of nuclear vibrations ($w_0 \approx 3\times10^{21}$ sec^{-1}). The sum of the kinetic energies of the fragments if found from the relation

$$E = E_1 + E_2 - E_0 - Q, \quad (2)$$

where E_0, E_1 and E_2 are the binding energies of the fissioning nucleus and of the two fragments, and Q the energy of their internal excitation[3]. The binding energies are calculated from the semi-empirical formula[4]. Formula (1) has been applied to the fission of uranium nuclei. The correct order of magnitude for the period of spontaneous fission, 10^{21} sec, is obtained with $E = 150$ mev and the radius $\rho = (A_1^{1/3} + A_2^{1/3})\ 1.5\times10^{-13}$ cm. (Here a preliminary approximate integration of Eq. (1) has been carried out over all fragments A_1 and A_2 satisfying the condition $A_1 + A_2 = A_0$). The hypothesis underlying the calculation of τ_{sp} assumes that the fission takes place through the intermediate stage of spheres in contact, which in no way contradicts the thermo-hydrodynamical mechanism of fission. An analogous calculation for Th^{232}_{90} gives the period of spontaneous fission as $\tau_{sp} > 10^{18}$ years. The asymmetry of the fission can be easily obtained from (1). Let us denote by $\bar{w}$ the probability of symmetric fission and let us expand $\ln(w/\bar{w})$ into a series according to powers of the variables

$$\xi = \left(\frac{Z_0 - 2Z_1}{Z_0}\right); \quad \eta = \frac{\bar{E} - E}{\bar{E}}$$

($\bar{E}$ is E for symmetric fission). We then obtain, with sufficient accuracy, the formula

$$\ln\frac{w}{\bar{w}} = \frac{Z_0^2 e^2}{\hbar \bar{v}}\left\{\xi \arccos(\bar{\sigma})^{-1/2} - \left(\arccos(\bar{\sigma})^{-1/2} + \frac{\sqrt{\bar{\sigma}-1}}{\bar{\sigma}}\right)\frac{\eta}{2}\right\}. \quad (3)$$

The results of calculations with this formula are given in Fig. 1; conversion of fragment charges into their mass numbers was carried out with the aid of the semi-empirical tables of Ref. 5; the same reference was used for comparison with experimental data. In these calculations