Attention is called here to the increase of the molar rotation ρ with frequency ν of the light which is well represented by the simple relations

Cations: $\nu = A'\rho^{1/2}$,

Anions: $\nu = A''\rho^{1/2} + B$,

A′ and A″ are constants for a given ion which decrease from ion to ion along with their crystal ionic radii.

Figure 1 shows the agreement of the measured values with the above relations. The values of $\sqrt{\rho}$ were calculated from the data of Okazaki[1] and are also given in the table below.

λ	Li^+	Na^+	K^+	NH_4^+	Cl^-	Br^-	I^-	OH^-
5900	0.378	0.412	0.490	0.632	1.000	1,396	2.057	0.655
5100	0.412	0.458	0.574	0,714	1.204	1.670	2.464	0.728
4300	0.458	0,600	0.707	0,877	1,487	2,145	3.162	0,949
3500	0,583	0.775	0.877	1.127	1.949	2.850	4,470	1.225

Distribution Function of a Non-ideal Bose Gas at the Temperature of Absolute Zero

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THE distribution function for the momentum molecules of an ideal Bose gas at the temperature of absolute zero has a δ-like character. All the molecules of the gas lie in the lowest energy level with zero momentum. For a non-ideal gas this no longer holds even at absolute zero, owing to the interaction of the molecule with the non zero momentum.

In the work of Bogoliubov and the author[1], the wave function of the lower state of a weakly non-ideal Bose-gas was calculated. In the first approximation it has the form

$$\varphi_0 = \exp\left\{\tfrac{1}{2}\sum_k (1-\lambda_k^{-2})\,\rho_k\rho_{-k}\right\}, \tag{1}$$

where

$$\rho_k = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\exp\{-i(kr_j)\}, \tag{2}$$

$$\lambda_k^4 = \frac{\hbar^2k^2}{V}\left(\frac{N\nu(k)}{4m} + \frac{\hbar^2k^2}{4m}\right)^{-1}, \tag{3}$$

$\nu(k) = \int \Phi(r)e^{i(kr)}dr$ = Fourier coefficient of energy of interaction $\Phi(r)$, N = number of particles, V = volume of the system, m = mass of the particle.

The function φ_0 corresponds to the zero vibrations of the collective variables ρ_k. By means of this wave function it is possible to find the distribution function for the momentum of molecules of a non-ideal Bose gas. It is possible to show this in the following way. The momentum distribution function permits us to calculate the mean value of operators of the additive type; this depends on the momenta of the particles. If one makes up an arbitrary function of additive type out of the particle momenta:

$$\sum_{j=1}^{N} f(\mathbf{p}_j), \tag{4}$$

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[1] A. Okazaki, Proc. Phys. - Math. Soc. (Japan) 24, 40 (1942); see also J. Parkington, *A Treatise on Physical Chemistry* (London, 1953) Vol. IV p. 604.

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then the determination of its average value leads to the calculation of the integrals over the coordinates of all N particles:

$$N\bar{f} = \frac{\sum\int \varphi_0 f(\mathbf{p}_j)\, \varphi_0 d\mathbf{r}_1 \dots d\mathbf{r}_N}{\int \varphi_0^2 d\mathbf{r}_1 \dots d\mathbf{r}_N} \tag{5}$$

$$= N\int w(\mathbf{p})\, f(\mathbf{p})\, d\mathbf{p},$$

where $w(p)$ is the unknown distribution function.

This problem from a mathematical point of view, is similar to the calculation of a contour integral, because we are able to apply the method of computation of integrals of such a type, given in reference 2. If the symmetry of function φ_0 is taken into account, it is sufficient to calculate only one integral, for example $j = 1$. It is convenient to represent the variables ρ_k in the form of a sum of variables ρ'_k (which are constructed exactly as before, but which do not include r_1) and $N^{-1/2}e^{i(kr_1)}$, and then to expand the function in series of these terms. For calculating the integral along $r_2, \dots, r_N$, it is convenient to consider the ρ'_k as auxiliary variables and take into account their connection with $r_2, \dots, r_N$, as this was done in reference 2. Finally, the desired integral is represented in a form of a single integral over p. Comparing the coefficients for the arbitrary function $f_{(p)}$, we obtain the distribution function $W(p)$.

Not stopping for details of calculation, which will be given in a later paper, we set forth only the final results of the computations. The distribution function of molecules of a Bose gas of momentum $p = \hbar k$ at the temperature of absolute zero has the form

$$w(\mathbf{p}) = C\delta(\mathbf{p}) + \frac{v}{4(2\pi\hbar)^3}(1 - \lambda_p^{-2})\lambda_p^2, \tag{6}$$

$$C = 1 - \frac{v}{4(2\pi\hbar)^3}\int (1 - \lambda_p^{-2})^2 \lambda_p^2\, d\mathbf{p}, \tag{7}$$

where $v = V/N$. The distribution function (6) was obtained on the assumption of weak interaction between particles. It agrees with the distribution function of a slightly non-ideal Bose gas, as determined by Bogoliubov[3]. Here we obtained it by another method, that is, we did not utilize a δ-like momentum distribution by way of a zero approximation.

The quantity λ^2_k is connected with the correlation function $g(r)$

$$g(\mathbf{r}) = 1 + \frac{1}{V}\sum_k (\lambda_k^2 - 1)\, e^{i(kr)}, \tag{8}$$

which is not difficult to determine, if one takes into account that the correlation function for the lower state is proportional to the variation derivative of the energy of the lower state for the interaction function[1]. It is possible to determine λ^2_k from experimental data on the scattering of slow neutrons in He II[4]; however, in this case it is impossible to compute the small interaction.

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[1] N. N. Bogoliubov and D. N. Zubarev, JETP 28, 129 (1955); Soviet Phys. 1, 83 (1955)

[2] D. N. Zubarev, Dokl. Akad. Nauk. SSSR 45, 757 (1954)

[3] N. N. Bogoliubov, Izv. Akad. Nauk SSSR, Ser. Fiz. 11, 77 (1947)

[4] R. P. Feynman, Phys. Rev. 94, 262 (1954)

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Towards a Statistical Theory of Fission Width

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An estimate of fission width has been obtained in a series of works[1,2]. This letter presents the possibility of examining the connection between different rates of fission width, and also of defining it more accurately and extending the field of its applicability.

In the work of Bohr and Wheeler[1], the rate was obtained by application of the methods of classical statistics:

$$\Gamma_f \cong \frac{D}{2\pi} N^*(E - E_f), \tag{1}$$

where D is the separation of levels; N^* is the number of levels which are bound. By $N^*(E)$ is understood the number of levels with energy of excitation not exceeding E. In the immediate neighborhood of the fission energy E_f we have $N^*(E-E_f) \cong 1$ and, therefore,

$$\Gamma_f \sim D/2\pi. \tag{2}$$

As is known[3], the classical statistics are applicable for the condition $T \gg \hbar\omega$, where T is the temperature and ω is the vibration frequency of the nuclear configuration, associated with fission. In the region $T \ll \hbar\omega$ it is impossible to use the estimates (1) and (2). In this region, in order to make an estimate of the fission width, valid in all temperature intervals, we represent it in the form $\Gamma f \cong (\hbar\omega/2\pi)w$ where w is the probability density on the dividing degree of freedom of the energy, exceeding the fission threshold E_f. We estimate this probability, having applied a microcanonical distribution[3] to the nucleus. This distribution takes into account not only the law of conservation of energy, but also the law of conservation of momentum—all states, compatible with the conservation laws, being equally probable. Finally, we obtain

$$\Gamma_f \cong \frac{\hbar\omega}{2\pi}\,\frac{N^*(E-E_f)}{N^*(E)} \tag{3}$$

As it was shown by Landau[4], the density of levels with fixed moment varies, in principle, according to the law $e^{S(E)}$ where S is the entropy. Since $dS/dE = 1/T$, we have $N^*(E) \cong T/D^*$ where D^* is the separation of the levels, which are bound. In view of the fact that, near the threshold, $N^*(E-E_f) \cong 1$, we obtain

$$\Gamma_f \sim (\hbar\omega/T)(D^*/2\pi). \tag{4}$$

In the classical region, $T \gg \hbar\omega$, the estimates (3) and (4) pass to that obtained by Bohr and Wheeler (1) and (2), since in this case, as it is not difficult to see, $D^* \cong (T/\hbar\omega)D$. On the other hand, in the opposite limiting case $T \ll \hbar\omega$ we have $D^* = D$ and therefore,

$$\Gamma_f \sim (\hbar\omega/T)(D/2\pi). \tag{5}$$

The actual values of the quantities $\hbar\omega$ and T for heavy nuclei are not known with any accuracy. We assume for computation that both these values are of the order of 1 mev; however, they can vary several fold. From this point of view the estimate (2) must be considered as the lower estimate of the fission width.

If the full energy of the composite nucleus E significantly surpasses the fission energy, then, as it is not difficult to see from (3)

$$\Gamma_f \sim \frac{\hbar\omega}{2\pi}\exp\left\{-\frac{E_f}{T}\right\}.$$

This rate was also obtained by Kramers[2]; evidently, it is not applicable in the immediate vicinity of the fission threshold.

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[1] N. Bohr and J. Wheeler, Phys. Rev. 56, 426 (1939)

[2] H. Kramers, Physica 7, 284 (1940)

[3] L. Landau and E. Lifshitz, *Statistical Physics*, State Technical Publication, 1951

[4] L. Landau, J. Exper. Theoret. Phys. USSR 7, 819 (1937)

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Radiation of Molecules in the Presence of a Strong High-Frequency Field

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THE absorption of high-frequency radiation in a molecular gas was examined in references 1,2. It will be shown in the present paper that, besides the absorption (and saturation of levels) at a frequency $\omega \approx \omega_{12} = (E_1 - E_2)/h \equiv \omega_0$ (E_1 and E_2 are energy levels of the molecules), there appears a radiation at a frequency Ω_0 [see Eq. (6)], which depends on the matrix element of the dipole moment, corresponding to the transition $E_1 \to E_2$, and on the field potential of the frequency ω.

We assume that the following conditions are satisfied[1-3]: a) $T \ll 1/\omega \ll \bar{\tau}$, where T is the duration of the molecular collision, and τ is the mean time between collisions; b) $|\omega - \omega_0| \lesssim 1/\tau$ and $|\omega - \omega_{mn}| \gg 1/\tau$, where $m, n = 1, 2$; c) molecular collisions re-establish the Boltzman energy