but is 40 to 50 times smaller than cross section σ_{10}.

There exists another possible method of computing σ_{0-1} using equation (7) which, taking $\sigma_{-11} = 0$, can be written in the form

$$\sigma_{0-1} = \{2\delta (kT/L)^2 - \sigma_{1-1}[\sigma_{1-1} - (\sigma_{-10} - \sigma_{10})]\}/\sigma_{10}. \quad (10)$$

Substituting here the value $\delta = 1.03 \times 10^3$, obtained from the curve in Fig. 1, and also the values of σ_{1-1}, σ_{10}, $(\sigma_{-10} - \sigma_{10})$ and kT/L we obtain for the energy 29 kev cross section $\sigma_{0-1} = 1.3 \times 10^{-17}$ cm^2. By computing σ_{0-1} according to Eq. (9) we obtain for this energy the value 0.8×10^{-17} cm^2. Considering the large errors in the measurements of the quantities entering into equations (9) and (10) the agreement between the values of σ_{0-1} computed by two different methods must be considered satisfactory.

In conclusion we consider it a pleasant duty to thank Prof. A. K. Val'ter for his constant interest and attention to this work and also V. Z. Surkov for his practical help in the construction and arrangement of the equipment.

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Nucleomesodynamics in Strong Coupling. II. The Ground and Isobar States, Nucleon Charge and Spin

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A nucleon is considered that interacts strongly with a pseudoscalar meson field. The interaction is assumed to be of the symmetric pseudovector type. The eigenvalues of the energy, charge and spin of the nucleon are determined, and also the explicit form of the wave function of the system. The ground and isobar states of the system are obtained.

1. WAVE EQUATION OF THE MESON FIELD IN THE ABSENCE OF THE NUCLEON

IN a previous paper[1] (which appears in this issue of the journal and which shall be referred to later as I), an approximate method is given for the consideration of a nucleon which interacts strongly with the meson field. The Hamiltonian of the system was simplified with the aid of a series of approximations and the spin-charge part of the wave function was determined. As a result the problem of the determination of the stationary quantum states of the system reduces to finding the eigenfunction and eigenvalues of the operator

$$\hat{H} = -G + {}^1/_2 \sum_{\alpha \vec{\varkappa}} \omega_{\vec{\varkappa}} \left[(q_{\alpha\vec{\varkappa}} - q^v_{\alpha\vec{\varkappa}})^2 - \partial^2/\partial q^2_{\alpha\vec{\varkappa}} \right]. \quad (1)$$

[1] S. I. Pekar, J. Exptl. Theoret. Phys.(U.S.S.R.) 30, 304 (1956); Soviet Phys. JETP 3, No. 3 (October, 1956).

Here, all the symbols of paper I have been retained. An operator similar to (1) has already been met in earlier works [see Ref. 2, p. 99, and Ref. 3, Sec. 3] and, as was shown therein, the eigenfunctions of this operator must be sought in the form

$$\Phi(q) = V(\vartheta, \beta, \delta)\exp\Big\{-\frac{1}{2}\sum_{\alpha\varkappa}\Big[(q_{\alpha\vec{\varkappa}} - q^{v}_{\alpha\vec{\varkappa}})^2 + \frac{1}{2}\ln\pi\Big]\Big\}\prod_{l=1}^{N} q_l\cdot 2^{N/2}. \quad (2)$$

Here q_i is one of the coordinates $q_{\alpha\vec{\varkappa}}$.

The arguments $q_{\alpha\vec{\varkappa}}$ enter directly into Φ, and, moreover, are contained in $v_i(q)$. Let $\partial/\partial^* q_{\alpha\vec{\varkappa}}$ denote differentiation only with respect to the $q_{\alpha\vec{\varkappa}}$ which do not enter into the argument of v_i. Then

$$\frac{\partial}{\partial q_{\alpha\vec{\varkappa}}} = \frac{\partial}{\partial^* q_{\alpha\vec{\varkappa}}} + \sum_{i=1}^{3}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}}\frac{\partial}{\partial v_i}, \quad (3)$$

$$\sum_{\alpha\vec{\varkappa}}\omega_{\vec{\varkappa}}\frac{\partial^2}{\partial q^2_{\alpha\vec{\varkappa}}} = \sum_{\alpha\vec{\varkappa}}\omega_{\vec{\varkappa}}\frac{\partial^2}{\partial^* q^2_{\alpha\vec{\varkappa}}} + \sum_{\alpha\vec{\varkappa}ij}\omega_{\vec{\varkappa}}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}}\frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}\frac{\partial^2}{\partial v_i\partial v_j} + \sum_{\alpha\vec{\varkappa}ij}\omega_{\vec{\varkappa}}\frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}\left(\frac{\partial}{\partial v_j}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}}\right)\frac{\partial}{\partial v_i} + 2\sum_{i\alpha\vec{\varkappa}}\omega_{\vec{\varkappa}}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}}\frac{\partial^2}{\partial v_i\partial^* q_{\alpha\vec{\varkappa}}}. \quad (4)$$

Taking into account Eqs. (59), (64), (67)-(69) of I we obtain

$$\sum_{\alpha\vec{\varkappa}ij}\omega_{\vec{\varkappa}}\frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}\left(\frac{\partial}{\partial v_j}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}}\right)\frac{\partial}{\partial v_i} = \frac{I^{(2n)}}{(I^{(n)})^2}\operatorname{ctg}\delta\cdot\frac{\partial}{\partial\delta}, \quad (5)$$

$$\sum_{\alpha\vec{\varkappa}ij}\omega_{\vec{\varkappa}}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}}\frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}\frac{\partial^2}{\partial v_i\partial v_j} = \frac{I^{(2n)}}{(I^{(n)})^2}\left[\frac{1}{\sin^2\delta}\left(\frac{\partial^2}{\partial\vartheta^2} + \frac{\partial^2}{\partial\beta^2} + 2\cos\delta\frac{\partial^2}{\partial\vartheta\,\partial\beta}\right) + \frac{\partial^2}{\partial\delta^2}\right] \quad (6)$$

It must be kept in mind that even in the development of Eqs. (67)-(69) in I, it was assumed that $\varphi_\alpha(\mathbf{r})$ differed slightly from $\varphi^v_\alpha(\mathbf{r})$; therefore if the function under the integral contained the factor φ_α and was significant only in the local region where φ^v_α differed appreciably from zero, then φ_α could be replaced by φ^v_α under the integral. On this this basis, we can neglect terms of the form

$$\sum_{\alpha\vec{\varkappa}}\left(q_{\alpha\vec{\varkappa}} - q^v_{\alpha\vec{\varkappa}}\right)\frac{\partial q^v_{\alpha\vec{\varkappa}}}{\partial v_i}; \quad \sum_{\alpha\varkappa}\left(q_{\alpha\vec{\varkappa}} - q^v_{\alpha\vec{\varkappa}}\right)\frac{\partial^2 q^v_{\alpha\vec{\varkappa}}}{\partial v_i\partial v_j} \quad (7)$$

in the calculation of the derivatives of Φ with respect to v_i and $q_{\alpha\vec{\varkappa}}$. We then have

$$\frac{\partial\Phi}{\partial v_i} = \frac{\partial V}{\partial v_i}e^{-s}\prod_{l=1}^{N} q_l\cdot 2^{N/2}, \quad (8)$$

$$s = \frac{1}{2}\sum_{\alpha\vec{\varkappa}}\left[\left(q_{\alpha\vec{\varkappa}} - q^v_{\alpha\vec{\varkappa}}\right)^2 + \frac{1}{2}\ln\pi\right], \quad (9)$$

$$\frac{\partial^2\Phi}{\partial v_i\partial v_j} = \frac{\partial^2 V}{\partial v_i\partial v_j}e^{-s}\prod_{l=1}^{N} q_l 2^{N/2} - R^{(1)}_{ij}\Phi, \quad (10)$$

$$\frac{\partial^2\Phi}{\partial v_i\partial^* q_{\alpha\vec{\varkappa}}} = -\frac{\partial V}{\partial v_i}\left(q_{\alpha\vec{\varkappa}} - q^v_{\alpha\vec{\varkappa}}\right)e^{-s}\prod_{l=1}^{N} q_l\cdot 2^{N/2} + \frac{\partial q^v_{\alpha\vec{\varkappa}}}{\partial v_i}\Phi + \frac{\partial V}{\partial v_i}e^{-s}\Pi^{\alpha\vec{\varkappa}}, \quad (11)$$

$$\Pi^{\alpha\vec{\varkappa}} \equiv \frac{\partial}{\partial q_{\alpha\vec{\varkappa}}}\prod_{l=1}^{N} q_l\cdot 2^{N/2}.$$

Taking into account Eqs. (4)-(6) and also Eqs. (59) and (64) of I, the effect of operator (1) on Φ can

[2] S. I. Pekar, *Investigations on the Electron Theory of Crystals*, Gostekhizdat, 1951.

[3] S. I. Pekar, J.Exptl.Theoret. Phys .(U.S.S.R.) 27, 411 (1954).

be put in the following form:

$$\hat{H}\Phi = \Big\{ -G + \frac{3}{2}\sum_{\vec{\varkappa}} \omega_{\vec{\varkappa}} + \sum_{l=1}^{N} \omega_{l} - 3\frac{I^{(n+1)}}{I^{(n)}} + \frac{3}{2}\frac{I^{(1)}I^{(2n)}}{(I^{(n)})^2} \Big\} \Phi + (\hat{\mathscr{H}} V) e^{-s} \prod_{l=1}^{N} q_l 2^{N/2} + \hat{H}_1 \Phi, \quad (12)$$

where

$$\hat{\mathscr{H}} = -\frac{1}{2}\frac{I^{(2n)}}{(I^{(n)})^2}\Big[\frac{1}{\sin^2\delta}\Big(\frac{\partial^2}{\partial\vartheta^2} + \frac{\partial^2}{\partial\beta^2} + 2\cos\delta\frac{\partial^2}{\partial\vartheta\,\partial\beta}\Big) + \frac{\partial^2}{\partial\delta^2} + \operatorname{ctg}\delta\frac{\partial}{\partial\delta}\Big], \quad (13)$$

$$\hat{H}_1\Phi = -\sum_{\alpha\vec{\varkappa}i} \omega_{\vec{\varkappa}} \frac{\partial V}{\partial v_i}\frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}} e^{-s}\Pi^{\alpha\vec{\varkappa}}. \quad (14)$$

The wave function (2) corresponds to a state of the system in which all the oscillators of the meson field are in the ground state, with the exception of N oscillators which are in singly excited states. In Eq. (12), the term $^3/_2 \sum_{\vec{\varkappa}} \omega_{\vec{\varkappa}}$ represents the energy of the zero point vibrations of all the oscillators and the term $\sum_{l=l}^{N} \omega_l$ the energy of the single excitations of the N oscillators. Physically, this means that there are N free mesons in addition to the nucleon.

In the case of the absence of free mesons, $N = 0$, $\hat{H}_1 \Phi = 0$, and if $V(v_i)$ is so chosen that it satisfies the equation

$$\hat{\mathscr{H}} V = \mathscr{H} V, \quad (15)$$

then Φ is an eigenfunction of the operator (1), as is seen from Eq. (12):

$$\hat{H}\Phi = H\Phi. \quad (16)$$

The eigenvalue of the energy of the system is

$$H = -G + \frac{3}{2}\sum_{\vec{\varkappa}} \omega_{\vec{\varkappa}} - 3\frac{I^{(n+1)}}{I^{(n)}} + \frac{3}{2}\frac{I^{(1)}I^{(2n)}}{(I^{(n)})^2} + \mathscr{H}. \quad (17)$$

In the absence of free mesons, Φ does not describe the stationary state of the system. The term $\hat{H}_1\Phi$ determines the scattering of the mesons on the nucleon. This scattering will be considered in a subsequent paper; here we shall limit ourselves to the case of the absence of free mesons ($N = 0$).

2. DETERMINATION OF THE ANGULAR PART OF THE WAVE FUNCTION

We now find the eigenfunctions and the eigenvalues of Eq. (15). Inasmuch as the variables θ and β do not enter explicitly into the operator (13), its eigenfunctions must, as is known, depend on these variables exponentially; therefore we must seek the solution of Eq. (15) in the form

$$V = D(\delta)\, e^{i(s\vartheta + p\beta)}. \quad (18)$$

Substituting (18) in (15) we get an equation for the function $D(\delta)$:

$$\frac{d^2D}{d\delta^2} + \operatorname{ctg}\delta\frac{dD}{d\delta} - \frac{1}{\sin^2\delta}(s^2 + p^2 + 2sp\cos\delta) D = -\lambda D, \quad (19)$$

$$\lambda I^{(2n)} / 2 (I^{(n)})^2 = \mathscr{H}.$$

If Eq. (52) or (53) of I is written in explicit form, then it is evident that they are trigonometric equations relative to the angles ϑ, β and δ. The roots of these equations, ϑ and β, for each given configuration of the meson field q are determined only with accuracy to shifts of multiples of 2π. If the given configurations of the meson field q correspond to values of the angles ϑ and β, then these configurations will also correspond to the angles $\vartheta + 2\pi$ and $\beta + 2\pi$. Usually, the requirement in quantum mechanics that the wave function of the whole system be a single valued function of the configuration q leads to the requirement that this function possess a periodicity of 2π in the arguments ϑ and β. Inasmuch as ψ^v changes sign upon change of ϑ or β by 2π (see Eq. 35, Ref. 1), then, according to Eq. (9)[1], Φ must also change sign. Therefore, the quantum numbers s and p in Eq. (18) must be half integers. The eigenfunctions and the eigenvalues of Eq. (19) will be found by the method of factorization (Ref. 4, part 4). Substitution of

$$Y(\delta) = \sin^{1/2}\delta\, D(\delta) \quad (20)$$

in Eq. (19) leads to the standard form suitable for factorization:

[4] L. Infeld and E. Hull, Revs. Mod. Phys. **23**, 21 (1951); Sec. 4.

$$\frac{d^2Y}{d\delta^2}-\frac{1}{\sin^2\delta}\Big(s^2+p^2+2sp\cos\delta-\frac{1}{4}\Big)Y+\Big(\lambda+\frac{1}{4}\Big)Y=0. \tag{21}$$

The integral index m, according to which factorization is carried out,[4] is connected with the quantum numbers by the relations

$$1)\ m=s-{}^1/_2 \text{ for } s\geqslant {}^1/_2, \qquad 2)\ m={}^1/_2-s \text{ for } s<{}^1/_2. \tag{22}$$

In the first case the factorization is carried out by the usual method, with the use of the recurrence form (2.6, Ib) of Ref. 4. In the second case, we take as the initial eigenfunction the function found by the first method with quantum number $s = 1/2$; we then apply the recurrence relation (2.6, Ia) of Ref. 4.

As a result, the eigenvalues λ in the equation (19) are given by

$$\lambda_j=j(j+1),\quad j={}^1/_2,\ {}^3/_2,\ {}^5/_2,\ {}^7/_2\ldots \tag{23}$$

For the eigenfunctions of Eq. (21) we get the recurrence relation

$$Y^j_{s-1,\,p}=\Big[\Big(j+\frac{1}{2}\Big)^2-\Big(s-\frac{1}{2}\Big)^2\Big]^{-1/2}\times\Big[\Big(s-\frac{1}{2}\Big)\operatorname{ctg}\delta+\frac{p}{\sin\delta}+\frac{d}{d\delta}\Big]Y^j_{s,\,p}. \tag{24}$$

The latter enables us to get the totality of eigenfunctions from the initial function

$$Y^j_{j,\,p}=\Big[\frac{\Gamma(2j+2)}{\Gamma(j+p+1)\Gamma(j-p+1)}\Big]^{1/2}\times\sin^{j+p+1/2}\frac{\delta}{2}\cos^{j-p+1/2}\frac{\delta}{2}. \tag{25}$$

We can obtain completely analogous recurrence relations with respect to the index p, inasmuch as s and p appear symmetrically in Eq. (21). The half integer quantum numbers s and p change in the following way

$$|s|\leqslant j,\quad |p|\leqslant j. \tag{26}$$

In the ground state

$$j=\frac{1}{2},\quad \mathcal{H}_{1/2}=\frac{\lambda_{1/2}}{2}\frac{I^{(2n)}}{(I^{(n)})^2}=\frac{3}{8}\frac{I^{(2n)}}{(I^{(n)})^2} \tag{27}$$

there exist 4 eigenfunctions of Eq. (15),

$$\begin{aligned} V^1_{1,1}&=\frac{1}{2\pi}\sin\frac{\delta}{2}e^{i(\vartheta+\beta)/2}, & V^1_{1,-1}&=\frac{1}{2\pi}\cos\frac{\delta}{2}e^{i(\vartheta-\beta)/2};\\ V^1_{-1,1}&=\frac{1}{2\pi}\cos\frac{\delta}{2}e^{i(\beta-\vartheta)/2}, & V^1_{-1,-1}&=-\frac{1}{2\pi}\sin\frac{\delta}{2}e^{-i(\vartheta+\beta)/2}.\end{aligned} \tag{28}$$

Here and later we shall write, for brevity, the double of the quantum numbers s, p and j: $V^{2j}_{2s,\,2p}$.

In the first excited state,

$$j={}^3/_2,\quad \mathcal{H}_{3/2}=15\,I^{(2n)}/8\,(I^{(n)})^2 \tag{29}$$

there are 16 eigenfunctions:

$$\begin{aligned}
V^3_{3,3}&=-V^{3*}_{-3,-3}=\frac{\sqrt{2}}{2\pi}\sin^3\frac{\delta}{2}e^{i(3\vartheta+3\beta)/2};\\
V^3_{1,3}&=V^{3*}_{-1,-3}=\frac{\sqrt{6}}{2\pi}\sin^2\frac{\delta}{2}\cos\frac{\delta}{2}e^{i(\vartheta+3\beta)/2};\\
V^3_{-1,3}&=-V^{3*}_{1,-3}=\frac{\sqrt{6}}{2\pi}\sin\frac{\delta}{2}\cos^2\frac{\delta}{2}e^{i(-\vartheta+3\beta)/2};\\
V^3_{-3,3}&=V^{3*}_{3,-3}=\frac{\sqrt{2}}{2\pi}\cos^3\frac{\delta}{2}e^{i(-3\vartheta+3\beta)/2};\\
V^3_{3,1}&=V^{3*}_{-3,-1}=\frac{\sqrt{6}}{2\pi}\sin^2\frac{\delta}{2}\cos\frac{\delta}{2}e^{i(3\vartheta+\beta)/2};\\
V^3_{1,1}&=-V^{3*}_{-1,-1}=\frac{\sqrt{2}}{2\pi}\sin\frac{\delta}{2}\Big(3\cos^3\frac{\delta}{2}-1\Big)e^{i(\vartheta+\beta)/2};\\
V^3_{-1,1}&=V^{3*}_{1,-1}=\frac{\sqrt{2}}{2\pi}\cos\frac{\delta}{2}\Big(1-3\sin^2\frac{\delta}{2}\Big)e^{i(-\vartheta+\beta)/2};\\
V^3_{-3,1}&=-V^{3*}_{3,-1}=-\frac{\sqrt{6}}{2\pi}\sin\frac{\delta}{2}\cos^2\frac{\delta}{2}e^{i(-3\vartheta+\beta)/2}.
\end{aligned} \tag{30}$$

For a given j, each of the quantum numbers s and p, being limited by the inequalities (26) have $2j+1$ values; therefore the multiplicity of the degeneracy of the level $\mathcal{H}_j$ will be $(2j+1)^2$.

Thus, in a system which consists of a nucleon that interacts strongly with the meson field, three types of excitation are possible:

1. Excitation of the spin-charge motion, with transition from a state defined by Eq. (28) of I. These excited states are evidently always unstable; they will not be considered in this paper.
2. Excitation of the oscillators of the meson field, i.e., creation of free mesons. The energy of such excitation is given by the term $\sum_{l=1}^{N} \omega_l$ of Eq. (17).
3. Excitation of rotational motion, described by Eqs. (15) and (13). The energy of this motion, in accordance with Eqs. (19) and (23), is given by the equation

$$\mathcal{H}_j = \frac{1}{2} \frac{I^{(2\,?)}}{(I^{(\ )})^2} j(j+1). \tag{31}$$

The wave equation of this motion coincides in form with the equation of motion of a symmetric rotator. However, in the latter case, s, p, and j are integers, while in Eqs. (18), (19) and (31) they are half integers. Such states, excited in the quantum number j, are appropriately called isobars.

Comparison of the energy (17) with the corresponding energies calculated by Pauli and Dancoff [see Ref. 5, Eq. (76)], shows that the term G, proportional to g^2, agrees exactly with the corresponding term of Pauli and Dancoff (in the comparison it should be noted that our g is equal to $2^{-1/2}g$ in Ref. 5). The energy of excitation of the isobars of $\mathcal{H}_j$, determined by Eq. (31), agrees exactly with Eq. (80) of Ref. 5, if we set $n = 0$, i.e., if the parameters $v_i(q)$ are chosen by approximating φ_α by the functions φ_α^v by the method of least squares (see Ref. 1, Sec. 5). In obtaining Eq. (76) of Ref. 5, Pauli and Dancoff evidently made an approximation. Without this approximation in their work, the energy of excitation of the isobars would have been obtained that would have agreed with that obtained by us in the case $n = 2$, i.e., in the case of most accurate energy approximation. Comparison of the remaining terms of the energy (17) is difficult, since they are not explicitly calculated in Ref. 5.

Equations (17) and (31) determine the energy of the system only in the zero approximation. Below we have also calculated the corrections to the energy in higher approximations.

[5] W. Pauli and S. M. Dancoff, Phys. Rev. 62, 85 (1942).

3. THE BASIS OF THE ADIABATIC APPROXIMATION AND ACCOUNT OF NONADIABATICITY

It was assumed in I that the spin-charge motion took place adiabatically because of the comparatively slow vibrations of the meson field. We shall now consider nonadiabaticity as a small perturbation and criteria will also be given for the adiabatic approximation.

We begin with the exact wave equation

$$\hat{H}\Psi \equiv (\hat{H}_0 + \hat{H}')\Psi = H\Psi. \tag{32}$$

We now introduce a complete set of orthogonal functions $\psi_s(q)$, which satisfy the equation

$$\hat{H}'(q)\,\psi_s(q) = H'_s(q)\,\psi_s(q), \quad \psi_s = \begin{vmatrix} C_{1s} \\ C_{2s} \\ C_{3s} \\ C_{4s} \end{vmatrix}, \tag{33}$$

in which the q appear as parameters. We expand the wave function of the system in the orthogonal functions $\psi_s(q)$:

$$\Psi(q) = \sum_{s=1}^{4} \Phi_s(q)\,\psi_s(q). \tag{34}$$

Substitution of this expansion in Eq. (32) leads to the following equations for the expansion coefficients $\Phi_s(q)$:

$$[\hat{H}_0 + H'_s(q)]\,\Phi_s - \sum_{s'\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}} \left\{\psi_s^*, \frac{\partial\psi_{s'}}{\partial q_{\alpha\vec{\varkappa}}}\right\} \frac{\partial\Phi_{s'}}{\partial q_{\alpha\vec{\varkappa}}} - \frac{1}{2}\sum_{s'\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}} \left\{\psi_s^*, \frac{\partial^2\psi_{s'}}{\partial q^2_{\alpha\vec{\varkappa}}}\right\} \Phi_{s'} = H\Phi_s; \quad s = 1, 2, 3, 4. \tag{35}$$

The second and third terms of the left hand side represent the nonadiabatic perturbation. In order to be able to use standard perturbation theory, we must put the system of equations (35) into form of the usual operator equation. For this purpose, we introduce the four vector $\Phi(q)$, whose components are the $\Phi_s(q)$. Furthermore, we introduce the matrices

$$A_{ss'}^{\alpha\vec{\varkappa}}(q) = -\omega_{\vec{\varkappa}} \left\{\psi_s^*, \frac{\partial\psi_{s'}}{\partial q_{\alpha\vec{\varkappa}}}\right\}; \tag{36}$$

$$B_{ss'} = -\frac{1}{2}\sum_{\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}} \left\{\psi_s^*, \frac{\partial^2\psi_{s'}}{\partial q^2_{\alpha\vec{\varkappa}}}\right\};$$

$$O_{ss'} = H'_s(q)\,\delta_{ss'}.$$

Then we can write the system of equations (35) in the form of a single vector equation of the operator type:

$$\left[\hat{H}_0 + O(q) + \sum_{\vec{\alpha\varkappa}} A^{\vec{\alpha\varkappa}} \frac{\partial}{\partial q_{\vec{\alpha\varkappa}}} + B\right]\Phi = H\Phi. \tag{37}$$

Here $O\Phi$, $A^{\vec{\alpha\varkappa}}\,\partial\Phi/\partial q_{\vec{\alpha\varkappa}}$ and $B\Phi$ are to be understood as the usual multiplications of matrices and vectors.

The nonadiabatic perturbation

$$\hat{\Omega} = \hat{M} + \hat{B},\ \hat{M} \equiv \sum_{\vec{\alpha\varkappa}} A^{\vec{\alpha\varkappa}}\,\partial/\partial q_{\vec{\alpha\varkappa}} \tag{38}$$

is omitted in zeroth approximation. Then the vector equation (37) breaks up into a series of independent equations for the different components of the vector Φ; these equations have the form:

$$[\hat{H}_0 + H_s'(q)]\,\Phi_s = H\Phi_s. \tag{39}$$

Since, in the different equations (39) (with different s), there enter different functions $H_s'(q)$, the spectra of the eigenvalues for these equations will in general be different. Therefore, the entire series of equations (39) can be satisfied only by such a vector Φ for which all the components are different from zero except the single component Φ_s; the latter must satisfy the corresponding Eq. (39). If we number the solutions of Eq. (39) by the quantum number m in the order of increasing H, then the solutions of the zeroth approximation will be denoted by Φ_{sm} and H_{sm}, where s is the number of the only component of the vector Φ different from zero.

In the zeroth approximation in the decomposition (34), there is only one term. Consequently, we can consider that the nucleon is found in the state $\psi_s(q)$, defined by Eq. (33), and possesses the energy $H_s'(q)$. In this case the index s takes on the sense of a quantum number of the nucleon.

The zeroth approximation described above coincides exactly with the adiabatic approximation used in the previous research (Eq. (11) of I). We now introduce the previously discarded terms $\hat{\Omega}$ as a small perturbation and compute the correction of the next order. In this case, we can replace ψ_s by ψ_s^v [see Eqs. (33) and (42) of I], inasmuch as (in addition to the adiabatic approximation) it has also been assumed that $\varphi_\alpha(\mathbf{r})$ differs only slightly from $\varphi_\alpha^v(\mathbf{r})$. With the help of Eq. (29) of I, we get

$$A_{11}^{\vec{\alpha\varkappa}} = -\omega_{\vec{\varkappa}} \frac{\partial}{\partial q_{\vec{\alpha\varkappa}}}[\,|c_1|^2 + |c_2|^2] = 0. \tag{40}$$

In accordance with Eqs. (4)-(6) and (13), the operator $-\frac{1}{2}\sum_{\vec{\alpha\varkappa}} \omega_{\vec{\varkappa}} \frac{\partial^2}{\partial q^2_{\vec{\alpha\varkappa}}}$ in application to the function of v_i is equivalent to the operator $\hat{\mathcal{H}}$. Further, we can represent the function ψ_1^v in the form

$$\psi_1^v = \frac{\pi}{\sqrt{2}}\begin{vmatrix} iV_{11}^1 + V_{1,-1}^1 + V_{-1,1}^1 - iV_{-1,-1}^1 \\ -V_{11}^1 + iV_{1-1}^1 - iV_{-1,1}^1 - V_{-1,-1}^1 \\ -V_{11}^1 - iV_{1,-1}^1 + iV_{-1,1}^1 - V_{-1,1}^1 \\ -iV_{11}^1 + V_{1,-1}^1 + V_{-1,1}^1 + iV_{-1,1}^1 \end{vmatrix} \tag{41}$$

It is evident from this that each of the four components ψ_1^v is an eigenfunction of $\hat{\mathcal{H}}$ and corresponds to the eigenvalue

$$\mathcal{H}_{1/2} = \frac{3}{8}\frac{I^{(2n)}}{(I^{(n)})^2}.$$

Therefore

$$B_{11} = +\{\psi_1^{v*},\ \hat{\mathcal{H}}\,\psi_1^v\} = \frac{3}{8}\frac{I^{(2n)}}{(I^{(n)})^2}. \tag{42}$$

Taking into account Eqs. (49) and (42), we find the correction due to nonadiabaticity of first order in the energy to be

$$\Delta_1 H = \int \Phi_{1,m}^* \hat{\Omega} \Phi_{1,m}\, dq\, dv = \frac{3}{8}\frac{I^{(2n)}}{(I^{(n)})^2}; \tag{43}$$

$$dv = \sin\delta\, d\vartheta\, d\beta\, d\delta.$$

It must be emphasized that the correction to the energy (43) is the same for all states of the system which are not excited by the spin-charge motion. Consequently, this correction does not change the energy of excitation of the isobar states and free mesons.

Here it should be clear why the integration in Eq. (43) was carried out over the variables $q_{\alpha\vec{\varkappa}}$ and also over v_i, in spite of the fact that v_i and $q_{\alpha\vec{\varkappa}}$ are not independent variables. In Eq. (37), only the variables $q_{\alpha\vec{\varkappa}}$ appear at first. Expressing $A_{ss'}^{\vec{\alpha\varkappa}}$ and $B_{ss'}$ in Eq. (36) by ψ_s^v, and also representing Φ in the form of Eq. (2), we introduce into Eq. (43) the arguments v_i which are definite functions of $q_{\alpha\vec{\varkappa}}$ (see Sec. 5 of I). Simultaneously, differentiation with respect to $q_{\alpha\vec{\varkappa}}$ leads to the form (3) and the operator $\hat{H}$ takes the form (12), (13). For this reason, Eq. (43) must be treated as a more general problem in which the variables v_i and $q_{\alpha\vec{\varkappa}}$ are considered as independent. If, in the solution of this generalized problem, we replace v_i by the above mentioned functions $v_i(q)$, then we obtain the solution of the original equation (43). Because of technical advantages, we introduce the nonadiabatic perturbation at that stage of the calculation when v_i and $q_{\alpha\vec{\varkappa}}$ are considered as independent variables. In connection with this, the eigenfunctions obtained above for the unperturbed problem are orthonormal for integration over $dvdq$; therefore, we can integrate in Eq. (43) over $dvdq$, considering v_i and $q_{\alpha\vec{\varkappa}}$ to be independent.

For a calculation of the correction to the energy of second order of smallness, we must consider the perturbation $\hat{M}$ in Eq. (38) and ignore $\hat{B}$, since, in the adiabatic approximation, as is well known, $\hat{B}$ is a quantity of second order of smallness in comparison with $\hat{M}$. The perturbation operator has the form:

$$\hat{M}_{1s} = \sum_{\alpha\vec{\varkappa}} A_{1s}^{\alpha\vec{\varkappa}} \frac{\partial}{\partial q_{\alpha\vec{\varkappa}}} \tag{44}$$

$$= - \sum_{ij=1}^{3} Q_{ij}^{(1)} \left\{ \psi_1^{*v}, \frac{\partial \psi_s^v}{\partial v_i} \right\} \frac{\partial}{\partial v_j}$$

$$- \sum_{i\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}} \left\{ \psi_1^{*v}, \frac{\partial \psi_s^v}{\partial v_i} \right\} \frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}} \frac{\partial}{\partial^* q_{\alpha\vec{\varkappa}}},$$

where

$$Q_{ij}^{(\nu)} = \sum_{\alpha\vec{\varkappa}} \omega_{\vec{\varkappa}}^{\nu} \frac{\partial v_i}{\partial q_{\alpha\vec{\varkappa}}} \frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}. \tag{45}$$

With the help of Eqs. (67)-(69), (59) and (64) of Ref. 1, we obtain

$$Q_{11}^{(\nu)} = Q_{22}^{(\nu)} = \frac{I^{(2n+\nu-1)}}{(I^{(n)})^2 \sin^2\delta}, \quad Q_{33}^{(\nu)} = \frac{I^{(2n+\nu-1)}}{(I^{(n)})^2}; \tag{46}$$

$$Q_{12}^{(\nu)} = Q_{21}^{(\nu)} = \frac{I^{(2n+\nu-1)} \cos\delta}{(I^{(n)})^2 \sin^2\delta},$$

$$Q_{13}^{(\nu)} = Q_{31}^{(\nu)} = Q_{23}^{(\nu)} = Q_{32}^{(\nu)} = 0.$$

In (44) there appear the eigenfunctions of Eq. (40) of I–ψ_s^v, which represent the perturbed spin-charge state of the nucleon in the fixed selfconsistent field configuration of the meson field $q_{\alpha\vec{\varkappa}}^v$ in the ground state. These ψ_s^v are put in the following form, with the aid of Eqs. (32)-(33), (42) of I:

$$\psi_2^v = \| T \| \cdot \| S \| \psi_2^0 e^{-i\gamma}, \tag{47}$$

$$\psi_3^v = \| T \| \cdot \| S \| \psi_3^0 e^{i\gamma},$$

$$\psi_4^v = \| T \| \cdot \| S \| \psi_4^0$$

Here we have multiplied ψ_s^v on the right by an arbitrary factor of modulus unity. This factor $(e^{\pm i\gamma})$ does not depend on the spin-charge degrees of freedom. As a result, the ψ_s^v are functions not of the four angles ϑ, φ, β, γ, but of only three angles ϑ, β, δ. Making use of Eq. (47), and taking the orthonormality of the φ_s^v into account, we obtain:

$$\left\{ \psi_1^{v*}, \frac{\partial \psi_2^v}{\partial \vartheta} \right\} = \frac{1}{2\sqrt{2}} e^{-i\delta}, \qquad \left\{ \psi_1^{v*}, \frac{\partial \psi_3^v}{\partial \vartheta} \right\} = -\frac{1}{2\sqrt{2}} e^{i\delta};$$
$$\left\{ \psi_1^{v*}, \frac{\partial \psi_3^v}{\partial \beta} \right\} = -\left\{ \psi_1^{v*}, \frac{\partial \psi_2^v}{\partial \beta} \right\} = \frac{1}{2\sqrt{2}} \left\{ \psi_1^{v*}, \frac{\partial \psi_4^v}{\partial \delta} \right\} = -\frac{i}{2} \tag{48}$$
$$\left\{ \psi_1^{v*}, \frac{\partial \psi_4^v}{\partial \vartheta} \right\} = \left\{ \psi_1^{v*}, \frac{\partial \psi_4^v}{\partial \beta} \right\} = \left\{ \psi_1^{v*}, \frac{\partial \psi_2^v}{\partial \delta} \right\} = \left\{ \psi_1^{v*}, \frac{\partial \psi_3^v}{\partial \delta} \right\} = 0.$$

In going on to the calculation of the second order energy correction, defined by the perturbation M, it should be noted that the perturbed ground state of the system is fourfold degenerate [see Eq. (28)], but the matrix elements of all transitions between these degenerate states are equal to zero, inasmuch as $A_{11}^{\alpha\vec{\varkappa}} = 0$. In this case the second order correction to the energy is determined by setting the determinant equal to zero:[6]

$$\left| \sum_{s,m_1}' \frac{(1m \mid M \mid sm_1)(sm_1 \mid \hat{M} \mid 1\hat{m}')}{H_{1m} - H_{sm_1}} - \Delta_2 H \delta_{mm'} \right| = 0. \tag{49}$$

Here the energy difference which appears in the denominator is basically the energy of excitation of spin-charge motion, and is approximately equal to $-(2/3)G$. Carrying out the averaging of the energy difference before the summation sign, we can treat the sum as a matrix element of $\hat{M}^2$ $[(1m \mid \hat{M} \mid 1m) = 0]$. It is equal to

[6] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Gostekhizdat, 1948, p. 165.

$$(1m\,|\,\hat{M}^2\,|\,1m') = \delta_{mm'}\left\{\frac{3}{16}\left[\frac{I^{(2n)}}{(I^{(n)})^2}\right]^2 + \frac{3}{4}I^{(1)}\left[\frac{I^{(2n)}}{(I^{(n)})^2}\right]^2 + \frac{3}{8}\,\frac{I^{(2n+1)}}{(I^{(n)})^2}\right\}. \qquad (50)$$

Here Φ_{1m} and $\Phi_{1m'}$ are two degenerate states of the ground level of the system.

The determinant (49) is diagonal. Inasmuch as the matrix elements (50) do not depend on m, the same correction to the energy is obtained in all degenerate states of the ground level of the system, namely

$$\Delta_2 H = -3\,(1m\,|\,\hat{M}^2\,|\,1m)\,/\,2G. \qquad (51)$$

The nonadiabaticity can be considered as a small perturbation if $\Delta_2 H$ is significantly less than the difference G between the levels of the unperturbed problem:

$$(1m\,|\,\hat{M}^2\,|\,1m)\,/\,G^2 \ll 1. \qquad (52)$$

This same inequality guarantees the smallness of the energy correction of first order $\Delta_1 H$ [see (43)] in comparison with G. If we assume a simple and monotonic path of the form factor $u(r)$ and characterize its effective radius by $a(\mu a \lessapprox 1)$, then the inequality (52) means

$$g\,/\,\mu a \gg 1. \qquad (53)$$

In completely analogous fashion, as was done in Ref. 7, it can be shown that the use in Ref. 1 of an assumption on the smallness of $\varphi'_\alpha(\mathbf{r})$ in comparison with $\varphi^v_\alpha(\mathbf{r})$ is correct for satisfaction of the inequality (53).

4. SPIN AND ISOTOPIC SPIN OF A SYSTEM

The momentum operator of a system

$$L_{ik} = -\sum_{\alpha=1}^{3}\int\left[x_i\frac{\partial\varphi_\alpha}{\partial x_k} - x_k\frac{\partial\varphi_\alpha}{\partial x_i}\right]\pi_\alpha dV + \frac{1}{2}\,\sigma_{ik};\quad \sigma_{12} = \sigma_3\ldots \qquad (54)$$

commutes with the energy operator of the system, i.e., the momentum is an integral of motion. With the help of Eq. 1[1], and also (3), we obtain

$$L_{ik} = L^q_{ik} + L^v_{ik} + \frac{1}{2}\,\sigma_{ik};$$
$$L^q_{ik} = i\sum_{\alpha\vec{\varkappa}}\int\left[x_i\frac{\partial\varphi_\alpha}{\partial x_k} - x_k\frac{\partial\varphi_\alpha}{\partial x_i}\right]\chi_{\vec{\varkappa}}(\mathbf{r})\,dV\,\sqrt{\omega_{\vec{\varkappa}}}\,\frac{\partial}{\partial^* q_{\alpha\vec{\varkappa}}}; \qquad (55)$$
$$L^v_{ik} = i\sum_{\alpha\vec{\varkappa}}\int\left[x_i\frac{\partial\varphi_\alpha}{\partial x_k} - x_k\frac{\partial\varphi_\alpha}{\partial x_i}\right]\chi_{\vec{\varkappa}}(\mathbf{r})\,dV\,\sqrt{\omega_{\vec{\varkappa}}}\,\sum_{j=1}^{3}\frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}\,\frac{\partial}{\partial v_j}.$$

In the calculation of L^v_{ik} the expressions $\partial v_j/\partial q_{\alpha\vec{\varkappa}}$ are put in the form of Eqs. (67)-(69)[1], and as a result of the use of Eq. (1), expressions are obtained which contain integrals of the form

$$\int\left[x_i\frac{\partial\varphi_\alpha}{\partial x_k} - x_k\frac{\partial\varphi_\alpha}{\partial x_i}\right]\hat{\omega}^n\,\frac{\partial\varphi^v_\alpha}{\partial v_j}\,dV. \qquad (56)$$

In the calculation of these integrals, φ_α is replaced by φ^v_α. For the latter, the expressions (60) and (38) are used[1]. It is convenient to express the operator $x_i\,\partial/\partial x_k - x_k\partial/\partial x_i$ in spherical coordinates as the derivative with respect to the corresponding angle. One should also note that the unit vector $\mathbf{r}/r$ is an eigenfunction of the operator $\hat{\omega}$. As a result, we obtain

$$L^v_1 \equiv L^v_{23} \equiv -L^v_{32} = -i\left(\operatorname{ctg}\delta\cos\vartheta\frac{\partial}{\partial\vartheta} + \frac{\cos\vartheta}{\sin\delta}\,\frac{\partial}{\partial\beta} + \sin\vartheta\frac{\partial}{\partial\delta}\right); \qquad (57)$$

$$L^v_2 \equiv L^v_{31} \equiv -L^v_{13} = i\,\frac{\partial}{\partial\vartheta}; \qquad (58)$$

$$L^v_3 \equiv L^v_{12} \equiv -L^v_{21} = -i\left(\operatorname{ctg}\delta\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{\sin\vartheta}{\sin\delta}\,\frac{\partial}{\partial\beta} - \cos\vartheta\frac{\partial}{\partial\delta}\right). \qquad (59)$$

Only one of these three expressions need be obtained by calculation; the other two can be obtained from

[7] S. I. Pekar, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 579 (1954).

the three commuting relations

$$L_1^v L_2^v - L_2^v L_1^v = iL_3^v; \qquad (60)$$

$$L_2^v L_3^v - L_3^v L_2^v = iL_1^v; \; L_3^v L_1^v - L_1^v L_3^v = iL_2^v.$$

One can prove the relations (60) without resorting to concrete expressions for the operators (57)-(59), if we first prove the analogous relations for L_{ik}^q:

$$L_1^q L_2^q - L_2^q L_1^q = iL_3^q; \qquad (61)$$

$$L_2^q L_3^q - L_3^q L_2^q = iL_1^q; \; L_3^q L_1^q - L_1^q L_3^q = iL_2^q.$$

The latter are proved by means of simple substitution in Eq. (61) of L_{ik}^q in the form (55) and division into products of integrals of sums of the form $\sum_{\vec{\varkappa}} \chi_{\vec{\varkappa}}(\mathbf{r})\chi_{\vec{\varkappa}}(\mathbf{r}') \quad \delta(\mathbf{r}-\mathbf{r}')$. Inasmuch as the commutation relations of the form (61), as is known, are valid even for L_i and $^1/_2\,\sigma_i$, Eqs. (60) follow at once, since L_i, L_i^q and σ_i commute amongst themselves.

The operator of the isotopic spin of the system

$$T_{\alpha\beta} = \int [\varphi_\alpha \pi_\beta - \varphi_\beta \pi_\alpha]\, dV + \frac{1}{2}\tau_{\alpha\beta};\; \tau_{12} \equiv \tau_3 \ldots \qquad (62)$$

commutes with the operator of the energy of the system and with the operators L_{ik}. Consequently, the isotopic spin is an integral of the motion. With the help of Eq. (1) of I, and also Eq. (3), we get

$$T_{\alpha\beta} = T_{\alpha\beta}^q + T_{\alpha\beta}^v + \frac{1}{2}\tau_{\alpha\beta};$$

$$T_{\alpha\beta}^q = -i\sum_{\vec{\varkappa}}\left(q_{\alpha\vec{\varkappa}}\frac{\partial}{\partial^* q_{\beta\vec{\varkappa}}} - q_{\beta\vec{\varkappa}}\frac{\partial}{\partial^* q_{\alpha\vec{\varkappa}}}\right); \qquad (63)$$

$$T_{\alpha\beta}^v = -i\sum_{\vec{\varkappa}j}\left[q_{\alpha\vec{\varkappa}}\frac{\partial v_j}{\partial q_{\beta\vec{\varkappa}}} - q_{\beta\vec{\varkappa}}\frac{\partial v_j}{\partial q_{\alpha\vec{\varkappa}}}\right]\frac{\partial}{\partial v_j}.$$

In the calculation of $T_{\alpha\beta}^v$, the expressions $\partial v_j / \partial q_{\alpha\vec{\varkappa}}$ are put in the form of Eqs. (67) - (69) of I. As a result of the use of Eq. (1) of I, we obtain the expression in the form

$$\int\left[\varphi_\alpha \hat{\omega}^n \frac{\partial \varphi_\beta^v}{\partial v_i} - \varphi_\beta \hat{\omega}^n \frac{\partial \varphi_\alpha^v}{\partial v_i}\right] dV. \qquad (64)$$

In the calculation of these integrals, φ_α is replaced by φ_α^v, and for the latter, use is made of Eqs. (60) and (38) of I. With the aid of Eqs. (63), the integrals (64) reduce to expressions of the form:

$$\frac{I^{(n)}}{2}\sum_\nu\left[\overline{\tau_\alpha \sigma_\nu}^{\psi^v}\frac{\partial}{\partial v_i}\overline{\tau_\beta\sigma_\nu}^{\psi^v} - \overline{\tau_\beta\sigma_\nu}^{\psi^v}\frac{\partial}{\partial v_i}\overline{\tau_\alpha\sigma_\nu}^{\psi^v}\right], \qquad (65)$$

which are calculated directly. As a result, we obtain

$$T_1^v \equiv T_{23}^v \equiv -T_{32}^v = -i\left(\operatorname{ctg}\delta\cos\beta\frac{\partial}{\partial\beta} + \frac{\cos\beta}{\sin\delta}\frac{\partial}{\partial\vartheta} + \sin\beta\frac{\partial}{\partial\delta}\right); \qquad (66)$$

$$T_2^v \equiv T_{31}^v \equiv -T_{13}^v = i\partial/\partial\beta; \qquad (67)$$

$$T_3^v \equiv T_{12}^v \equiv -T_{21}^v = -i\left(\operatorname{ctg}\delta\sin\beta\frac{\partial}{\partial\beta} + \frac{\sin\beta}{\sin\delta}\frac{\partial}{\partial\vartheta} - \cos\beta\frac{\partial}{\partial\delta}\right). \qquad (68)$$

The components of the operators T_i, T_i^q, T_i^v and $(1/2)\tau_i$ satisfy the general commutation relations of the form (60).

Comparison of L_i^v and T_i^v shows that they interchange with one another upon substitution of $\vartheta \rightleftarrows \beta$. This means that the eigenfunctions of these operators are identical, and their eigenfunctions are obtained from one another by exchange of the angles $\vartheta \rightleftarrows \beta$. Direct calculation shows that

$$L^{v^2} \equiv \sum_{i=1}^{3} L_i^{v^2} = T^{v^2} \equiv \sum_{i=1}^{3} T_i^{v^2} = \frac{2\,(I^{(n)})^2}{I^{(2n)}}\hat{\mathscr{H}}, \qquad (69)$$

where $\hat{\mathscr{H}}$ is the operator of the energy of rotational motion, determined by Eq. (13).

It is interesting to note that the operators L_i^v and T_i^v coincide with the operators of infinitely small rotations in three dimensional space, while the Euler angles of these rotations are identified with the angles ϑ, β and δ. These operators of infinitely small rotations have been suitably investigated in the theory of the representation of three dimensional rotation groups (see Ref. 8). The eigenfunctions of these operators were investigated and a series of interesting relations among them were established.

It should be emphasized that the operators L_{ik} $T_{\alpha\beta}$ represent operators of infinitely small rotations, respectively, of ordinary and isotopic space, and therefore commute. The operators L_{ik}^v and $T_{\alpha\beta}^v$ are operators of infinitely small rotations of the same three dimensional space, in which the Euler angle belongs to ordinary space, the Euler angle β to isotopic space, and the σ angle $\delta = \varphi + \gamma$ is related to both spaces. Nontheless, L_{ik}^v and $T_{\alpha\beta}^v$ also commute. The operators L_{ik}^v and $T_{\alpha\beta}^v$ differ from those operators of the zero field introduced by Pauli and Dancoff[5], because the latter are operators of rotation corresponding to ordinary and isotopic spaces. However, the operators L_{ik}^0 and $T_{\alpha\beta}^0$ also satisfy a condition analogous to Eq. (69).

We now proceed to the determination of such eigenfunctions of the energy operator which are related to the lowest level ($j = 1/2$), which would simultaneously be eigenfunctions of the operators L_{12} and T_{12}. The latter is related to the charge operator of the system $\hat{e}$ by the expression

$$\hat{\varepsilon} = T_{12} + 1/2. \tag{70}$$

First, let us consider the result of the action of the operators L_{12} and T_{12}, which are defined by Eqs. (55) and (63), on the wave function of the ground state of the system:

$$\Psi_{2s,2p}^1 = \psi^v V_{2s,2p}^1 (\vartheta\beta\delta) \exp\left\{-\frac{1}{2}\sum_{\alpha\vec{\varkappa}}\left[(q_{\alpha\vec{\varkappa}} - q_{\alpha\vec{\varkappa}}^v)^2 + \frac{1}{2}\ln\pi\right]\right\}, \tag{71}$$

where $V_{2s,2p}^1$ is defined by Eq. (28) and ψ^v by Eq. (41). In the application of the operators L_{12}^q and T_{12}^q to $\Psi_{2s,2p}^1$, we are obliged to differentiate only the exponential factor in Eq. (71). We can neglect the result of this differentiation, inasmuch as we have previously neglected terms of the form

$$\sum_{\vec{\varkappa}} (q_{\alpha\vec{\varkappa}} - q_{\alpha\vec{\varkappa}}^v) q_{\beta\vec{\varkappa}}^v = \int (\varphi_\alpha - \varphi_\alpha^v) \hat{\omega} \varphi_\beta^v dV. \tag{72}$$

The result of the action of operators L_{12}^v and T_{12}^v on the function $V_{2s,2p}^1$ has the following form:

$$\begin{aligned} &L_{12}^v V_{11}^1 = \frac{i}{2} V_{-11}^1; && T_{12}^v V_{11}^1 = \frac{i}{2} V_{1-1}^1; \\ &L_{12}^v V_{1-1}^1 = \frac{i}{2} V_{-1-1}^1; && T_{12}^v V_{-11}^1 = \frac{i}{2} V_{-1-1}^1; \\ &L_{12}^v V_{-11}^1 = -\frac{i}{2} V_{11}^1; && T_{12}^v V_{1-1}^1 = -\frac{i}{2} V_{11}^1; \\ &L_{12}^v V_{-1-1}^1 = -\frac{i}{2} V_{1-1}^1; && T_{12}^v V_{-1-1}^1 = -\frac{i}{2} V_{-11}^1. \end{aligned} \tag{73}$$

With the help of these formulas, and keeping in mind Eq. (8) of I and Eq. (41), we get

$$L_{12}^v \psi^v = -\frac{1}{2}\sigma_{12}\psi^v; \quad T_{12}^v \psi^v = -\frac{1}{2}\tau_{12}\psi^v. \tag{74}$$

From this it is evident that the action of the operators L_{12} and T_{12} on $\Psi_{2s,2p}^1$ reduces to the action of L_{12}^v and T_{12}^v on the factor $V_{2s,2p}^1$ in Eq. (71).

The problem consists in looking for such linear combinations of the four functions $V_{2s,2p}^1$, which would simultaneously be eigenfunctions of the operators L_{12}^v and T_{12}^v. These linear combinations, fortunately, exactly coincide with the components ψ^v [see Eq. (41)], because Eqs. (74) show that each of the four components ψ^v are eigenfunctions of the operators L_{12}^v and T_{12}^v, inasmuch as σ_{12} and τ_{12} are diagonal.

Thus the eigenfunctions of the charge and spin operators of the system are given by an expression of the form (71), in which we must put the appropriate linear combination in place of the function $V_{2s,2p}^1$. This combination can be taken from the table below:

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[8] I. M. Gel'fand and Z. Ia. Shapiro, Usp. matemat. nauk 7, 1 (1952).

Projection of the spin of the system L_{12}	Charge of the system $(T_{12}+{}^1/_2)$	
	0 (neutron)	1 (proton)
$-{}^1/_2$	$\frac{1}{\pi\sqrt{2}}C_1^v$	$\frac{1}{\pi\sqrt{2}}C_3^v$
$+{}^1/_2$	$\frac{1}{\pi\sqrt{2}}C_2^v$	$\frac{1}{\pi\sqrt{2}}C_4^v$

The C_k^v are defined by Eq. (35) of I.

The calculated eigenvalues of the charge and spin projections of the system consisting of a nucleon that strongly interacts with the vacuum vibrations of the meson field, coincide with the observed values.

In subsequent papers we shall consider, on the basis of the theory developed above, the scattering of π mesons on nucleons, the magnetic moments of nucleons, quasi-statistic nuclear forces and other phenomena.

Translated by R. T. Beyer
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Meson Component of the Cosmic Radiation at an Altitude of 3200 m Above Sea Level

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The momentum spectrum of μ-mesons was measured in the momentum range $0.4 \leq p \lesssim 14$ bev/c at an altitude of 3200 m above sea level. The ratio of the number of protons to μ-mesons at this altitude was determined. The ratio of positive to negative μ-mesons was found as a function of momentum.

In 1951 we began an investigation of the proton and meson components of the cosmic radiation at an altitude of 3200 m above sea level. For this purpose there was constructed a special magnetic spectrometer, a description of which was given in Ref. 1. The proton component was determined as a result of the investigation. The actual shape of the spectrum was determined for the meson component. By using a series of improvements, the momentum of the particles in the magnetic field was determined with great precision.[1] The relative error in determining the momentum of the particles is equal to

$$\sigma = [(0.035\,p)^2 + (0.018\,/\,\beta)^2]^{1/2}. \qquad (1)$$

Here and everywhere below the momentum p of the particles is measured in units of bev/c; β is the velocity of the particles, measured in units of the velocity of light.

1. Protons in the Hard Component

In a second variation of the determination of Ref. 1, in which there was no lead above the magnet and under the magnet was located $x_1 = 45.2$ gm/cm^2 of lead and $x_2 = 139$ gm/cm^2 of [1]copper, then simultaneously with particles which were stopped in the absorbers, we also detected those particles which had gone through the complete system of absorbers. In contrast with the former particles the latter are called the "hard" component. The hard component consists principally of μ-mesons with momenta greater than 0.370 bev/c (i.e., kinetic energy $E \geq 260$ mev), and a certain

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[1] N. M. Kocharian, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 160 (1955); Soviet Phys. JETP 1, 128 (1955).