Actually, $\Delta\Phi = 0$ under these conditions, in agreement with Eq. (2.2). The relations (2.2) and (2.3) were used by the author[4] in the investigation of the destruction of the superconducting state in a magnetic field.

We can raise the question as to the determination of the minimum work done by an external source on a body placed in a medium where the magnetic field is constant and homogeneous. It is not difficult to find that

$$R_{\min} = \Delta(E - T_0 S) - \frac{1}{4\pi}\int \mathbf{H}_0 \Delta \mathbf{H} dV. \qquad (2.4)$$

Similar considerations permit us to obtain the following generalized form of (1.5):

$$(\partial H_0^i / \partial \bar{H}^i)_T > 0. \qquad (2.5)$$

Here $\bar{H}^i = V^{-1}\int H^i dV$ (V is the volume enclosed by the surface on which the magnetic field is taken to be equal to H_0). Conclusions can also be drawn from (2.5) as to the dependence of $\bar{H}$ on H_0, as was done above for the dependence of B on H.

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* Equation (1.2) permits us to determine the probability of thermodynamic fluctuations in the following manner:

$$w \sim \exp\left\{-\frac{1}{2kT}\left[\Delta T \Delta S - \frac{1}{4\pi}\int dV(\Delta \mathbf{H}\Delta \mathbf{B} + \Delta \mathbf{E}\Delta \mathbf{D})\right]\right\}.$$

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On the First Appearance of Atomic Electrons With l, n, n_r and $n + l$ Given

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EARLIER it was shown[1] that the expression for Z_l, the atomic number of an element [an expression obtained on the basis of the rule of systematic filling (Aufbau principle) of the $(n - l)$ groups[2]], in the electronic shell of neutral unexcited atoms of which the first electron appears with a given value for the orbital quantum number l, can be reduced to the form:

$$Z_l = {}^1/_6(2l+1)^3 + {}^1/_6(5 - 2l). \qquad (1)$$

The second member of the right side of this equation represents a relatively small value and the dependence of Z_l on l is determined principally by the first member, which is proportional to $(2l + 1)^3$; this agrees well with the conclusions of statistical theory.[3] One can also reduce to a similar form a series of other equations[4] obtained on the basis of that same rule, equations for the first appearance of atomic electrons with a given value for the principal quantum number, for the radial quantum number and with a given value for the sum of the principal and orbital quantum numbers.

For the simplification of further exposition we shall introduce the following notation. Let

$$K(y) = {}^1/_6 y^3 + \delta;\ \delta = \begin{cases} -{}^1/_6 y, & \text{if } y \text{ is odd} \\ +1/3\, y, & \text{if } y \text{ is even.} \end{cases} \qquad (2)$$

From the condition $l \leq n - 1$ and from the integral value of l it follows that the maximum value of the orbital quantum number (l_{max}) in the presence of the given value of $n + l$ is equal to 0.5 $(n + l - 1)$ if $n + l$ is odd and equal to 0.5 $(n + l - 2)$ if $n + l$ is even. Therefore, the number of different quantum positions in the limits of one $(n + l)$ group is equal to:[5]

$$\sum_{l=0}^{l_{max}} 2(2l+1) = 2(l_{max}+1)^2 = \begin{cases} 0.5(n+l+1)^2, & \text{if } n+l \text{ is odd} \\ 0.5\,(n+l)^2, & \text{if } n+l \text{ is even} \end{cases} \qquad (3)$$

From this it follows that the number of different quantum positions in the limits of an aggregate, including the $(n + l)$ groups with all the values of $n + l$ less than a certain odd number $y = 2q + 1$ $(q = 0,1,2,3,..)$ is equal to

$$\sum_{q=0}^{0.5\,(y-1)} (2q)^2 = {}^1/_6 y^3 - {}^1/_6 y. \qquad (4)$$

For the aggregate itself, including the $(n + l)$ groups with the values of $n + l$ less than a certain even number $y = 2q + 2$, we have, consequently,

$$\sum_{q=0}^{0.5\,(y-2)} (2q)^2 + 1/2y^2 = 1/6y^3 + 1/3y. \qquad (5)$$

By such means, taking into consideration the

determination of the operator K in accordance with Eq. (2), we can say that the total number of different quantum positions for which $n + l < y$ is equal to $K(y)$.

In conformity with the rule of systematic filling (Aufbau principle) of the $(n + l)$ groups, the electron adjoining the singly ionized atom occupies in the ground state the level with the minimum possible value of $n + l$[6] without violating the Pauli exclusion principle. Therefore, the transition to the filling of the levels with a new value of $n + l$ proceeds (with the increase of Z) after all levels with smaller values of $n + l$ are charged. From this it is possible to conclude that the value of Z_{n+l} equal to

$$Z_{n+l} = \mathrm{K}(n + l) + 1. \tag{6}$$

must correspond to the first appearance of the electron with given $n + l$.

In conformity with that same general rule, the filling of n of the l-subgroups in the limits of each $(n + l)$ group proceeds from the subgroups with large l's to the subgroups with smaller l's.[2] Therefore the first appearance of the electron with given l must have a place in the beginning of the filling of each $(n + l)$ group with an odd value for $(n + l)$ equal to $2l + 1$. Consequently,

$$Z_l = \mathrm{K}(2l + 1) + 1. \tag{7}$$

This expression is the same as Eq. (1).

The filling of the levels with a new value for the principal quantum number must begin in accordance with that same rule, in the midst of the filling of levels belonging to the $(n + l)$ group with a given value for $n + l$, the levels of the s-subgroup of this $(n + l)$ group remaining unfilled. From this it follows that:

$$Z_n = \mathrm{K}(n + 1) - 1. \tag{8}$$

The filling of the levels with a given value for the radial quantum number must begin likewise from the s-subgroup because, with n_r given, the levels with the least l are always more penetrating with regard to energy. Keeping in mind that $n = n_r + l + 1$ and that, consequently, in the case of the s-levels $n_r = n - 1$, we can write:

$$Z_r = \mathrm{K}(n_r + 2) - 1. \tag{9}$$

where Z_r is the atomic number of the element in the electronic shell of the neutral unexcited atoms, of which the first electron appears with a given value of the radial quantum number n_r.

Taking into consideration the fact that the change of the value of $K(y)$ with the increase of y is determined chiefly by the term $(1/6)y^3$ independent of the evenness of y, one can write the expression for Z_n, Z_r and $Z_n + l$ in the form:

$$Z_n = {}^1/_6(n + 1)^3 + \delta_n;\ \delta_n \tag{10}$$

$$= \begin{cases} +{}^1/_3(n - 2), & \text{if } n \text{ is odd} \\ -{}^1/_6(n + 7), & \text{if } n \text{ is even} \end{cases}$$

$$Z_r = {}^1/_6(n_r + 2)^3 + \delta_r;\ \delta_r \tag{11}$$

$$= \begin{cases} -{}^1/_6(n_r + 8), & \text{if } n_r \text{ is odd} \\ +{}^1/_3(n_r - 1), & \text{if } n_r \text{ is even} \end{cases}$$

$$Z_{n+l} = {}^1/_6(n + l)^3 + \delta_{n+l};\ \delta_{n+l} \tag{12}$$

$$= \begin{cases} -{}^1{}_6(n + l - 6), & \text{if } n + l \text{ is odd} \\ +{}^1{}_3(n + l + 3), & \text{if } n + l \text{ is even} \end{cases}$$

It is not difficult to be convinced that in the determination of the dependence of Z_l on l, Z_n on n, etc., the additional term (δ) plays a secondary role.

By such means it has been shown that the expressions for Z_l, Z_n, Z_r and Z_{n+l} can be reduced to a general binomial form for which it is found the principal dependence of Z_l, Z_n, Z_r and Z_{n+l} on one of the terms which is proportional, respectively, to $(2l + 1)^3$, $(2l + 1)^3$, $(n + 1)^3$, $(n_r + 2)^3$ and $(n + l)^3$.

Earlier it was noted[1] that Fq. (1) gives the values of Z_l for $l = 0$, 1 and 2 which correspond with the experiments and a value of Z_l, which is very close to the experimental value for $l = 3$. As concerns the equations (10)-(12) developed above, they then give the values of Z which correspond exactly with the experimentally obtained values without exceptions; Z_n and Z_r take the values 1, 3, 11, 19, 55 and 87 and Z_{n+l}, the values 1, 3, 5, 13, 21, 39, 57, and 89. The series of elements which correspond to the first appearance of the electron with given n and given n_r are identical because the filling of the levels with a new value for n_r, as also of the levels with a new value for n, always begins with the s-subgroup. In connection with this, the elements whose electron for the first time appears with a new value for n (and with a new value for n_r) are those elements of the first group of the system of Mendeleev, the elements lying at the beginning of each period. Since the chief quantum number of the s-subgroup of the

levels with the filling of which (group) the period begins, is equal to the number of the period, one can consider that Eq. (10) as coming from the rule of systematic filling (Aufbau principle) of the $(n + l)$ groups, an expression of the relation between the number of the period in the system of Mendeleev and the charge of the nucleus of the atoms of the element beginning the period.

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Translated by B. Abbott
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Gamma Radiation Accompanying the Absorption of Fast Protons by Nuclei

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THE absorption of fast protons by nuclei can be accompanied by the radiation of photons through two mechanisms. On the one hand, the absorption of a proton by the nucleus causes a diffraction perturbation of the proton wave, enabling it to radiate (diffraction radiation)[1]; on the other hand, radiation can be caused by the direct absorption of the proton (bremsstrahlung radiation)[2]. The second mechanism is more essential. In the present note we wish to estimate the role of the anomalous magnetic moment of the proton in bremsstrahlung radiation, our earlier calculation of which[2] was not exact.

We will assume that for an estimate of the bremsstrahlung the proton can be described by the Dirac equation with an anomalous magnetic moment μ':

$$(\gamma_\nu \partial/\partial x_\nu - ie\gamma_\nu A_\nu - i\mu'\gamma_\nu\gamma_\rho F_{\nu\rho} + m)\,\psi = 0, \qquad (1)$$

here $F_{\nu\rho} = \partial A_\rho/\partial x_\nu - \partial A_\nu/\partial x_\rho$ is the field tensor and A_ν is the vector potential, equal to $A_\nu = (2\omega)^{-1/2} e_\nu \exp\{-i(\mathrm{kr} - \omega t)\}$ (e_ν is the unit polarization vector, ω the frequency, k the wave vector of the photon; we employ the system of units in which $c = \hbar = 1$). Since $e \ll 1$, in terms containing the electromagnetic field, it is possible to replace ψ by $\psi_0 = ue^{i(\mathbf{pr} - Et)}$, where u is the spinor amplitude of the incident plane wave of the proton with momentum p and energy E. In this way we obtain an inhomogeneous equation for

$$(\vec{\gamma}\partial/\partial \mathbf{r} - \gamma_4 E' + m)\,\Phi = (2\omega)^{-1/2}(ie\hat{e} + 2\mu'\hat{k}\hat{e})\,\Phi_0(\mathbf{r})\,e^{-i\mathbf{kr}}, \qquad (2)$$

where

$$E' = E - \omega,\quad \hat{a} = \gamma_\nu a_\nu \quad (\nu = 1, 2, 3, 4),\quad \Phi_0(\mathbf{r}) = ue^{i\mathbf{pr}}.$$

Obtaining $\Phi(\mathbf{r})$ from this equation on the surface of the nucleus, which is assumed completely black with respect to the incident proton, it is possible to define the current of protons, absorbed by the nucleus, which has one photon at infinity. This current density is defined by the formula

$$j = \left(\overline{\Phi}\frac{\vec{\gamma}\mathbf{p}}{p}\Phi\right)_{r=R},\quad \overline{\Phi} = \Phi^*\gamma_4,$$

and the bremsstrahlung cross-section is equal to

$$d\sigma = (j\pi R^2/v)\,(2\pi)^{-3}\omega^2 d\omega d^2\theta \qquad (3)$$

where v is the proton velocity, $d^2\theta$ the solid angle in which the photon is emitted.

The solution of Eq. (2) has the form

$$\Phi(\mathbf{r}) = -(2\omega)^{-1/2}\int G_0(\mathbf{r}, \mathbf{r}') \times (ie\hat{e} + 2\mu'\hat{k}\hat{e})\,e^{-i\mathbf{kr}'}\Phi_0(\mathbf{r}')\,d\mathbf{r}', \qquad (4)$$

where G_0 is the Green's function for the Dirac equation[1]:

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi}\left(\vec{\gamma}\frac{\partial}{\partial \mathbf{r}} - \gamma_4 E' - m\right)\frac{e^{ip'|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \qquad (5)$$

$$p' = \sqrt{E'^2 - m^2}.$$

Employing Eq. (5) it is possible to show that $\Phi(\mathbf{r})$ is given by the formula