tions for the amplitudes gives the dispersion relation

$$1 = P_0 k^2 (G(k)/m) \sum_j [(\omega - \mathbf{k}\mathbf{v}_j^0)^2 - \hbar^2 k^4 / 4m^2]^{-1}, \qquad (3)$$

where $G(k)$ is the Fourier component of the interaction potential $G(r)$.

Equation (3), whose solution gives the dependence of ω on k, coincides with the dispersion relation derived in Ref. 3 by another method.

The advantage of the collective description of the interaction by Eqs. (1) is that these equations permit the formulation of the limiting problem of an isolated system of interacting particles confined to a bounded region of space (in analogy to the limiting problem of hydrodynamics of an ideal fluid).

In particular, we can use Eqs. (1) to formulate the problem of free surface oscillations of heavy nuclei (given the potential of interaction between nucleons). So far, we have not considered the effect of the symmetry of the wave function on Eq. (3). In the case of the Fermi statistics, when each fermion state is filled by two particles, i.e., the resulting spin is equal to zero, the Hartree-Fock equations (taking into account the antisymmetry of the wave function) differ from the Hartree Eq. (1) by the additional term

$$-\int G(|\mathbf{r} - \mathbf{r}|) \sum_j \psi_j^*(\mathbf{r}) \psi_j(\mathbf{r}') \psi_i(\mathbf{r}')\, d\mathbf{r}',$$

which takes into account the exchange effect.

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Some Remarks Concerning the Macroscopic Theory of Superconductivity

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IN the macroscopic theory of superconductivity developed in Ref. 1 (see also Ref. 2, in which further pertinent works are cited), the free energy density for a superconductor in the absence of a magnetic field is taken to be

$$F_{s0}(T) = F_{n0}(T) + \alpha(T)|\Psi|^2 + {}^1\!/_2\beta|\Psi|^4, \qquad (1)$$

where F_{n0} is the free energy density in the normal state and $|\Psi|^2$ is the concentration of "superconducting electrons". Further, α and β are coefficients expressible in terms of the critical magnetic field for the bulk metal H_{cm} and the weak field penetration depth δ_0:

$$H_{cm}^2 = 4\pi\alpha^2/\beta; \quad \delta_0^2 = mc^2\beta/4\pi e^2|\alpha|. \qquad (2)$$

Equation (1) represents an expansion in powers of $|\Psi|^2$; in general, however, it is possible to break off the expansion after the $|\Psi|^4$ term only in the immediate vicinity of the second-order phase transition under consideration, i.e., for $T_c - T \ll T_c$, where T_c is the critical temperature. Under these conditions it is also possible to set $\alpha = (d\alpha/dT)_c(T - T_c)$ and $\beta = \beta(T_c)$, as was done in Ref. 1 and subsequently. As we move away from T_c and, in particular, as $T \to 0$ it becomes impossible to write an expression for $F_{s0}(T)$ based upon general considerations; on the other hand, it would be desirable to obtain even semi-empirical formulas which would permit comparison of theory with experiment for all temperatures. For this purpose Bardeen[3] adopted the expression:

$$F_{s0}(T) = F_{n0}(T) + \frac{H_0^2}{4\pi}\left\{\left(\frac{T}{T_c}\right)^2\left(1 - \sqrt{1 - \left|\frac{\Psi}{\Psi_0}\right|^2}\right) - \frac{1}{2}\left|\frac{\Psi}{\Psi_0}\right|^2\right\}, \qquad (3)$$

(Ψ_0 being the equilibrium value of Ψ at $T = 0$) which is used in connection with the so-called two-fluid model for a superconductor.[4] The two-fluid model, however, meets with serious objections[2,4], and the use of Eq. (3) is actually based only upon the usually adequate relations*

$$H^2_{\mathrm{cm}} = H_0^2\,[1-(T/T_{\mathrm{c}})^2]^2, \qquad (4)$$

$$\delta_0^2 = \delta_{00}^2\,[1-(T/T_{\mathrm{c}})^4]^{-1}.$$

At the same time, one can also, taking Eq. (2) into consideration, arrive at Eq. (4) by using Eq. (1) for all T and setting:

$$\alpha = -\frac{e^2\delta_{00}^2H_0^2}{mc^2}\left[1-\left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]\cdot\left[1+\left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]^{-1}; \qquad (5)$$

$$\beta = \frac{4\pi e^4\delta_{00}H_0^2}{m^2c^4}\left[1+\left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]^{-2}.$$

(this has been pointed out previously[2,3]).

Eq. (3) has no general advantages as compared with Eqs. (1) and (5). At the same time, Eq. (1) is considerably simpler than Eq. (3), and its applicability at all temperatures should cause no particular surprise. This comes from the fact that the ratio of the superconducting electron concentration $n_s = |\Psi|^2$ to the total concentration of conduction electrons n_0 is, even at $T = 0$, on the order of 1/5 (this is for Sn and Al, approximate data on the value of n_0 being used[2]). Therefore, the ratio n_s/n_0, in terms of which the expansion in Eq. (1) is evidently performed, is small for all T, which makes it possible to stop the expansion after the $|\Psi|^4$ term with good accuracy. We note finally that by modifying Eqs. (5) in a corresponding fashion one can readily, starting from Eqs. (1) and (2), arrive at expressions for H_{cm} and δ_0 which agree in the closest way with experiment; Eq. (3), on the other hand, is closely tied up with the relations (4), which are known to be in error for $T \to 0$. If we use Eqs. (1) and (5), all of the basic formulas obtained in Ref. 1 and subsequently (see Ref. 2) can be left unaltered, since the temperature dependence of α and β plays no part in these calculations. Thus, the characteristic parameter κ has the form

$$\varkappa = \frac{\sqrt{2}\,e}{\hbar c}H_{\mathrm{cm}}\delta_0^2 = 2.16\times10^7\cdot H_0\delta_{00}^2\left[1+\left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]^{-1} \qquad (6)$$

$$= \varkappa_0\left[1+\left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]^{-1};$$

we shall disregard here the extremely dubious possibility that the charge e in Eq. (6) may be unequal to the charge of the free electron (see Ref. 2). From Eq. (6) it is clear that $\kappa_0 = 2\kappa_{\mathrm{c}} \equiv 2\kappa(T_{\mathrm{c}})$. Further, following Ref. 1, the surface energy on the boundary between the normal and superconducting phases for $\sqrt{\kappa} \ll 1$ is equal to

$$\sigma_{ns} = \Delta H^2_{\mathrm{cm}}\ 8\pi; \quad \Delta = 1{,}89\varkappa\delta_0/\varkappa; \quad \sqrt{\varkappa} \ll 1. \qquad (7)$$

Even for $\kappa = 0.02$ the limiting expression (7) is accurate to no better than ~ 15%; for larger κ it is necessary to use the results obtained numerically (see Table I)** The limiting expression for σ_{ns} obtained in Ref. 3 from Eq. (3) is considerably more involved than Eq. (7), into which it transforms for $T \to T_{\mathrm{c}}$. Even for $T = 0$, however, the value obtained for σ_{ns} in Ref. 3 is only 17% greater than that found from Eq. (7). A comparison of the theory with experiment[7] is presented in Table II, in which data from Refs. 4, 8 and 9 are also used.

TABLE I

$\varkappa$	Δ/δ_0 numerical calculation	Δ/δ_0 From Eq. (7)
0.01	168.4	189
0.02	79.8	94.5
0.05	28.5	37.8
0.08	16.2	23.6
0.165	6.25	11.5
0.25	3.39	7.56
0.30	2.39	6.30
0.4	1.30	4.73
0.5	0,676	3.78
0.6	0.286	3.15
0.70	0.020	2.70
0.7071	0.000	2.67
0.80	—0.171	2.36

TABLE II

	Al	Sn
δ_{00} (CM)	4.93×10^{-6}	5.1×10^{-6}
H_0	96	304.5
$\varkappa_0$	0,050	0.171
Δ_0 (CM):		
Theory	$28.5\ \delta_{00} = 14.1\times10^{-5}$	$6.0\ \delta_{00} = 3.06\times10^{-5}$
Experiment	15.3×10^{-5}	1.8×10^{-5}

For Al the agreement between theory and experiment is excellent; this evidently applies as well to the temperature dependence of Δ, which is clear from Eqs. (4), (6) and (7) or Table I.

In the case of Sn, however, it is difficult to judge the degree of agreement between theory and experiment, since tin is anisotropic and the quantity Δ is measured relative to an interphase boundary parallel to the tetrad axis[9], while δ_{00} refers to appolycrystal. Further, due to this anisotropy, one cannot assume that the possible errors in determining Δ by the method of Refs. 7 and 9 will be the same for Al and Sn. In order to verify the theory on the present plan it will be necessary to make measurements of $\Delta(T)$ for a number of cubical metals. As regards the determination of the anisotropy of δ_0 and Δ, this appears to be an independent and, moreover, extremely important problem (see Refs. 2 and 10).

For Al and other cubic crystals it is necessary also to determine the dependence of the penetration depth upon the intensity of the magnetic field. For the case in which $\kappa \ll 1$ and the penetration depth δ_1 for a weak alternating field is measured in the presence of a strong external field H we have[1,11]

$$(\delta_1 - \delta_0) / \delta_0 = (3\varkappa / 4 \sqrt{2}) (H / H_{cm})^2. \tag{8}$$

As follows from the numerical calculations for $\kappa \leq 0.4$ and $H = H_{cm}$, Eq. (8) is correct to within 15%, the true values for δ_1 exceeding those obtained from Eq. (8). If the penetrating depth δ, and not δ_1, is measured directly for a strong field H, $(\delta - \delta_0)\delta_0$ will be smaller by a factor of three than is indicated by Eq. (8). The temperature dedpenence for δ_1 and δ can be seen from Eqs. (6) and (8). Using a variational method, Bardeen obtained for δ the expression (5.29) of Ref. 3, which for $T \to T_c$ coincides with ours. (In Ref. 3 the quantity δ is confused with the experimentally-measured[12] quantity δ, and in consequence the deviation from experiment by a factor of 2 to 3 noted in Ref. 3 does not exist.) For $T < T_c$ the expression for δ from Ref. 3 differs from that obtained in a more accurate manner from Eq. (3)[1]. Here there is obtained from Eq. (3) an expression which differs fundamentally from Eq. (8); this might, in principle, permit one to choose between Eqs. (1) and (3) on the basis of experimental data [from Eq. (3) it follows that as $T \to 0$ the depth δ_1 ceases to depend upon H; this result is connected with the fact that, according to Eq. (3), in a state of equilibrium $\partial^2 F_{s0}/\partial\Psi^2 \to \infty$ as $T \to 0$].

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* The limitations of Eq. (3) in this connection are evident from the fact that the expression (4) for H_{cm} leads to the relation $c_e = aT^3$ for the electronic component of the specific heat in the superconducting state. At the same time it follows, from both theoretical considerations[5] and, more important, from experimental data[6], that the dependence of c_e upon T is exponential, although at not too low temperatures Eq. (4) can be used for H_{cm} as a good approximation.

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Many Electron Quantum Theory Dispersion Formulas for Metals in the Infrared Region

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SOKOLOV et al[1,2] have derived general quantum theory dispersion formulas by the method of the density operator of the many electron theory. However, the authors have limited themselves to the visible and ultraviolet spectral ranges where the major role in light absorption is played by the quantum transitions of the electron system into