of the phase magnetization density, which is assymetric in p-space is small.** In first approximation equation (7) can be written in the following form:

$$F(p^2)\left(\frac{\mathbf{p}}{m}\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{M} + \mathbf{M}e\left(\mathbf{E}\frac{\partial}{\partial \mathbf{p}}\right)F = -\mathbf{J}_\tau(\vec{\sigma}_0) - \mathbf{J}_\tau(\vec{\Sigma}). \tag{8}$$

The form of the right hand side of (8) is possible because of the linearization of the integral J_τ. The approximations made to get (8) from (7) are in any case legitimate if $J_\tau(\vec{\Sigma})$ is much larger than all the other members containing $\vec{\Sigma}$ in the left hand side of (7). This condition is realized in the region of small deviations from homogeneity, of small frequencies and weak fields. Furthermore, it is assumed that $J_\tau(\vec{\sigma}_0) \lesssim J_\tau(\Sigma)$. The latter condition is satisfied in spite of the inequality $\sigma_0 >> \Sigma$, because $J_\tau(\vec{\sigma}_0)$ is a quantity assymetric in momentum space.

The solution of (8) can be written in the form

$$\Sigma_i = \alpha_{ikl}\frac{\partial M_k}{\partial r_l} + \beta_{ikl}M_k E_l + \gamma_{ik}M_k, \tag{9}$$

where α, β and γ are tensors depending only on the vector **p**. Neglecting terms of the form $E_i\partial M_i/\partial r_k$, we get the following equation for the space density of magnetization:

$$\frac{\partial M_i}{\partial t} - c_1\Delta M_i - c_2\frac{\partial^2 M_k}{\partial r_i \partial r_k} + \frac{\beta}{\hbar}[\mathbf{M}\mathbf{x}\mathbf{H}]_i = -\frac{1}{U}(\mathbf{M}-\mathbf{M}_0)_i. \tag{10}$$

Note that the approximation $J_\tau(\vec{\Sigma}) = \vec{\Sigma}/\tau$ which satisfies (3) because of the relation (5), gives the following values for the coefficients c_1 and c_2:

$$c_1 = \frac{1}{3}\int dp\,\frac{p^2}{m^2}\tau F,\quad c_2 = 0. \tag{11}$$

For metallic electrons in a degenerate state, F differs from zero only for momenta lying on the Fermi surface, hence:

$$c_1 = {}^1/_3(p_0/m)^2\tau(p_0) = {}^1/_3 v_0\Lambda,$$

where v_0 is the electron velocity on the Fermi surface, and $\Lambda = v_0 t$ is the mean free path. In this case Eq. (10) takes the form

$$\frac{\partial \mathbf{M}}{\partial t} - \frac{1}{3}v_0\Lambda\Delta\mathbf{M} + \frac{\beta}{\hbar}[\mathbf{M}\mathbf{x}\mathbf{H}] = -\frac{1}{U}(\mathbf{M}-\mathbf{M}_0). \tag{12}$$

Equation (12) is an equation for the magnetization density discovered by Dyson[2] in his theory of electron diffusion in metals.

To finish, let us study the limiting conditions for which one has to solve Eqs. (10) and (12). For this purpose, let us consider the limiting conditions for the phase magnetization density $\vec{\sigma}$. Let **n** be the normal to the surface of the body. Denote $\vec{\sigma}$ by $\vec{\sigma}^{(1)}$ for pn> 0 and by $\vec{\sigma}^{(2)}$ for pn< 0. The conservation law of the magnetic moment of the particle during the collision with the surface can then be written in the form

$$\vec{\sigma}^{(1)}(-\mathbf{pn};\,0) = \vec{\sigma}^{(2)}(\mathbf{pn};\,0). \tag{13}$$

Here, the zero value of the argument corresponds to a magnetization of the surface of the body. From condition (13) it follows that

$$\int d\mathbf{p}\,(\mathbf{pn})\,\vec{\sigma}(\mathbf{p},\,0) = 0. \tag{14}$$

In particular, for the linear approximation of $J_\tau(\vec{\Sigma})$ mentioned above, the relation (14) gives (n grad) M=0 which corresponds to the limiting condition used by Dyson[2] when he neglected the surface relaxation.

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*For sake of simplicity, we shall assume a quadratic dependence of the kinetic energy on the momentum. Generalization to any case is evident. The derivation of equation (1) is absolutely identical to the derivation of the equation for the quantum distribution function-the density matrix in the mixed representation (see for instance Ref. 1)

** When Σ is neglected, Eq. (6) becomes the well known Bloch[3] equation for $T_1 = T_2 = U$.

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[2] F. J. Dyson, Phys. Rev. 98, 349 (1955).

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The Conservation of Isotopic Spin and the Scattering of Antinucleons by Nucleons

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IN recent times, evidence for the conservation of isotopic spin in the interaction of nucleons with nucleons has been obtained. With the discovery of the antiproton[1], it becomes expedient to determine the consequences of the conservation of isotopic spin in the interaction of nucleons with antinucleons In particular, such calculations can enable us to check the hypothesis itself of the conservation of isotopic spin in the interaction of nucleons (neutrons n, protons p) and antinucleons (antiprotons $\overline{p}$, antineutrons $\overline{n}$). Let us consider collisions in which mesons do not occur:

1) $\overline{p}+p\rightarrow\overline{p}+p$ (elastic scattering of antiprotons by protons)

2) $\overline{p}+n\rightarrow\overline{p}+n$ (elastic scattering of antiprotons by neutrons)

3) $\overline{p}+p\rightarrow\overline{n}+n$ (conversion of a proton and antiproton into a neutron and antineutron)

4) $\overline{n}+n\rightarrow\overline{n}+n$ (elastic scattering of antineutrons by neutrons)

5) $\overline{n}+p\rightarrow\overline{n}+p$ (elastic scattering of antineutrons by protons)

6) $\overline{n}+n\rightarrow\overline{p}+p$ (conversion of an antineutron and neutron into an antiproton and proton).

From charge symmetry, if we neglect electromagnetic forces:

$$\sigma_1=\sigma_4;\ \sigma_2=\sigma_5;\ \sigma_3=\sigma_6.$$

Thus only processes 1, 2 and 3 need be considered.

Keep in mind, that all the particles entering into the reactions have isotopic spin $T=\frac{1}{2}$ and the following isotopic spin projections, T_3:

p	n	$\overline{p}$	$\overline{n}$
$^1/_2$	$-^1/_2$	$-^1/_2$	$^1/_2$

The nucleon-antinucleon system has, in general, two isotopic states: $T=1$ and $T=0$. If we denote the scattering amplitudes in these states respectively by f and g, then it is easy to see that the differential cross sections of reactions 1-3 are expressed in terms of f and g as follows:

$$\sigma_1=\tfrac{1}{4}|f+g|^2;\quad \sigma_2=|f|^2;\quad \sigma_3=\tfrac{1}{4}|f-g|^2.$$

Note that f and g are complex functions of the initial and final momenta of the particles and their spins. Thus the three cross sections are expressed in terms of three independent quantities: the moduli of the amplitudes, $|f|$ and $|g|$, and their relative phase. It follows from this, that, in general, there are no equations linking these cross sections.

However, in the scattering of antinucleons by nucleons at high energies, one can consider that $|f-g|\ll|f|$. This has to do with the fact that σ_3 is the cross section of a peculiar inelastic process (double "overcharging"). Such an "inelastic" process must have a significantly smaller probability than that of "genuine" inelastic processes. Considering nucleons and antinucleons as "gray" bodies (in particular cases they will be black bodies) of radius $\rho\sim10^{-13}$ cm, one gets an elastic cross section (mainly of a diffraction type) of the order of the inelastic (i.e., of the order of the annihilation cross section). Therefore, from the fact that the process $\overline{p}+p\rightarrow\overline{n}+n$ embraces a small portion of all the inelastic processes, it follows that its cross section is small compared to the elastic scattering $\overline{p}+p\rightarrow\overline{p}+p$ ($\overline{p}+n\rightarrow\overline{p}+n$), but this means that the inequality $|f-g|\ll|f|$ is satisfied.

On the basis of this result, we obtain $\sigma_1=\sigma_2$, i.e., the differential cross sections of the elastic scattering of antiprotons by protons and by neutrons are equal. Applying this to the forward direction, 0°, and recalling the connection between the imaginary part of the scattering amplitude and the total cross section σ_t, we find

$$\sigma_t(\overline{p}+p)\approx\sigma_t(\overline{p}+n).$$

Together with the equality of the elastic cross sections, this means that the total annihilation cross sections for the collision of high energy antiprotons (antineutrons) with protons and with neutrons are equal.

In conclusion, I thank L. D. Landau for an interesting discussion.

Translated by M. Rosen
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The Conservation of Isotopic Spin and the Cross Section of the Interaction of High-Energy π-Mesons and Nucleons with Nucleons

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IN reference 1, the consequences of the conservation of isotopic spin in the scattering of antinucleons by nucleons were considered. The fact was used that the charge exchange cross section for the collision of antinucleons with nucleons is much less than the cross section for the other inelastic processes. For the scattering of π-mesons and nucleons, there are analogous results, justified at least for those energies at which inelastic reactions (creation of mesons) occur with high probability.

In the scattering of protons, the following reactions are possible: