$$\gamma_k = \beta_k + \beta'_k, \quad \bar{\gamma}_k = \beta_k - \beta_k. \tag{62}$$

It must be pointed out that the four-dimensional quantities of the type $A = A_0 + \vec{\gamma}\mathbf{A}$ are not real quaternions, similar to Hamilton's quaternions in three-dimensional space, because of the special treatment of the time. This is due to the pseudo-Euclidean character of the Minkowski world. For the same reason the result of the multiplication of the Cauchy-Riemann equations (53) is a hyperbolic equation.

In this way the present formulation of the theory may, with good justification, be called a pseudo-quaternion theory, at least in the case of non-static fields.

CONCLUSIONS

1. There exist relations which make it possible to use anticommuting matrices, not only in the theory of mixtures of meson fields, but also for "pure" fields.
2. It is possible to replace the sixteen-dimensional reducible representation of the Dirac algebra by an eight-dimensional one.
3. One may use, as kinematic matrices of photon or vector meson theory, the reflection matrices of electron theory, γ_4, $\gamma_1\gamma_2\gamma_3$ and $\gamma_1\gamma_2\gamma_3\gamma_4$. The connection of these with the Tamm matrices has been found.

Translated by R. Peierls
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SOVIET PHYSICS JETP VOLUME 3, NUMBER 2 SEPTEMBER, 1956

Interrelation between the Anisotropy of the Hall Effect and the Change in Resistance of Metals in a Magnetic Field. I. Investigation of Zinc

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(Submitted to JETP editor January 15, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 262-271 (1956)

The dependence of the resistance and the Hall effect for zinc on the magnitude of the angle between the axis of symmetry of the sixth order and the magnetic field is investigated for magnetic fields of up to 25,000 oersteds and for temperatures of 4 and 20°K. The possibility of explaining the observed regularities within the framework of present day theory is considered.

INTRODUCTION

A VERY large anisotropy of the resistance in a magnetic field has been observed for a number of metals. The resistance of a single crystal in a transverse magnetic field may change by 15 to 30 times when it is turned about an axis parallel to the current. Such a strong anisotropy, observed for certain orientations of single crystals of gallium, zinc, cadmium and tin[1-5], is unexpected, since in the absence of a magnetic field the anisotropy in the conductivity is small--of the order of tenths of a percent.[6]

The strong anisotropy is manifested by the occurrence of deep narrow minima, which we may call "anomalous minima," in the resistance, as plotted against angle of rotation. The position of the crystal which corresponds to the appearance of such an anomalous minimum is distinguished, as a rule, not only by the magnitude of the resistance, but also by the character of the dependence of the resistance on the magnetic field. In large fields the dependence on the field is not found to be quadratic, but weaker, approximately linear.[2-5]

It has previously been shown[7-9] that the delayed growth of the resistance in a magnetic field is

[1] W. J. de Haas and J. W. Blom, Physica 2, 952 (1935).

[2] B. G. Lazarev, N.M. Nakhimovich and E. A. Parfenova, J. Exptl. Theoret. Phys. (U.S.S.R.) 9, 1169 (1939).

[3] E. Justi, J. Kramer and R. Schulze, Physik. Z. 41, 308 (1940).

[4] E. S. Borovik, Dokl. Akad. Nauk SSSR 69, 767 (1949).

[5] E. S. Borovik, J. Exptl. Theoret. Phys. (U.S.S.R.) 23, 91 (1952).

[6] Halis, *Collection of Physical Constants*, ONTI, Moscow-Leningrad, 1937.

[7] E. S. Borovik, J. Exptl. Theoret. Phys. (U.S.S.R.) 23, 83 (1952).

[8] E. S. Borovik, Dokl. Akad. Nauk SSSR 75, 639 (1950).

[9] E. S. Borovik, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 355 (1954).

connected with the influence of the large Hall field. We might expect that the delayed growth of the resistance for orientations corresponding to anomalous minima would also be connected with the influence of the Hall field. Hence an investigation of the anisotropy of the Hall effect and of its relation to the anisotropy of the resistance in a magnetic field was undertaken. Zinc, a metal with a large anisotropy in its resistance[2], was chosen as one of the first objects of the investigation.

The anisotropy in the Hall effect in zinc was investigated for the temperatures 300 and 78 °K by Noskov[10] and Lazarev[11], and for the temperature 20°K by Logan and Marcus.[12] The Hall effect in zinc has been investigated[13] at temperatures down to 4°K for one orientation of a single crystal. A brief communication of our results on the investigation of the anisotropy of the Hall effect in zinc and tin at 4°K has been published.[14,15]

1. METHODS OF MEASUREMENT AND CHARACTERISTICS OF THE OBJECTS OF THE INVESTIGATION

In works on the investigation of the anisotropy in the Hall effect either the magnitude of the Hall field for directions of the magnetic field parallel to the principal directions of the crystal is investigated or else, if the crystal is turned during the measurements, only one of the components of the Hall field is measured.[10-12] These methods may be considered satisfactory in the region of small magnetic fields.

In the region of large magnetic fields the situation becomes more complex. In the case, for example of the variation of the resistance in a magnetic field, it is clear that it is quite insufficient to know the magnitude of the effect for the principal directions in order to determine it for an arbitrary direction. One might expect that a similar situation would occur for the Hall field.

In order to obtain complete information concerning the magnitude and direction of the electric field in a metal located in a magnetic field when a current is passed through it, it is in general necessary to measure all three components of this electric field, E_x, E_y, E_z. The Hall field, in accordance with the mechanism by which it arises, must be perpendicular to the current but not necessarily perpendicular to the magnetic field. The components of the Hall field must change sign when the direction of the magnetic field is reversed. In order to measure both components (E_y and E_z) of the Hall field and the electric field (E_x) in the direction of the current, the following setup was used.

Using a cylindrically shaped sample, we mounted two pairs of electrodes on the surfaces cut perpendicular to the axis of the cylinder. The electrodes were placed approximately at the ends of two mutually perpendicular diameters of these cuts. With this arrangement, a measurement of the difference in potentials for the two pairs of electrodes permits the determination of both components of the transverse electric field. In order to determine the component, E_x, of the electric field parallel to the current, two more electrodes were mounted at distances of ~ 1/4 of the length of the crystal from its ends. The electrodes were made from thin wire (d=0.03-0.05 mm) and were attached by spot welding. The positions of the transverse electrodes were determined by measurements made while the crystal was rotated under a microscope.

The samples, placed in a cooling liquid, could be turned about their longitudinal axis, perpendicular to the magnetic field.

Later, when we had confirmed the obvious fact that for a direction of the magnetic field parallel to one of the principal directions of the crystal the Hall field perpendicular to the magnetic field was zero ($E_z = 0$), we used a more convenient electrical method of determining the positions of the Hall electrodes. If the pairs of Hall electrodes are situated approximately along mutually perpendicular principal directions in the crystal, then from the results of measurements of the difference in the Hall potentials for both pairs of electrodes and their possible combinations in pairs for two positions of the crystal, with the magnetic field parallel to a principal direction, we may easily find the position of the electrodes corresponding to the condition that $E_z = 0$.

The setting up of a magnetic field parallel to a principal direction was done in accordance with the magnitude of the resistance in the magnetic field. The great sharpness of the minima which occur on rotation easily allows the setting up of the required direction with an accuracy of as little as 20 min. The error in the magnitude of the Hall

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[10] M. M. Noskov, J. Exptl. Theoret. Phys. (U.S.S.R.) 8, 717 (1938).

[11] B. Lazarev, Nature 134, 139 (1934).

[12] J. K. Logan and J. A. Marcus, Phys. Rev. 88, 1234 (1952).

[13] E. S. Borovik, Dokl. Akad. Nauk SSSR 70,601 (1950).

[14] E. S. Borovik, Dokl. Akad. Nauk SSSR 95, 485 (1954).

[15] E. S. Borovik, Dopovidi A. N. URSR 4, 354 (1955).

field on account of the inaccuracy in the determination of the position of the electrodes is 1-3%. Differences in the Hall potentials were measured by a low resistance potentiometer with a scale of 10^{-8} volt per division. Secondary effects were excluded by reversing the direction of the current in the sample and the direction of the magnetic field.

The single crystal of Zn-1 0 which was investigated had the form of a cylinder of elliptical cross section with transverse dimensions of 0.9 and 1.00 mm and a length (between the potential leads) of 9 mm. The axis of symmetry of the sixth order was perpendicular to the longitudinal axis of the sample, while one of the binary axes was parallel to it. The deviation from the indicated directions of orientations was less than the error in the measurements. The accuracy of the determination of orientation was within 1-2°.

The following are the data for the dependence of the resistance of Zn-1 0 on the temperature:

T°K:	20.38	4.22	1.81
r_{0T}/r_{00}:	$6.06\cdot10^{-3}$	$5.30\cdot10^{-4}$	$5.16\cdot10^{-4}$

(r_{00} is the resistance for 0°C; r_{0T} is the resistance for the temperature of the experiment and for $H=0$.)

2. RESULTS OF THE MEASUREMENTS

In the sample of Zn-10 the axis of symmetry of the sixth order was perpendicular to the longitudinal axis of the sample. If we direct the x axis along the longitudinal axis of the crystal, according to the direction of the current in it, then when the sample is turned about its longitudinal axis the axis of symmetry of the sixth order will turn in the y,z plane. We designate as ψ the angle between the axis of symmetry of the sixth order and the magnetic field directed along the z axis.

The conditions of the experiment for zinc in the investigated temperature interval correspond to the region of essential change in the resistance in a magnetic field. Hence we will characterize the Hall effect by the magnitudes E_y/E_x and E_z/E_x[10,13]. Figure 1 shows the dependence of these quantities on the angle ψ between the axis of symmetry of the sixth order and the magnetic field for $T=4.22°$ K and a field H = 20, 200 oersteds. The variation of E_y/E_x is shown by curve 1 and that of E_z/E_x by curve 2. Curve 3 of the same figure shows, for comparison, the variation in resistance. Curve 3 is similar to curves obtained in reference 2 for crystals of corresponding orientation.

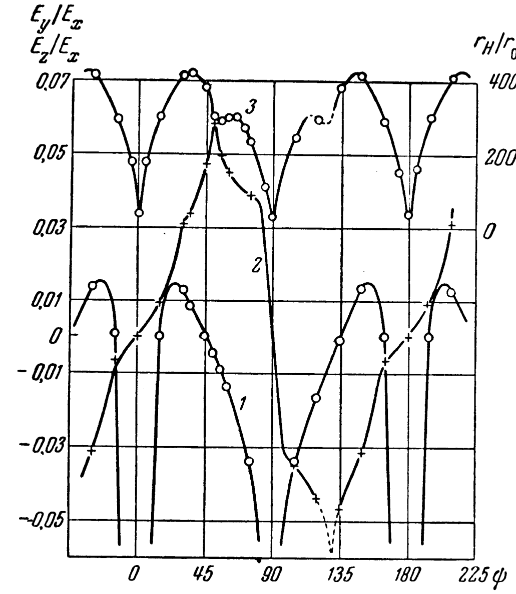

FIG. 1. Dependence of the Hall effect and the change in resistance on angle of rotation of the sample for Zn-10. ψ is the angle between the axis of symmetry of the sixth order and the direction of the magnetic field, $T=4.22$ ° K, $H=20{,}200$ oersteds. Curve 1 gives E_y/E_x; Curve 2, E_z/E_x; curve 3, r_H/r_{0T}.

The absolute magnitudes of the Hall field have sharp maxima for values of the angle ψ corresponding to the principal minima in the resistance, that is, for $\psi=0$ and 90°, and a less sharply defined maximum for the secondary minimum in the resistance at $\psi=55$°.

The Hall field vector is perpendicular to the direction of the magnetic field only when it is directed along one of the main crystallographic directions. The variation in the Hall field components E_y/E_x and E_z/E_x when the crystal is rotated is shown in Figs. 2 and 3 for a number of values of the magnetic field. Since the diagram for Zn-10 is sufficiently symmetrical (Fig. 1), we shall, in what follows, limit ourselves to a study of the interval in ψ lying between 0 and 90°.

It is curious to note that the Hall effect for a crystal with an axis of sixth order parallel to the current is positive down to helium temperatures, but for crystals with an orientation such as obtains in Zn-1 0 it is positive to $T=20.4$° K[11,12]. For Zn-10 at $T=4.22$ ° K in a very high field (20, 200 oersteds) the magnitude of E_y/E_x is positive only in a narrow interval of angles ($\psi=15$-45°). The magnitude of the Hall field for $\psi=0$°, which is

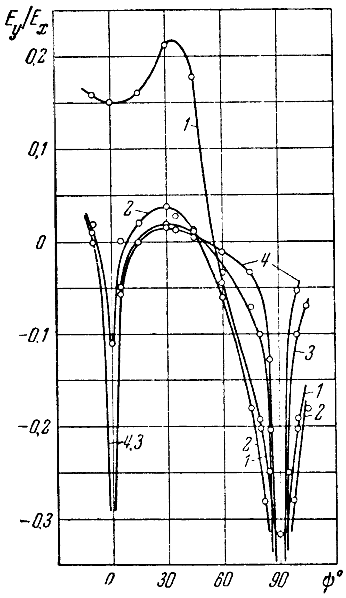

FIG. 2

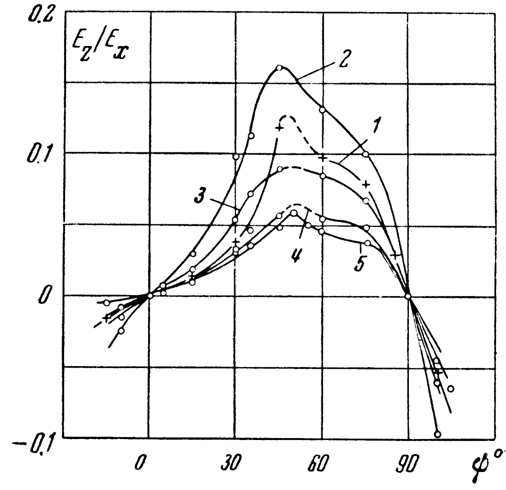

FIG. 3

FIG. 2. Dependence of the Hall field on angle of rotation for Zn-10. E_y/E_x for T=4.22 °K. Curve 1 is for H=2350, curve 2 for H=5000, curve 3 for H=9850, and curve 4 for H=15, 200 oersteds. At $\psi=0°$, E_y/E_x=-0.465 for curve 3 and E_y/E_x=-0.56 for curve 4. At ψ=90°, E_y/E_x= -0.54 for curve 2, E_y/E_x=-0.575 for cuve 3 and E_y/E_x=-0.515 for curve 4.

FIG. 3. Dependence of the Hall field on angle of rotation for Zn-10. E_z/E_x for T=4.22 °K. Curve 1 is for H=2350, 2 for H=5000, 3 for H=9850, 4 for H=15,200, and 5 for H=20,200 oersteds.

negative for high fields, first decreases rapidly in absolute value as the magnetic field is decreased, then changes sign and at $H = 2350$ oersteds already has a rather large positive value. For H = 2350 oersteds the magnitude of E_y/E_x is positive in the greater part of the angular interval.

The curves for the longitudinal component of the Hall field E_z/E_x (Fig. 3) give, as they should, zero values for directions of the magnetic field coinciding with the principal directions in the crystal: $\psi = 0°$ and $\psi = 90°$. The magnitude of E_z/E_x has a maximum in the region ψ=40-50°. As the magnitude of the magnetic field is decreased, the value of E_z/E_x for all orientations first increases, then passes through a maximum and begins to decrease.

The complex character of the dependence of the Hall field on the angle ψ shows that in a study of the anisotropy of the Hall effect in zinc we cannot limit ourselves to a study of the magnitude of the effect in the principal directions, but must obtain the entire picture of its variation with angle of rotation.

Diagrams showing the variation of the resistance of Zn-10 with angle of rotation in magnetic fields $H <$ 20,000 oersteds are presented in Fig. 4. The curves in Fig. 4 are similar to the curves obtained in Ref. 2 for crystals of the same orientation, but are distinguished from them by a rather greater value of the increase in the resistance, in conformity with the smaller value of the residual resistance of our sample.

Figure 5 shows the variation in the magnitude of the Hall effect and of the resistance with the angle ψ for Zn-10 at 20.38° K and H = 20,200 oersteds. Here E_y/E_x is positive for almost the entire angular interval and only becomes negative close to 90°. The conditions of the experiment from zinc at T = 20.4 °K still correspond to a region of essential increase in the resistance in a magnetic field ($\Delta r/r_{0T}$=2.5 -5). The character of the angular dependence of the Hall field is still sufficiently complicated that the study of the Hall effect cannot be limited to the principal directions.

The dependence of the Hall field and of the change in resistance in a magnetic field on the magnitude of the field is shown in Fig. 6 for T = 20.38 ° K. The Hall field for $\psi = 0°$ reaches its

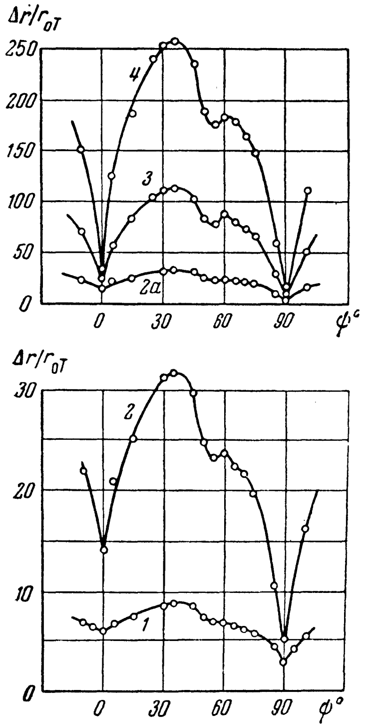

FIG. 4.

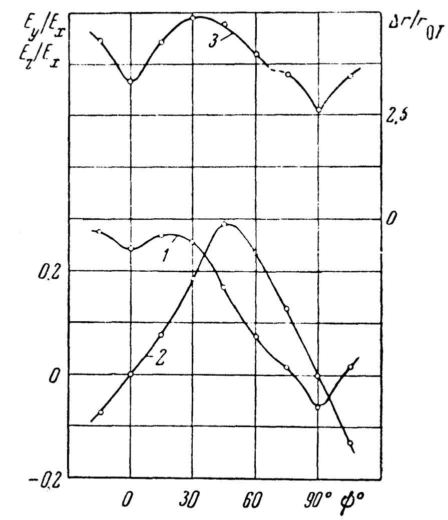

FIG. 5

FIG. 4. Dependence of the change in resistance of Zn-10 in a magnetic field on the angle of rotation. $T = 4.22\,^{\circ}$ K. Curve 1 is for H=2350. 2 and 2a for H=5000, 3 for H= 9850 , and 4 for H= 15, 200 oersteds.

FIG. 5. Dependence of the Hall field and of the change in resistance on the angle of rotation for Zn-10. T=20.38 $^{\circ}$K, H=20,200 oersteds. Curve 1 gives E_y/E_x, 2 gives E_z/E_x, and 3 gives $\Delta r/r_{0T}$.

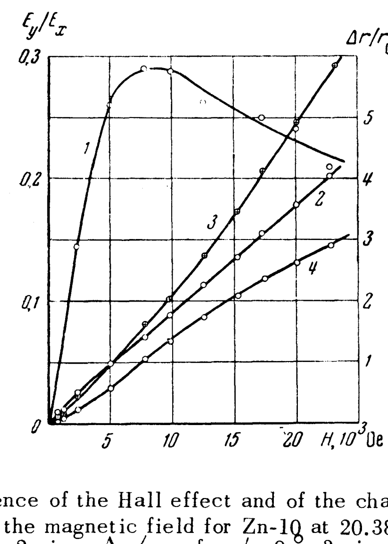

FIG. 6. Dependence of the Hall effect and of the change in resistance on the magnitude of the magnetic field for Zn-10 at 20.38 $^{\circ}$K. Curve 1 gives E_y/E_x for $\psi = 0^{\circ}$, 2 gives $\Delta r/r_{0T}$ for $\psi = 0^{\circ}$, 3 gives $\Delta r/r_{0T}$ for $\psi = 35^{\circ}$ 4 gives $\Delta r/r_{0T}$ for $\psi = 90^{\circ}$.

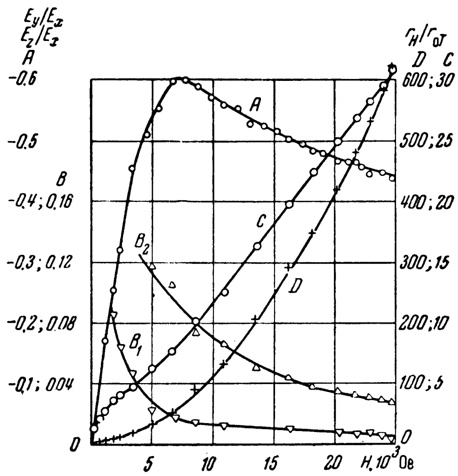

FIG. 7. Dependence of the magnitude of the Hall field and of the change in resistance on the magnitude of the magnetic field for Zn-10 at $T=4.22$ °K, Curve A gives E_y/E_x for $\psi=90$°, B_1 gives E_y/E_x for $\psi=35$°, B_2 gives E_z/E_x for $\psi=35$°, C gives r_H/r_{0T} for $\psi=90$°, D gives r_H/r_{0T} for $\psi=35$.° The scales for the curves are designated by the same letters.

maximum value $E_y/E_x=0.29$ for $H\approx 9000$ oersteds. The values measured for $\psi=90°$ had a large error and were not put on the graph. For $\psi=90°$ the Hall field changes sign at $H\approx 9\times10^3$ oersteds and becomes positive at small fields, so that for small fields the magnitude of $E_y/E_x > 0$ for all values of the angle ψ. The crossing of curves 3 and 2 of Fig. 6 at $H\approx 5000$ oersteds is also noteworthy. It indicates that the minimum at $\psi=90°$ on the diagram showing the variation of resistance with angle of rotation in a magnetic field disappears for $H < 5000$ oersteds.

Figure 7 shows the dependence of the magnitude of the Hall field and of the change in the resistance on the magnitude of the magnetic field for $T=4.22°$ K and for two values of the angle ψ. For an orientation of the hexagonal axes of the crystal perpendicular to the field ($\psi=90°$) the curve for the Hall field (curve A in Fig. 7) has the maximum which is typical of metals having equal numbers of holes and electrons.[15] At its maximum E_y/E_x has a large magnitude ($E_y/E_x\approx 0.6$), and after the maximum the Hall field falls slowly. The curve showing the variation of the resistance in a magnetic field is linear at large fields (curve c in Fig. 7).

For $\psi=35°$, corresponding to the maximum in the plot of resistance *versus* angle of rotation (Fig.1), the magnitude of the Hall field is characteristically much smaller. Cf. curves B_1 and B_2 of Fig. 7, in which we observe only the decreasing portion of the curve after the maximum, this maximum lying somewhere in the region of small fields. The curve for the change in resistance is almost quadratic. In a field of 25,000 oersteds the resistance is increased by more than 600 times its magnitude in the absence of field and exceeds by more than 20 times the value of the resistance at the same field for $\psi=90°$.

Figure 8 gives the results obtained for $\psi=0°$ (hexagonal axis parallel to the magnetic field) for temperatures of 4.22 and 1.81 °K. The curves for the change in resistance (curves 3 and 4) are similar to the curves obtained in references 2 and 16; the irregularities are obviously connected with quantum oscillations which occur at the particular values of the field involved. These quantum oscillations are most noticeable on the curves for the Hall field (curves 1 and 2). Just as for $\psi=90°$, the absolute magnitude of the Hall field is large (the maximum observed values of $E_y/E_x\approx 0.8$) and in the region of fields investigated E_y/E_x increases steadily with field. The change in sign of the Hall field at $H=3600$ oersteds is noteworthy.

Figure 8 also gives, for comparison, the dependence of the magnetic susceptibility on the field (curves 5 and 6), according to results obtained in references 17 and 18. Although there is not complete correspondence between the oscillations in

[16] N. M. Nakhimovich, J. Exptl. Theoret. Phys. (U.S.S.R.) 12, 539 (1942).

[17] J. A. Marcus, Phys. Rev. 76, 413 (1949).

[18] B. I. Verkin, Dokl. Akad. Nauk SSSR 81, 529 (1951).

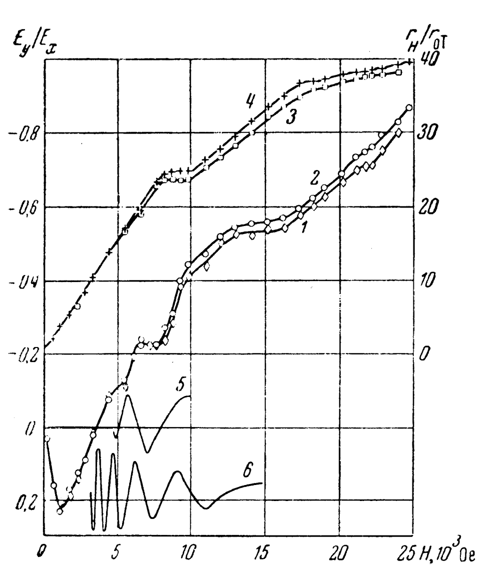

FIG. 8. Dependence of the magnitude of the Hall field and of the change in resistance on the magnitude of the magnetic field for Zn-10. $\psi=0°$. Curve 1 gives E_y/E_x for $T=1.81$, 2 gives E_y/E_x for $T=4.22$, 3 gives r_H/r_{0T} for $T=1.81$, 4 gives r_H/r_{0T} for $T=4.22$ °K; 5 is the curve of the magnetic susceptibility χ for $\psi=0°$ at $T=14$ °K[17]; 6 is the curve of the difference of the principal susceptibilities for $T=4.22$ °K.[18]

the two curves, the periods of the oscillations in the curves for the Hall field and for the magnetic susceptibility are quite the same in order of magnitude. Results involving quantum oscillations have been discussed in previous communications.[15]

3. DISCUSSION OF RESULTS

The results obtained indicate that for zinc, just as for tin,[14] the minima in the resistance correspond to increased values of the Hall field. However, these increased values of the Hall field cannot completely explain the slow increase in resistance in a magnetic field for $\psi=0°$ and $\psi=90°$, since, for example, for $\psi=90°$, the slow increase in resistance in a magnetic field continues even after E_y/E_x has passed through a maximum and has begun to decrease.

We shall try to explain, at least qualitatively, the connection between the electronic structure of the metal and the character of the diagrams showing the variation of properties with angle of rotation (Figs. 1 through 5).

A model in which account was taken of the anisotropy of the effective mass for the electrons in the upper energy zone has already been considered in the early works of Blochinzev and Nordheim[19] and Kontorova and Frenkel'[20]; the mass of the holes was considered isotropic. A much more general consideration of the question of the effect of the anisotropy on galvanometric phenomena in metals with overlapping Brillouin zones has been given in an article by Dimitriev[21]; unfortunately the author came to the mistaken conclusion that the phenomenon of saturation should be observed for all metals. The question in which we are interested, that of the anomalous minima and, in general, of the form of the diagrams showing the variation of properties with angle of rotation, has not been treated in the literature.

In considering a metal in which the overlapping of zones is small and the number of electrons in the upper zone is small, it is expedient to consider separately in the upper zone groups of electrons the excited members of which lie close to each pair of opposite surfaces of the polyhedron bounding the first zone. Of course, such a division of the electrons into groups is possible only if the Fermi surface is cut into separate isolated sections by the polyhedron bounding the first zone, that is, if the number of holes as well as electrons is small.

If the binding of the electrons in the lattice may be considered weak, then for such a separate group the components of the reciprocal mass tensor for directions parallel to the bounding surface of the zone must be only slightly different from the free electron values $1/m_0$. The component of the reciprocal mass tensor for the direction perpendicular to the bounding surface $h^{-2}\,\partial^2 E/\partial k_z^2$ must be larger, and the corresponding component of the effective mass $m_z{}^*$ must be smaller, than the free electron mass. It is clear that if the condition of weak binding is not observed, we may still consider as true in the general case the statement that the effective mass for acceleration in a direction perpendicular to the surface of discontinuity in the energy must, for such individual groups of electrons, be smaller than in other directions. A decrease in the value of the effective mass also obtains for holes situated near the surface of discontinuity in the energy. However, in their case the value of the effective mass grows very quickly

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[19] D. Blokhintsev and L. Nordheim, Z. Physik 84, 168 (1933).

[20] T. A. Kontorova and Ia. I. Frenkel', J. Exptl. Theoret. Phys. (U.S.S.R.) 11, 666 (1941).

[21] V. A. Dimitriev, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 1019 (1950).

with distance from the surface of discontinuity and, in the case of weak binding, may become substantially greater than the free electron mass even at a fairly small distance from this surface. Hence we may expect an especially large anisotropy of the effective mass for holes.

Under experimental conditions of very low temperature, when we are in the region where the resistance is almost independent of temperature, the electron distribution is determined mainly by alterations of the lattice due to impurities. In this case we may with sufficient accuracy consider the probability of collision of an electron with the lattice as isotropic, independent of the electron's direction or movement. If the average time between collisions is isotropic, then we may generalize the formulas given in Refs. 19 and 20 and consider the general case with certain groups of electrons with anisotropic effective masses. However, as is clear from the work of Dimitriev,[21] these general formulas will be of only limited applicability. On the other hand, in the present state of the theory excessive detailization might seem premature. Hence we limit ourselves to the particular cases which we need.

Let us consider the properties of a metal which has one group of holes with concentration n_1 and one group of electrons in the upper zone with concentration n_2. We suppose that our coordinate axes are chosen directed along the principal axes of the tensor $\frac{1}{\hbar^2}\frac{\partial^2 E(\mathbf{k})}{\partial k_2 \partial k_1}$. The current flows along the x axis, and the magnetic field is parallel to the z axis ($H=H_z$). We designate the magnitudes of the effective masses for the holes by μ_{ii} and for the electrons by m_{ii} ($i=1,2,3$). The average time between collisions we call τ_1 for the holes and τ_2 for the electrons. Then in the absence of the magnetic field the electric conductivity is

$$\sigma_{0T} = \frac{n_1 e^2 \tau_1}{\mu_{11}} + \frac{n_2 e^2 \tau_2}{m_{11}} = \sigma'_{11} + \sigma''_{11}. \tag{1}$$

The electric conductivity in the magnetic field is

$$\sigma_H = \frac{\sigma'_{11}}{1+\varphi_1^2} + \frac{\sigma''_{11}}{1+\varphi_2^2} + \left(\frac{\sigma'_{22}}{1+\varphi_1^2} + \frac{\sigma''_{22}}{1+\varphi_2^2}\right)\left(\frac{E_y}{E_x}\right)^2 \tag{2}$$

and the Hall field, from the condition $j_y = 0$, is

$$\frac{E_y}{E_x} = \frac{n_1\varphi_1^2/(1+\varphi_1^2) - n_2\varphi_2^2/(1+\varphi_2^2)}{n_1\sqrt{\mu_{11}/\mu_{22}}\,\varphi_1/(1+\varphi_1^2) + n_2\sqrt{m_{11}/m_{22}}\,\varphi_2/(1+\varphi_2^2)}, \tag{3}$$

where $\sigma'_{11} = n_1 e^2 \tau_1/\mu_{11}$ and $\sigma''_{11} = n_2 e^2 \tau_2/m_{11}$ are the electric conductivities in the direction of the x axis, with σ'_{11} depending on the holes and σ''_{11} depending on the electrons, while $\sigma'_{22} = n_1 e^2 \tau_1/\mu_{22}$; and $\sigma''_{22} = n_2 e^2 \tau_2/m_{22}$ are the corresponding conductivities in the direction of the y axis. Here

$$\varphi_1 = eH\tau_1/c\sqrt{\mu_{11}\mu_{22}}; \qquad \varphi_2 = eH\tau_2/c\sqrt{m_{11}m_{22}}. \tag{4}$$

Formulas (1) through (4) are not very different from the formulas for the isotropic model.

The effective magnitude of the magnetic field, which is determined by the dimensionless variables φ depends on the geometric mean of the effective masses for the directions x and y.

If the principal directions of the reciprocal effective mass tensor do not coincide with the chosen coordinate axes, a z-component of the electric field will occur, and the expressions will become notably more complicated. Although we have not determined the exact form of these expressions, we may make some general remarks.

From what has been seen above, the solutions of the equations of motion must contain periodic terms, functions of dimensionless parameters φ' and φ'', of the type of the magnitudes φ_1 and φ_2 introduced in (4). These magnitudes φ' and φ'' must now also be contained in the non-diagonal components of the reciprocal effective mass tensor. The magnitudes φ', φ'' characterize the effective magnitude of the magnetic field. In the general case of arbitrary direction of coordinate axes, it follows from considerations of symmetry and dimensionality that

$$\sigma_H = \frac{\sigma'_{11}}{1+(\varphi')^2} + \frac{\sigma''_{11}}{1+(\varphi'')^2} + \left(\frac{\sigma'_{22}}{1+(\varphi')^2} + \frac{\sigma''_{22}}{1+(\varphi'')^2}\right)\left(\frac{E_y}{E_x}\right)^2 + \left(\frac{\sigma'_{33}}{1+(\varphi')^2} + \frac{\sigma''_{33}}{1+(\varphi'')^2}\right)\left(\frac{E_z}{E_x}\right)^2, \tag{5}$$

where the magnitudes φ' and $\varphi'' \sim eH\tau/c$ go over into φ_1 and φ_2, which were introduced in (4), when the coordinate axes coincide with the principal directions of the reciprocal effective mass tensor.

It should be noted that for orientations of magnetic field and current coinciding with the principal directions of the effective mass tensor, the geometrical average of the effective masses in a direction perpendicular to the surface of discontinuity in the energy will enter into the magnitude φ. If the conclusion that the effective mass in a direction perpendicular to the surface of discontinuity in the energy should be less than in other directions is correct, then for a direction of the magnetic field perpendicular to the boundary of the zone, $\sqrt{m_{11}m_{22}}$ will have a maximum value in comparison with orientations of the field parallel to other principal directions. The magnitude of the effective value for this direction will be less than for other directions, and the increase in resistance in the magnetic field will also be less.

In the general case of several groups of electrons and holes a sum of terms similar to those introduced in (5), each of which will correspond to a single group of holes or electrons, will enter into the expression for σ_H. We will have a minimum on the diagram showing change in resistance in the magnetic field *versus* angle of rotation each time one of the surfaces of the polyhedron of the first zone becomes perpendicular to the field. The minima will be obtained only for those surfaces which intersect the Fermi surface. These qualitative observations give a picture which corresponds in general to the experimental rules for the change in resistance in a magnetic field formulated in Refs. 5 and 14, and to the results obtained here for zinc. An exact comparison with the results of experiment is made difficult by the fact that because of the behavior of the remaining groups the position of the minimum in the resistance may be displaced and field-dependent. Metals which have a very complicated structure in their angular-dependence diagrams, with a great number of minima (Sn, Ga) have a complex crystalline cell and the form of their energy zones has not been calculated.

This explanation seems to be contradicted to a certain extent by the disappearance of most of the minima in the angular-dependence diagrams at low fields.

With respect to the Hall effect, it follows from the indicated considerations only that for points of minimum resistance E_y / E_x will have an extreme value, but it is impossible to say whether it will be a maximum or a minimum, since the Hall field is determined by a difference in terms corresponding to the behavior of groups of electrons and holes.

We limit ourselves here to just these qualitative indications as to the possibility of theoretical explanation of the experimental results.

CONCLUSIONS

As a result of this investigation of the interrelation between the anisotropy of the Hall effect and the change in resistance in a magnetic field, it has been established that the minimum in the plot of resistance *versus* angle of rotation corresponds to a maximum in the Hall field. It is shown that for zinc at $T \leq 20\,^\circ$K it is impossible to derive complete information about the Hall effect from measurements of its magnitude made only for the principal crystallographic directions, and that it is necessary to study the complete variation with angle of rotation. By qualitative consideration of the properties of a zone model for a metal, the Fermi surface of which intersects the polyhedron bounding the Brillouin zone, it is shown that this model can explain the experimentally observed form of the angular-dependence diagrams.

In conclusion the author expresses his thanks to Professor B. G. Lazarev for his interest in the work and to V. I. Sharonov for help in the measurements.

Translated by M. G. Gibbons
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