In the case of Sn, however, it is difficult to judge the degree of agreement between theory and experiment, since tin is anisotropic and the quantity Δ is measured relative to an interphase boundary parallel to the tetrad axis[9], while δ_{00} refers to appolycrystal. Further, due to this anisotropy, one cannot assume that the possible errors in determining Δ by the method of Refs. 7 and 9 will be the same for Al and Sn. In order to verify the theory on the present plan it will be necessary to make measurements of $\Delta(T)$ for a number of cubical metals. As regards the determination of the anisotropy of δ_0 and Δ, this appears to be an independent and, moreover, extremely important problem (see Refs. 2 and 10).

For Al and other cubic crystals it is necessary also to determine the dependence of the penetration depth upon the intensity of the magnetic field. For the case in which $\kappa \ll 1$ and the penetration depth δ_1 for a weak alternating field is measured in the presence of a strong external field H we have[1,11]

$$(\delta_1 - \delta_0) / \delta_0 = (3\varkappa / 4\sqrt{2})(H / H_{cm})^2. \tag{8}$$

As follows from the numerical calculations for $\kappa \leq 0.4$ and $H = H_{cm}$, Eq. (8) is correct to within 15%, the true values for δ_1 exceeding those obtained from Eq. (8). If the penetrating depth δ, and not δ_1, is measured directly for a strong field H, $(\delta - \delta_0)\delta_0$ will be smaller by a factor of three than is indicated by Eq. (8). The temperature dedpenence for δ_1 and δ can be seen from Eqs. (6) and (8). Using a variational method, Bardeen obtained for δ the expression (5.29) of Ref. 3, which for $T \to T_c$ coincides with ours. (In Ref. 3 the quantity δ is confused with the experimentally-measured[12] quantity δ, and in consequence the deviation from experiment by a factor of 2 to 3 noted in Ref. 3 does not exist.) For $T < T_c$ the expression for δ from Ref. 3 differs from that obtained in a more accurate manner from Eq. (3)[1]. Here there is obtained from Eq. (3) an expression which differs fundamentally from Eq. (8); this might, in principle, permit one to choose between Eqs. (1) and (3) on the basis of experimental data [from Eq. (3) it follows that as $T \to 0$ the depth δ_1 ceases to depend upon H; this result is connected with the fact that, according to Eq. (3), in a state of equilibrium $\partial^2 F_{s0}/\partial\Psi^2 \to \infty$ as $T \to 0$].

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* The limitations of Eq. (3) in this connection are evident from the fact that the expression (4) for H_{cm} leads to the relation $c_e = aT^3$ for the electronic component of the specific heat in the superconducting state. At the same time it follows, from both theoretical considerations[5] and, more important, from experimental data[6], that the dependence of c_e upon T is exponential, although at not too low temperatures Eq. (4) can be used for H_{cm} as a good approximation.

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Many Electron Quantum Theory Dispersion Formulas for Metals in the Infrared Region

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SOKOLOV et al[1,2] have derived general quantum theory dispersion formulas by the method of the density operator of the many electron theory. However, the authors have limited themselves to the visible and ultraviolet spectral ranges where the major role in light absorption is played by the quantum transitions of the electron system into higher energy states. Meanwhile, it is not difficult to consider the absorption of infrared light by the same method. Vonsovskii[3] has reviewed the effect of oscillating electric field of low frequency on a system of coupled electrons in a crystal by the method of the density operator. However, since he did not take damping into account, he obtained only the expression for the polarization current, while the conduction current turned out to be zero.

We can include the damping of the electron motion by replacing in the calculations the energy of the excited state E by the complex quantity $E - i\hbar\Gamma$, where Γ is the damping factor. Using the wavefunction of a system of N interacting electrons in the form given by Sokolov[1], it is not difficult to generalize the calculations to this case. We obtain the following expression for the dielectric constant ϵ and the conductivity σ:

$$\varepsilon = 1 - 4\pi e^2 S^* / m(\omega^2 + \Gamma^2), \qquad (1)$$

$$\sigma = e^2 \Gamma S^* / m(\omega^2 + \Gamma^2),$$

where e is the electron charge, m is the electron mass, ω the frequency of light and S^* a tensor representing the effective number of conduction electrons whose components are

$$(S^*)_{\alpha\beta} = \frac{1}{\hbar V} \qquad (2)$$

$$\times \sum_{j=1}^{N} \int (k \,|\, \hat{\rho}(0) \,|\, k) \frac{\partial}{\partial k_{j\alpha}} \left(k \,\middle|\, \sum_{i=1}^{N} \hat{P}_{i\beta} \,\middle|\, k \right) dk \quad (\alpha,\ \beta = x,\ y,\ z).$$

In this expression V is the volume of the unit crystal cell, $\hat{\rho}(0)$ is the density matrix at the initial moment, ΣP_i is an operator of the total momentum, and the index k represents all of the quantum numbers $k_1, \ldots, k_N$ that determine the state of the system.

If we discard in Eq. (1) the damping terms by setting $\Gamma = 0$, we obtain the formula derived by Vonskovii[3] for the dielectric constant. In the case of noninteracting electrons, Eq. (2) becomes a corresponding expression of the one electron band theory.

Thus, the dispersion formulas for the infrared spectral range have the same form in the many electron theory as in the one electron band theory of metals. The concept of an effective number of conduction electrons retains the same meaning in the many electron theory. However, the effective number of conduction electrons is determined by a density matrix of the entire system of the metal, and also by the matrix elements of the total momentum of the system. Therefore, a correct description of the optical properties of metals in the infrared spectral range should include the interactions between electrons.

We note that Eq. (1) cannot be obtained by a simple substitution $\omega \to \omega - i\Gamma$ in the expression for ϵ given by Vonskovii[3], because such a substitution has a meaning only for the natural frequency of the system and not for the light frequencies.

To determine the numerical values of S^* it is necessary to apply the described scheme in any particular many electron model of a metal. That allows us then to solve the problem of the effect of the coupling of electrons among themselves on the value of the effective number of conduction electrons.

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Elastic Scattering of 18.7 MEV Protons by Nickel and Copper Nuclei

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A COMPARISON of experimental data for elastic scattering of nucleons with energies from a few mev to a few tens of mev by nuclei [1-10] with the existing theoretical calculations[11,12] shows that the differential cross section for elastic scattering qualitatively corresponds with the "black body" model, but there is no quantitative agreement. Calculations based on the optical model with sharp boundaries for the potential well[13] give too large a value for the differential cross section for large angles as compared with the experimental data. Only the consideration of the diffuse boundary of the nucleus[14] gave results closer to the experimental data of the calculations for heavy nuclei.

In the present work the elastic scattering of 18.7 mev protons from the neighboring nuclei —