Angular Momentum Distribution and the Spatial Distribution of Nucleons in Nuclei

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WITH the help of the Thomas-Fermi model, a relation is established between the spatial distribution of particles $\rho(r)$ and the distribution of the angular momentum $n(L)$ giving the number of particles with a given angular momentum L[1] From the condition $n(L) = 0$, $\rho(r)$ being known, the numbers of particles Z_l can be found for which l-states occur first, and the mean square of the angular momentum[2] can be determined as well. The latter can be written in the form:

$$\overline{L^2} = \frac{6}{25}\left(\frac{9\pi}{4}\right)^{2/3} Z^{2/3}\Xi; \quad \Xi = \frac{5\int\limits_0^\infty \rho^{5/3}(r)\, r^4 dr}{\left[3\int\limits_0^\infty \rho(r)\, r^2 dz\right]^{5/3}} \tag{1}$$

Knowing $\rho(r)$, $\overline{L^2}$ can be found. In the case of a nucleus, however, $\rho(r)$ of nucleons is not well enough known. Then, vice versa, from the empirical data about the first appearance of odd protons* in states with $l = 1, 2, 3, 4, 5, \ldots$ for $Z = 3, 9, 21, 41, 71, \ldots$ we can obtain information about $\rho(r)$[3-5] (strictly speaking, for nuclei with Z close to Z_l). Information about $\rho(r)$ for nuclei with any Z can be obtained, in principle, from the comparison of $\overline{L^2}$ calculated from Eq. (1), with the empirical data for $\overline{L^2}$ found from the scheme of level filling according to the formula

$$\overline{L^2_{\rm emp}} = \frac{1}{Z}\sum_i l_i\,(l_i+1) \quad (\hbar = 1). \tag{2}$$

The most acceptable scheme of level filling of odd protons and neutrons is the following one[6,7]:

$$\begin{gathered}
\underset{2}{1s\,|}\;2p_{3/2}\;\underset{(6)?}{\vdots}\;\underset{8}{2p_{1/2}\,|}\;\underset{14}{3d_{5/2}\,\vdots}\;\underset{20}{2s3d_{3/2}\,|}\;\underset{28}{4f_{7/2}\,|}\;\underset{34}{3p\,\vdots}\;\underset{40}{4f_{5/2}\,\vdots}\;\underset{50}{5g_{9/2}\,|}\;4d_{5/2},\;\underset{64}{5g_{7/2}\,\vdots}\;3s,\;4d_{3/2}\;\underset{(70)?}{\vdots}\\
\underset{82}{6h_{11/2}\,|}\;5f_{7/2},\;6h_{9/2}\;\underset{100}{\vdots}\;4p_{3/2},\;5f_{5/2},\;4p_{1/2}\;\underset{(112)?}{\vdots}\;\underset{126}{7i_{13/2}\,|}\;6g_{9/2},\;7i_{11/2},\;5d_{5/2},\;6g_{7/2},\;4s,\\
5d_{3/2},\;8j_{15/2}\;\underset{(184)}{|}\;\ldots
\end{gathered} \tag{3}$$

The scheme (3) is (in the section 126-184) an extrapolation of the Meyer-Jensen scheme. Comparing $\overline{L^2}$ from Eq. (1) with $\overline{L^2_{\rm emp}}$ from Eq. (3) and assuming a certain $\rho(r)$, we can find a relation between the parameters of $\rho(r)$. Taking as the simplest $\rho(r)$ a constant density distribution within a sphere of radius R, we obtain $\xi = 1$ and $\overline{L^2} = 0.885Z^{2/3}$ much larger than $\overline{L^2_{\rm emp}}$. On the other hand, lowering L^2 results in the density $\rho_0 e^{-br^2}$ or $\rho_0 e^{-\alpha r}$ ($\xi = 0.472$, $\overline{L^2} = 0.418Z^{2/3}$). The density giving the correct $\overline{L^2}$ should therefore be in between a constant and $\rho_0 e^{-\alpha r}$.

The density distributions dependent on one parameter result in constant values of ξ and cannot be in agreement with empirical $\overline{L^2}$ for all nuclei (see Ref. 5). Taking the effect of nuclear density saturation into account and neglecting for the time being the influence of Coulomb forces, we shall assume the nuclear density to be constant in the central part of the nucleus and falling off on the periphery. This condition is met by the following density distributions[5,8,9]:

$$\rho^{(\rm I)}(x) = \begin{cases}\rho_0^{(\rm I)} & (x \leqslant x_0)\\ \rho_0^{(\rm I)}\cdot e^{-(x-x_0)} & (x > x_0),\end{cases} \quad (x = \alpha r,\ x_0 = \alpha R_0^{(\rm I)}); \tag{4}$$

$$\rho^{(\rm II)}(y) = \begin{cases}\rho_0^{(\rm II)}, & (y \leqslant y_0)\\ \rho_0^{(\rm II)}(y_0/y)^2 e^{-(y-y_0)}, & (y > y_0),\end{cases} \quad (y = \beta r,\ y_0 = \beta R_0^{(\rm II)}). \tag{5}$$

The parameters x_0 and y_0 represent here the ratio of the radii of the region of constant density $R_0^{(\rm I)}$

and $R_0^{(II)}$ to the effective thicknesses of the surface layer $1/\alpha$ and $1/\beta$.

From the requirement of agreement between $\overline{L}^2$ calculated for the densities (4) and (5) and $\overline{L}^2_{emp}$ we have found the values of x_0 and y_0 for different Z. The resultant dependence of x_0 and y_0 on Z has a saw-tooth character (see the Figure, curve *1*) analogous to the dependence of $\overline{L}^2_{emp}$ on Z according to (3). In view of the fact that y_0 changes in a relatively smaller range than x_0, $\rho^{(II)}(r)$ should be given first choice. The parameters x_0 and y_0 rise in the regions of completion of the levels $3d_{5/2}$, $4f_{7/2}$, $5g_{9/2}$, etc., attaining maxima for nuclei with magic Z, which corresponds to the relatively thinner surface layer in magic nuclei. Additional maxima of x_0 and y_0 appear as the result of the completion of the levels $5g_{7/2}$, $6h_{9/2}$, $7i_{11/2}$, . . (N=64, 100, ~150,...). We note that the above treatment is applicable only in the case of spherically symmetric magic nuclei. The parameters x_0 and y_0 can, however, be correlated with the effective thickness of the surface layer, obtained by averaging $\rho(r)$ over the angles in real (generally) deformed nuclei and therefore, with the qualitative values of the eccentricity of the nuclei, found from the quadrupole moment (see Figure, curve *4*). Thus, the minima of x_0 and y_0 correspond, in the region of heavy nuclei, to the most strongly deformed nuclei.

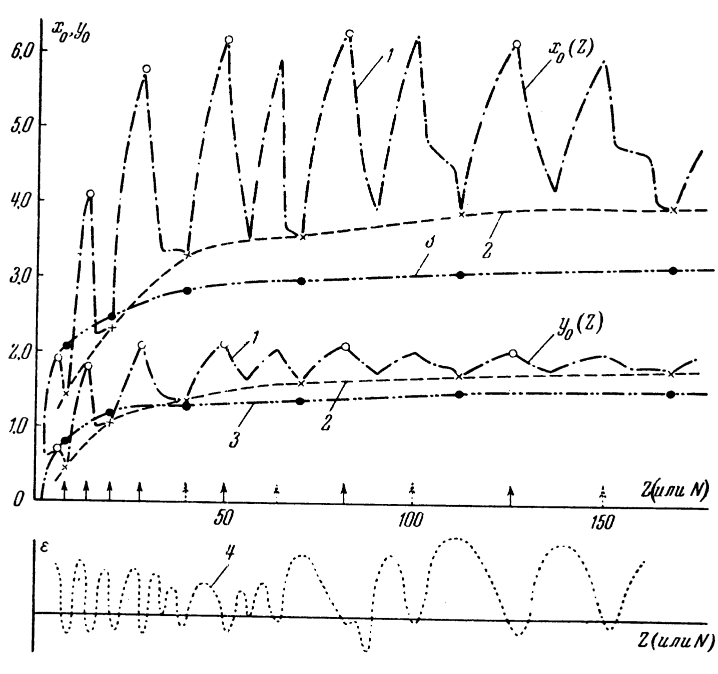

FIG. 1.

It is interesting to note that the values of x_0 and y_0 found from $\overline{L}^2$ and the numbers of first occurrence of Z_l for identical nuclei (see Figure, curves *3* and *4*, respectively) do not coincide. This is connected with the fact that L^2_{emp}, in contrast to Z_l, is determined by all completed states for given Z. Taking into account that the scheme of level filling is not finally established and bearing in mind the statistical character of Eq. (1), the obtained values of x_0 and y_0 should be regarded as qualitative only, expecially for non-magic nuclei.

Disregarding other details, we shall determine the remaining parameters $\rho_Z(r)$ of the protons (4) and (5) using the empirical Coulomb energies of nuclei[10]. Taking the mean values $x_0 \approx 4.8$ and $y_0 \approx 1.8$ (for $Z > 20$) and assuming $\rho_Z(r)/\rho_N(r) = Z/N$, we obtain

$$R_{0Z}^{(I)} = 0.92\times10^{-13}\ A^{1/3}; \qquad (6)$$

$$R_{0Z}^{(II)} = 0.73\times10^{-13}\ A^{1/3}.$$

On the basis of nonuniform density of nucleons, of the type (4) and (5), we can analyze the effective empirical nuclear radii, finding the mean values of different powers of r from $\rho(r)$, on which the various effects depend**. Thus, the mean value of r^2 from $\rho_Z^{(II)}(r)$ is

$$\overline{R_Z^{2(II)}} = 3/5\left[(y_0^3 + 5y_0^2 + 10y_0 + 10)/y_0^2(y_0+3)\right] R_{0Z}^{2\,(II)}. \qquad (7)$$

Finding the mean square radius $\{\overline{R_Z^{2(II)}}\}^{1/2}$ for the average $y_0 \approx 1.8$ and introducing the equivalent radius of constant density proton distribution that gives the same $\{\overline{r^2}\}^{1/2}$ $(R_Z = (5/3)^{1/2}\{\overline{R_Z^2}\}^{1/2})$, we find

$$R_Z^{(II)} \cong 1{,}21\times 10^{-13}\ A^{1/3} \qquad (8)$$

in good agreement with empirical electromagnetic nuclear radii[11].

Since for the densities (4) and (5) the nucleon density differs markedly from zero at distances larger than $R_Z \sim 1.2\times 10^{-13}A^{1/3}$ one can understand qualitatively, on the basis of the examined $\rho(r)$, the considerably larger values of nuclear radii $(1.5\times 10^{-13}A^{1/3})$ obtained from the cross section in processes in which nucleons (and evidently π-mesons) take part and data found from α-decay where the effective radii are connected with the region of action of nuclear forces.

If we shall assume the same $\rho(r)$[(4) and (5)] and the same level scheme (3) for neutrons, the corresponding parameters x_{0N} and y_{0N} for $\rho(r)$ will be correlated with N as x_0 and y_0 with Z.

Finally, we note that both $\{\overline{R_Z^{2\,(II)}}\}^{1/3}$ and the effective radii

$$\tilde{R}_Z^{(II)} = R_{0Z}^{(II)} + 1/\beta = (1 + 1/y_{0Z})\,R_{0Z}^{(II)}$$

will change nonmonotonically because of the sawtooth like change of y_{0Z}. Magic nuclei will have lower $\{\overline{R_Z^2}\}^{1/2}$ and $\tilde{R}_Z$. Therefore, the relative drop in the value of the radius should be more pronounced for the doubly-magic nuclei. The effective empirical nuclear radii show also relative drops for the magic and some sub-magic nuclei[12]. Such nonmonotonic character of the effective nuclear radii can be regarded as caused by deviations from spherical symmetry.

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Note added in Proof: For the density of the form $\rho_z(r) = \rho_0[1 + e^{K(r-c)}]^{-1}$ the parameter Kc calculated from $\bar{L}$ for Au ($Z = 79$) is in sufficient agreement with the value $Kc \approx 12.0$, for which best agreement between theory and experiment is observed for the cross section angular dependence for the scattering of high-energy electrons on Au_{79} nuclei[13].

* In the following, dealing with protons, we shall keep in mind that unless otherwise mentioned, the results are valid for neutrons as well.

** The parameters x_0 and y_0 were determined from the numbers of first occurrence and the r^2 also in Ref. 7.

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111

Some Cases of Generation of Heavy Unstable Particles on Beryllium Nuclei

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In recent years, it has been made evident[1-3] that hyperons and K-mesons can be created by