fields fit the following formulas well:

$(H_k/28.5) + (T/0.54)^2 - 1$ for $p = 0$

$(H_k/26.5) + (T/0.495)^2 = 1$ for $p = 1550 \pm 50$ atm.

H_k is shown as a function of T^2 on Fig. 3, indicating the accuracy of the derived equations. H_0 and T_k have the following values for cadmium:

at $p = 0$, $H_0 = 28.5$ oersteds, $T_k = 0.540$°K;

at $p = 1550$ atm, $H_0 = 26.5$ oersteds, $T_k = 0.495$.

Thus:

$$\partial T_k/\partial p = 3 \times 10^{-11}\text{°K dyne}^{-1}\text{ cm}^2$$

$$\partial H_0/\partial p = 1.27 \times 10^{-9}\text{oersteds dyne}^{-1}\text{cm}^2,$$

i.e., they have the same order of magnitude as for other superconductors [6,10]. Thus we note the change of T_k by a pressure ~1500 atm is 8.3%, i.e., slightly larger than the corresponding value for other superconductors.

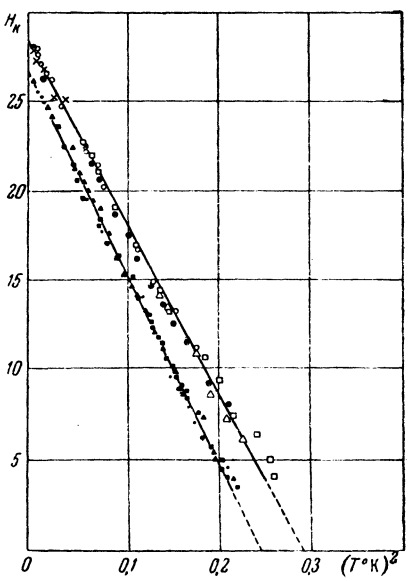

FIG. 3. Same meaning of symbols as in Fig. 2.

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Translated by S. D. Sydoriak
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The Quantum Theory of the Magnetostriction of Cubic Monocrystals of Ferromagnetics at Low Temperatures

A. A. GUSEV
Moscow State University

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IN an earlier paper[1] we presented a general method for the quantum treatment of the magnetostriction of ferromagnetic single crystals at low temperatures. The results were applied to crystals of hexagonal symmetry.

All the basic physical assumptions of the theory outlined in reference 1 are also valid for the cubic lattice. The special peculiarity of the latter is its much higher symmetry, which in particular, does not permit us to extend the theory of magnetic anisotropy, developed for hexagonal crystals, to cubic crystals.[2] In the case of magnetostriction, this difficulty arises to a significant degree, since the deformation of the lattice by the magnetic forces eliminates the symmetry and there is introduced into the Hamiltonian the operators of magnetoelastic interaction which are quadratic in the spin operators and linear in the components of the deformation tensor. This difficulty affects the consideration of the magneto-elastic anisotropy which appears in the phenomenon of magnetostriction.

The equivalent Hamiltonian of the system in this case has the form

$$\mathcal{H} = G_0 - {}^1/_2 \sum G_{\alpha\beta}(f_1, f_2)\, S^{\alpha}_{f_1} S^{\beta}_{f_2} \qquad (1)$$

$$- {}^1/_2 \sum A^{\alpha\beta}_{ij}\, u_{ij}\, S^{\alpha}_{f_1} S^{\beta}_{f_2} - \mu H \sum \gamma_{\alpha} S^{\alpha}_{f},$$

where the summations are carried out over all upper and lower indices appearing in them. The difference from the hexagonal case consists of the fact that the form of the tensors $G_{\alpha\beta}$ and $A^{\alpha\beta}_{ij}$ will be different, corresponding to cubic symmetry of the lattice. In the same way as in reference 1, we write the Hamiltonian in diagonal form:

$$\mathcal{H} = E_0 + \Delta E_0 + \sum_k E_k\, \hat{\xi}^{+}_{k}\, \hat{\xi}_k. \qquad (2)$$

(All the symbols in Eqs. (1) and (2) have the same meaning as in reference 1). In the case under consideration, the quantity ΔE_0 is small at low temperatures and strong fields in comparison with the remaining terms in Eq. (2), and can be neglected for our purposes. Under these conditions we have for E_0 and E_k, approximately,

$$E_0 = -(N/2)\,[\bar{G}_a + \xi(\gamma_{\alpha}, \gamma_{\beta}, u_{ij}) + 2\mu H], \qquad (3)$$

$$E_k = 2\mu H + \xi(\gamma_{\alpha}, \gamma_{\beta}, u_{ij}) + \beta k^2. \qquad (4)$$

Here $\bar{G}_a$ are the mean values of the diagonal terms of the tensor of electron interaction (the exchange integral), $\beta = G_a f^2$, k = wave number, and

$$\xi(\gamma_{\alpha}, \gamma_{\beta}, u_{ij}) = \left(u_{11}\overline{A^{11}_{11}} + u_{27}\overline{A^{11}_{22}} + u_{33}\overline{A^{11}_{33}}\right)\gamma_1^2 \qquad (5)$$

$$+ \left(u_{11}\overline{A^{22}_{11}} + u_{22}\overline{A^{22}_{22}} + u_{33}\overline{A^{22}_{33}}\right)\gamma_2^2$$

$$+ \left(u_{11}\overline{A^{33}_{11}} + u_{22}\overline{A^{33}_{22}} + u_{33}\overline{A^{33}_{33}}\right)\gamma_3^2$$

$$+ u_{12}\overline{A^{12}_{12}}\gamma_1\gamma_2 + u_{23}\overline{A^{23}_{23}}\gamma_2\gamma_3 + u_{13}\overline{A^{13}_{13}}\gamma_1\gamma_3,$$

where $\overline{A^{\alpha\beta}_{ij}}$ are the mean values of the components of the tensor of magneto-elastic interaction.

Having Eq. (2), it is not difficult to calculate the partition function, and hence the free energy, the magnetoelastic part of which in the same approximation has the form

$$\Psi_{\text{M.y}} = -\frac{N\xi(\gamma_{\alpha}, \gamma_{\beta}, u_{ij})}{2V}\varphi(\vartheta, H), \qquad (6)$$

where

$$\varphi(\vartheta, H) = 1 - \frac{V}{4N}\left(\frac{\vartheta}{\beta\pi}\right)^{3/2} e^{-2\mu H/\vartheta}, \quad \vartheta = kT. \qquad (7)$$

For cubic symmetry of the lattice, the components of the tensor $A^{\alpha\beta}_{ij}$ have the following properties:

$$\overline{A^{11}_{11}} = \overline{A^{22}_{22}} = \overline{A^{33}_{33}} = A_1, \quad \overline{A^{12}_{12}} = \overline{A^{23}_{23}} = \overline{A^{13}_{13}} = A_2;$$

$$\overline{A^{\alpha\beta}_{ij}} = 0 \text{ for } (ij) \neq (\alpha\beta).$$

Computing $\xi(\gamma_{\alpha}, \gamma_{\beta}, u_{ij})$ and substituting in Eq. (6) we get for $\Psi_{\text{M. y}}$ an expression from consideration of which it is evident that, because of its dependence on $\mu_{\alpha} u_{ij}$ coincides with the classical expression for the magneto-elastic energy of a cubic crystal. Hence the quantities

$$\left(\overline{A^{\alpha\beta}_{ij}}\, N/2V\right)\varphi(\vartheta, H) \qquad (8)$$

play the role of magneto-elastic coefficients; however, in contrast to the classical expressions, they are functions of the temperature and the magnetic field.

In order to obtain expressions for the constants of magnetostriction, we introduce the elastic energy and find the equilibrium values of the components of the deformation tensor $u_{ij} = u^0_{ij}$ from the condition of a minimum. If we substitute these in the formula for the relative extension, we get

$$\lambda_{[100]} = \lambda^0_{[100]}\, \varphi(\vartheta, H), \quad \lambda^0_{[100]} = c_1 A_1 N/V, \qquad (9)$$
$$\lambda_{[111]} = \lambda^0_{[111]}\, \varphi(\vartheta, H), \quad \lambda^0_{[111]} = c_2 A_2 N/V.$$

The coefficients c_1 and c_2 depend on the elastic constants of the cubic lattice. As also in the case of the hexagonal crystals, the quantum calculation permits the a priori explanation of both positive and negative value of $\lambda_{[100]}$ and $\lambda_{[111]}$, which the classical theory could not do. Thus we have obtained in clear form the dependence of the constants of magnetostriction of cubic single crystals on temperature and, which is very important, on the external magnetic field.

The quantum theory of magnetostriction of ferromagnetics with a cubic lattice is extremely weakly

developed. There is on this problem only the work of Vonsovskii[3], where a theory is given for the high temperature region. In this same work an expression is given (without derivation) for the temperature factor $\varphi(\vartheta, H)$ at low temperatures (for $H = 0$). In the corresponding case one can introduce the expression for $\varphi(\vartheta, 0)$ from the quasi-classical theory of Akulov. There dependence is of the form $\vartheta^{3/2}$ in these expressions; however, our dependence (7), while containing $\vartheta^{3/2}$, possesses a more complicated character (for details, see reference 4).

In the comparison of the expressions for the constants of Anisotropy[3] and the constants of magnetostriction of a hexagonal crystal[1] and a cubic crystal Eq. (9), a definite analogy is noted in their temperature behavior at low temperatures. This circumstance points up the deep connection between the phenomenon of anisotropy and that of magnetostriction, and is determined by the character of the energy spectrum of ferromagnetics at low temperatures. The great variation observed in the temperature dependence of the constants of magnetostriction and anisotropy for medium and high temperatures is connected with the fact that the "gas" of quasi-particles (elementary excitations) ceases to be ideal upon increase in temperature, and other quasi-particles (especially phonons), their interaction with ferromagnons begins to play an important role, and the individuality of the lattice appears strongly. These circumstances disrupt the apparent universality at low temperatures and lead to a series of complicated effects.

The theory developed by us, as noted in reference 1, applies primarily to liquid hydrogen temperatures. The development of a systematic quantum mechanical theory of magnetoelastic phenomena for intermediate temperatures and in the region of the Curie point is an important problem of the quantum theory of magnetism that is still unsolved.

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The Study of Relativistic Particles by the Use of Nuclear Emulsions in a Pulsed Magnetic Field

V. M. LIKHACHEV, A. V. KUTSENKO AND V. P. BORONKOV
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

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IN recent years thick photographic emulsions have been used widely in a large number of physical problems. The study of the tracks left by particles in their passage through an emulsion enable us to derive information about the properties of the particles, the nature of nuclear reactions etc. However, the sign of a particle is seldom determined by this method and in many instances (when the entire track is not contained in the emulsion) the energy is not determined with sufficient accuracy.

A more complete solution of these problems can be achieved if the nuclear track plate is placed in a strong magnetic field during the period of irradiation. A calculation shows that an analysis of the momenta and signs of the particles can achieve sufficiently accurate results only in strong magnetic fields of the order of $1\text{-}1.5 \times 10^5$ gauss and above. It is known that such strong fields can be set up, at least at the present time, only in the form of pulses. However, in working with accelerators which also produce pulsed beams of particles, the use of pulsed magnetic fields seems to us especially advantageous because of the possibility of synchronizing the particle beam with the field*. We have used this kind of pulsed magnetic field to measure the spectrum of photons from the synchrotron of the Physical Institute of the Academy of Sciences of the USSR.

The apparatus consists of a current source[1] a control circuit and a solenoid within which the photographic films are placed. As a source of energy we use a bank of IM-3/100 pulse capacitors with a capacity of 3300 μf. The current-switching element is an IG-100/5000 ignitron. The pulse which fires the ignitron is timed so that the field peak coincides with the arrival of the particle beam from the accelerator. In order to avoid exposure of the nuclear emulsion in the absence of the magnetic field the accelerator operates under a so-called single-pulse regime**. The coil, which was made according to our own design, enabled us to set up a magnetic field of $1\text{-}1.5 \times 10^5$ gauss with a 5 mm space between