Asymptotic Behavior of Electromagnetic Vacuum-Polarization in the Presence of Meson Interactions

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It is proved that when $e^2 \ll g^2 \ll 1$ the asymptotic form of the photon propagation function is the same as in the case $g^2 = 0$.

1. INTRODUCTION

1. LANDAU, Abrikosov and Khalatnikov[1] found the asymptotic form of the photon propagation function (Green's function) $D^{F'}_{\alpha\beta}$. If this function is written in the transverse gauge

$$\frac{i}{2} D^{F'}_{\alpha\beta}(k) \equiv D_{\alpha\beta}(k) = \frac{1}{k^2}\left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right)\tilde{d}_t,$$

then as proved in reference 1,

$$\tilde{d}_t^{-1} = 1 - {}^4/_3\, \varepsilon y, \tag{1}$$

with

$$y = \ln\frac{k^2}{m^2}; \quad \varepsilon = \frac{e^2}{(4\pi)^2} = \frac{1}{4\pi}\frac{1}{137}$$

Equation (1) takes into account only the interaction between a photon and the electron-positron vacuum. Landau[1] already observed that it must be supplemented with terms corresponding to other types of charged particle. In addition it is necessary to take account of the nonelectrodynamic interactions of these particles. Since we have at present no way to handle strong interactions, it is not, strictly speaking, possible to evaluate the effects of mesons and nucleons. It seems nevertheless interesting to consider these effects, assuming that the meson-nucleon coupling constant g is large compared with the electric charge but still small compared with 1,

$$e^2 \ll g^2 \ll 1.$$

Then we may use the method of reference 1 to derive asymptotic expressions.

We shall show that, in spite of the stronger nonelectromagnetic interaction, the particles give the same contribution to the photon propagation function as if the interaction were absent, i.e., the asymptotic form of $D_{\alpha\beta}$ is independent of g. With the assumptions we have made, the nonelectromagnetic interaction does not produce a "form-factor" which changes the interaction of the particles with the Maxwell field.

2. When $e^2 \ll g^2$, we may expect that the nucleon propagation function $S^{F'} = (2/i)G$, the meson function $\Delta^{F'} = (2/i)\Delta$, and the vertex function Γ_5 of the pseudoscalar meson-nucleon interaction, may be determined without considering electromagnetic interactions. The asymptotic expressions for these functions were found by Abrikosov, Galanin and Khalatnikov[2], and by Galanin, Ioffe and Pomeranchuk[3]. They are as follows*

$$\begin{gathered} G(p) = \frac{i\hat{p}}{p^2}\, b(x); \\ \Delta(p) = \frac{1}{p^2}\, c(x), \\ \Gamma_5(p;\ 0) = \Gamma_5(0;\ p) = \Gamma_5(p;\ p) = \gamma_5\, a(x), \end{gathered} \tag{2}$$

with $x = \ln(p^2/m^2)$. Explicit expressions for $a(x)$, $b(x)$ and $c(x)$ are given in references 2 and 3, but are not needed here.

The calculations can be done in two different ways. One may operate with unrenormalized functions d_t, a, b, c etc. and then renormalize the final expressions[1,2]. Or one may renormalize the starting equations[3] and then operate only with renormalized functions (we denote these by $\tilde{d}_t$, $\tilde{b}$ etc). It is easy to show that the two methods are equivalent, since the transition from unrenormalized to renormalized functions can be made at any stage of the calculation.

We shall use the first method since it is more convenient. In the divergent integrals we cut off

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[1] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497, 1177 (1954)

* We use the following notations: $p^2 = \mathbf{p}^2 - p_0^2$; $\hat{p} = \sum_{\alpha=1}^{4} \gamma_\alpha p_\alpha$; $\gamma_j = -i\beta\alpha_j$ for $j = 1, 2, 3$; $\gamma_4 = \beta$, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ $(\gamma_5^2 = 1)$; $\Gamma_5(p;k) \equiv \Gamma_5(p, p-k; k)$ with p the nucleon and k the meson momentum. We use Heaviside units for e and g and set $\hbar = c = 1$.

[2] A. A. Abrikosov, A. D. Galanin and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR 97, 793 (1954).

[3] A. D. Galanin, B. L. Ioffe and I. Ia. Pomeranchuk, J. Exper. Theoret. Phys. USSR 29, 51 (1955); Soviet Phys. 2, 37 (1956).

the range of the logarithmic variable x with an upper limit L which will disappear from the results after renormalization. The renormalized quantities are defined by the requirement that for $x = 0$ they become equal to the corresponding free-particle propagation functions, i.e., the normalized functions $\tilde{b}(x)$, $\tilde{d}_t(x)$ etc. are equal to unity at $x = 0$. Thus

$$\tilde{d}_t(x) = \frac{d_t(x)}{d_t(0)}; \tag{3}$$

$$\tilde{a}(x) = \frac{a(x)}{a(0)}; \ \tilde{b}(x) = \frac{b(x)}{b(0)}; \tilde{c}(x) = \frac{c(x)}{c(0)} \tag{3a}$$

The renormalization of electric charge and of the meson-nucleon coupling constant are defined as follows:

$$e_1^2 d_t(0) = e^2; \tag{4}$$

$$g_1^2 a^2(0)\, b^2(0)\, c(0) = g^2, \tag{4a}$$

where e_1 and g_1 are the unrenormalized, e and g the renormalized constants+.

2. INTERACTION WITH NEUTRAL MESONS

1. We consider protons interacting with the Maxwell field A_α and with a neutral meson field φ. In the Lagrangian of the system the interactions are represented by the following terms (the meson interaction being pseudoscalar)

$$U_1 = ig_1 \bar{\psi}\gamma_5\psi\varphi, \qquad U_2 = ie_1 \bar{\psi}\gamma_\alpha \psi A_\alpha.$$

The electromagnetic polarization of the proton vacuum is represented by the graph shown in Fig. 1. The wiggly lines denote the propagation of photons, the continuous lines denote protons. The propagation of the protons here includes the meson-proton interaction. The ordinary vertex denotes a matrix γ_α; the "blob" vertex denotes a function Γ_β, the vertex function describing the interaction of a proton with the Maxwell field in the presence of the meson interaction. The vacuum-polarization tensor $P_{\alpha\beta}$ corresponding to Fig. 1 has the following form**

FIG. 1

$$P_{\alpha\beta}(k) = \frac{ie_1^2}{(2\pi)^4}\int \mathrm{Sp}\, \gamma_\alpha G(p)\Gamma_\beta(p;\ k)G(p-k)\, d^4k. \tag{5}$$

The function $G(p)$ has the form (2), while Γ_β is determined by the integral equation

$$\Gamma_\beta(p;\ k) = \gamma_\beta + \frac{ig_1^2}{(2\pi)^4}\int \Gamma_5(p;\ p-q) \times G(q)\Gamma_\beta(q;\ k)G(q-k) \times \Gamma_5(q-k;\ q-p)\Delta(p-q)\, d^4q. \tag{6}$$

Eq. (6) is represented graphically in Fig. 2. The dotted line denotes a meson+, and the vertices with dotted lines incident denote the vertex function Γ_5. Eq. (6) is valid for $g^2 x \lesssim 1$. We assume also $e^2 x \ll 1$, i.e., perturbation theory is valid for the electromagnetic interaction.

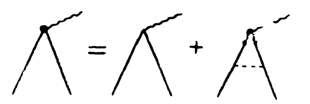

FIG. 2

Since the integral in Eq. (5) diverges for large p, we separate the integrand into terms of various orders in (k/p). Then $G(p-k)$ and $\Gamma_\beta(p,k)$ take the form

$$G(p-k) = G(p) + G^{(1)}(p,\ k) + G^{(2)}(p,\ k), \tag{7}$$

$$\Gamma_\beta(p;\ k) = \Gamma_\beta^{(0)}(p) + \Gamma_\beta^{(1)}(p;\ k) + \Gamma_\beta^{(2)}(p,\ k).$$

According to Eq. (2)

$$G(p) = ib(x)(\hat{p}/p^2),$$
$$G^{(1)}(p,\ k) = ib(x)(\hat{p}\hat{k}\hat{p}/p^4), \tag{8}$$
$$G^{(2)}(p,\ k) = ib(x)(\hat{p}\hat{k}\hat{p}\hat{k}\hat{p}/p^6).$$

We may write for $\Gamma_\beta^{(0)}$

$$\Gamma_\beta^{(0)} = \gamma_\beta \xi(x), \tag{9}$$

+Eq. (3), (3a), (4), (4a) coincide with the usual renormalization conditions of Dyson[4]. Because of the introduction of the cutoff L, the renormalization constants Z_1, Z_2, Z_3 here become functions of L. In electrodynamics $Z_3(L) = d_t(0)$; $Z_1 = Z_2 = 1$. In meson theory $Z_3(L) = c(0)$; $Z_2(L) = b(0)$; $Z_1^{-1}(L) = a(0)$.

[4] F. J. Dyson, Phys. Rev. 75, 1736 (1949).

** $P_{\alpha\beta}$ is defined in such a way that the Dyson-Schwinger equation for the photon Green's function $D_{\alpha\beta}$ takes the form $(-\Box + P)D = 1$.

+Equation (6) is obtained from Fig. 2 by first writing down the scattering matrix by the ordinary rules of perturbation theory, going as far as third-order terms, and then substituting Γ_5 for γ_5, $(2/i)G$ for S^F, and $(2/i)\Delta$ for Δ^F.

while $\Gamma^{(1)}_{\beta}$ and $\Gamma^{(2)}_{\beta}$ consist of slowly varying functions of $x = \ln(p^2/m^2)$ and $y = \ln(k^2/m^2)$, multiplied by (k/p) and by $(k/p)^2$ respectively. The structure of these functions will be discussed in detail later. Substituting Eq. (7) into (5) we obtain

$$P_{\alpha\beta}(k) = \frac{ie_1^2}{(2\pi)^4}\int \gamma_\alpha G(p)\,[\Gamma^{(0)}_\beta(p)G(p) + \Gamma^{(0)}_\beta(p)G^{(1)}(p,k) + \Gamma^{(1)}_\beta(p,k)G(p) + \Gamma^{(1)}_\beta(p,k)G^{(1)}(p,k) + \Gamma^{(0)}_\beta(p)G^{(2)}(p,k) + \Gamma^{(2)}_\beta(p,k)G(p) + \ldots]\,d^4p.$$

Here the first (quadratically divergent) term is independent of k and must be removed by the usual appeal to gauge-invariance. The next two terms contain odd powers of the vector p_α and therefore vanish after integration[+]. The remaining terms are logarithmically divergent. The higher terms in the expansion in powers of (k/p), which are here omitted, would give convergent integrals and therefore are negligible in the asymptotic region.

2. We observe that $\Gamma^{(0)}_\sigma(p) = \Gamma_\sigma(p;0)$ can be immediately fixed by Ward's identity[5], which states that

$$\Gamma_\sigma(p;\,0) = i\frac{\partial G^{-1}(p)}{\partial p_\sigma}.$$

With Eq. (2) this gives

$$\Gamma^{(0)}_\sigma(p) = \frac{1}{b(x)}\gamma_\sigma$$

or by comparison with Eq. (9)

$$\xi(x) = 1/b(x). \tag{10}$$

However, we shall also derive equations which determine ξ directly from Eq. (6), since this is necessary for the later work.

We substitue Eqs. (7), (8), (9) into (6). This gives three equations, of degrees zero, one and two in k. The zero-order equation is

$$\gamma_\sigma\xi(x) = \gamma_\sigma + \frac{ig_1^2}{(2\pi)^4}\int\Gamma_5(p;\,p-q)\,b(z)\frac{i\hat{q}}{q^2} \times \xi(z)\gamma_\sigma b(z)\frac{i\hat{q}}{q^2}\Gamma_5(q-k;\,q-p) \times \Delta(p-q)\,d^4q \qquad \left(z = \ln\frac{q^2}{m^2}\right)$$

The last integral diverges logarithmically at large q, hence we may neglect k^2 and p^2 in it in comparison with q^2, and set the lower limit of integration at $q^2 = p^2 (k^2 \ll p^2)$. Then using Eq. (2) we obtain

$$\xi(x)\gamma_\sigma = \gamma_\sigma + \frac{iq_1^2}{(2\pi)^4} \times \int_{q^2>p^2} a^2(z)\,b^2(z)\,c(z)\,\xi(z)\,\gamma_5\frac{i\hat{q}}{q^2}\gamma_\sigma\frac{i\hat{q}}{q^2}\gamma_5\frac{d^4q}{q^2}. \tag{11}$$

The integrand of Eq. (11) may be simplified. Since γ_5 anticommutes with $\hat{q}$ and γ_σ,

$$\gamma_5 i\hat{q}\gamma_\sigma i\hat{q}\gamma_5 = \hat{q}\gamma_\sigma\hat{q}.$$

By rotating the q_4 axis through an angle $\pi/2$ in the complex plane (see reference 1), we bring the integral into a euclidean space in which the averaging over angles is simple and gives

$$\overline{q_\alpha q_\beta} = {}^1/_4\, q^2\delta_{\alpha\beta},$$

i.e.

$$\overline{\hat{q}\gamma_\sigma\hat{q}} = {}^1/_4\, q^2\gamma_\alpha\gamma_\sigma\gamma_\alpha = -{}^1/_2\, q^2\gamma_\sigma.$$

Then we write for the volume-element

$$d^4q = i\pi^2 q^4\,dz \qquad \left(z = \ln\frac{q^2}{m^2}\right), \tag{12}$$

and Eq. (9) takes the form

$$\xi(x) = 1 + \frac{\lambda_1}{2}\int_x^L a^2(z)\,b^2(z)\,c(z)\,\xi(z)\,dz, \tag{13}$$

with

$$\lambda_1 = g_1^2/(4\pi)^2.$$

3. The equation of first degree in k obtained from (6) is the following

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[+] These terms depend linearly on k_α, and it is clear that the tensor $P_{\alpha\beta}$ cannot be a linear function of k_α.

[5] J. C. Ward, Phys. Rev. 78, 182 (1950); A. Salam, Phys. Rev. 79, 910 (1950); N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).

$$\Gamma_\sigma^{(1)}(p,\ k)=\frac{ig_1^2}{(2\pi)^4}\int\Gamma_5(p;\ p-q)\,b(z)\frac{i\hat{q}}{q^2}\Big[\xi(z)\gamma_\sigma b(z)\frac{i\hat{q}\,\hat{k}\,\hat{q}}{q^4}$$
$$+\Gamma_\sigma^{(1)}(q,\ k)\,b(z)\frac{i\hat{q}}{q^2}\Big]\Gamma_5(q-k;\ q-p)\,\Delta(p-q)\,d^4q.$$

This integral converges for $q^2 \gg p^2$. The important range of integration is $k^2 < q^2 < p^2$ (the situation is exactly the same as in reference 1 when $k^2 \ll p^2$). Hence in the integrand we may neglect k and q compared with p . Then using Eq. (2) we find

$$\Gamma_\sigma^{(1)}(p,\ k)-\frac{ig_1^2}{(2\pi)^4}\frac{a^2(x)\,c(x)}{p^2}\int\limits_{k^2<q^2<p^2}b^2(z)\,\gamma_5\frac{i\hat{q}}{q^2}\Gamma_\sigma^{(1)}(q,\ k)\frac{i\hat{q}}{q^2}\gamma_5\,d^4q$$
$$=\frac{ig_1^2}{(2\pi)^4}\frac{a^2(x)\,c(x)}{p^2}\int\limits_{k^2<q^2<p^2}b^2(z)\xi(z)\,\gamma_5\frac{i\hat{q}}{q^2}\gamma_\sigma\frac{i\hat{q}\,\hat{k}\,\check{q}}{q^4}\gamma_5\,d^4q.$$

The inhomogeneous term on the right of this linear integral equation contains an odd power of q in the integrand and therefore vanishes. So the equation is homogeneous and implies[++]

$$\Gamma_\sigma^{(1)}(p,\ k)=0. \tag{14}$$

4. The equation of second degree in k obtained from Eq. (6) takes the following form after using Eqs. (14), (8) and (9):

$$\Gamma_\sigma^{(2)}(p,\ k)=\frac{ig_1^2}{(2\pi)^4}\int\Gamma_5(p;\ p-q)\,b(z)\frac{i\hat{q}}{q^2}\Big[\xi(z)\gamma_\sigma b(z)\frac{i\hat{q}\,\hat{k}\,\hat{q}\,\hat{k}\,\hat{q}}{q^6}$$
$$+\Gamma_\sigma^{(2)}(q,\ k)\,b(z)\frac{i\hat{q}}{q^2}\Big]\Gamma_5(q-k;\ q-p)\,\Delta(p-q)\,d^4q.$$

In this integral the important range is again $k^2 < q^2 < p^2$ Hence using (2) we obtain

$$\Gamma_\sigma^{(2)}(p,\ k)-\frac{ig_1^2}{(2\pi)^4}\frac{a^2(x)\,c(x)}{p^2}\int\limits_{k^2<q^2<p^2}b^2(z)\,\gamma_5\frac{i\hat{q}}{q^2}\Gamma_\sigma^{(2)}(q,\ k)\frac{i\hat{q}}{q^2}\gamma_5\,d^4q$$
$$=\frac{ig_1^2}{(2\pi)^4}\frac{a^2(x)\,c(x)}{p^2}\int\limits_{k^2<q^2<p^2}b^2(z)\,\xi(z)\,\gamma_5\frac{i\hat{q}}{q^2}\gamma_\sigma\frac{i\hat{q}\,\hat{k}\,\hat{q}\,\hat{k}\,\hat{q}}{q^6}\gamma_5\,d^4q. \tag{15}$$

We evaluate the right side Eq. (15). First we remove the matrix γ_5 as before

$$\gamma_5 i\hat{q}\gamma_\sigma i\hat{q}\,\hat{k}\hat{q}\hat{k}\,\hat{q}\,\gamma_5=\hat{q}\gamma_\sigma\hat{q}\,\hat{k}\hat{q}\hat{k}\,\hat{q}.$$

Next an averaging over angles gives*

$$\overline{q_\alpha q_\beta q_\gamma q_\delta}=Bq^4(\delta_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}), \tag{16}$$
$$B=1/24$$

and therefore

$$\overline{\hat{q}\gamma_\sigma\,\hat{q}\,\hat{k}\,\hat{q}\,\hat{k}\,\hat{q}}={}^1/_3\,q^4\,(k^2\gamma_\sigma-\hat{k}k_\sigma).$$

Substituting this into Eq. (15), we reduce the right side to the form

$$-\frac{\lambda_1}{3}\frac{k^2\gamma_\sigma-\hat{k}\,k_\sigma}{p^2}\,a^2(x)\,c(x)\int\limits_y^x b^2(z)\,\xi(z)\,dz. \tag{17}$$

From Eq. (17) it is clear that we should look for a solution $\Gamma_\sigma^{(2)}(p,k)$ of the form

$$\Gamma_\sigma^{(2)}(p,\ k)=\frac{1}{3}\frac{k^2\gamma_\sigma-\hat{k}\,k_\sigma}{p^2}s(x,\ y) \tag{18}$$
$$\left(x=\ln\frac{p^2}{m^2};\quad y=\ln\frac{k^2}{m^2}\right)$$

Substituting Eq. (18) into (15) and using the same arguments which we applied in the derivation of Eq. (17), we find

[++] In the equation for $\Gamma_\sigma^{(1)}$ there are terms of order k/p^2 but none of order k/p, and the higher terms in the development of Γ_5 and Δ also give extra powers of $(1/p)$. From this it is obvious that the first-order term is absent in the expansion of Γ_σ in powers of (k/p).

* The form of Eq. (16) follows from considerations of symmetry. The value of B is easily obtained as follows. Since $\overline{q_1^4}= q^4 3B$; $\overline{q_1^2 q_2^2} = \overline{q_1^2 q_3^2} = \overline{q_1^2 q_4^2} = Bq^4$, we have $\overline{q_1^2}\,q^2= 6Bq^4$. But $\overline{q_1^2} = \frac{1}{4}q^2$, hence $B = 1/24$.

$$s(x,\ y) = -\frac{\lambda_1}{2}a^2(x)\,c(x)\int_y^x b^2(z)\,[\xi(z) - s(z,\ y)]\,dz. \tag{19}$$

5. We substitute Eqs. (7), (8), (9), (14) and (18) into the expression (6) for the polarization tensor, carry out the angular integrations, and introduce logarithmic variables. The result is

$$P_{\alpha\beta}(k) = {}^4/_3\,\varepsilon_1\,(k^2\delta_{\alpha\beta} - k_\alpha k_\beta)\int_y^L b^2(x)\,[\xi(x) - s(x,\ y)]\,dx,$$

with

$$\varepsilon_1 = e_1^2/(4\pi)^2$$

or alternatively[+]

$$d_t^{-1}(y) = 1 + {}^4/_3\,\varepsilon_1\int_y^L b^2(x)\,[\xi(x) - s(x,\ y)]\,dx. \tag{20}$$

Comparing Eqs. (20) and (19), we see that the function $d_t^{-1}(y)$ is simply related to $s(x,y)$. If we write

$$s(x,\ y) = -\frac{\lambda_1}{2}a^2(x)\,c(x)\,q(x,\ y), \tag{21}$$

then

$$d_t^{-1}(y) = 1 + {}^4/_3\,\varepsilon_1\,q(L,\ y). \tag{22}$$

By Eqs. (21) and (19), the function $q(x,\ y)$ satisfies the equation

$$q(x,\ y) = \int_y^x b^2(z)\left[\xi(z) + \frac{\lambda_1}{2}a^2(z)\,c(z)\,q(z,\ y)\right]dz. \tag{23}$$

[+]We observe that if at this point we carry out the renormalization according to Eqs. (3), (3a), (4), (4a), then Eq. (20) and (10) give

$$\tilde{d}_t^{-1}(y) = 1 + {}^4/_3\,\varepsilon Z_1'\left\{\int_y^L l^2(x)[\tilde{\xi}(x) - \tilde{s}(x,\ y)]\,dx - \int_0^L \tilde{b}^2(x)\,[\xi(x) - \tilde{s}(x,0)]\,dx\right\},$$

with

$$b(x) = b(0)\,\tilde{b}(x);\quad \xi(x) = \xi(0)\,\tilde{\xi}(x);\quad s(x,\ y) = \xi(0)\,\tilde{s}(x,\ y);$$

$$Z_1' = \frac{1}{\xi(0)} = \tilde{\xi}(L) = Z_2 = b(0)$$

Compare the analogous expressions in reference 3.

The integral equation (23) can be transformed into a differential equation by differentiating it with respect to x. This gives

$$\frac{\partial q(x,\ y)}{\partial x} = b^2(x)\left[\xi(x) + \frac{\lambda_1}{2}a^2(x)\,c(x)\,q(x,\ y)\right], \tag{24}$$

with the initial condition

$$q(x,\ x) = 0. \tag{24a}$$

We combine Eq. (24) with the differential equation for $\xi(x)$ obtained by differentiating Eq. (13),

$$\frac{d\xi(x)}{dx} = -\frac{\lambda_1}{2}a^2(x)\,b^2(x)\,c(x)\,\xi(x), \tag{25}$$

with the boundary condition

$$\xi(L) = 1. \tag{25a}$$

We multiply Eq. (24) by $\xi(x)$, Eq. (25) by $q(x,\ y)$, and add. The result is

$$\frac{\partial}{\partial x}[\xi(x)\,q(x,\ y)] = b^2(x)\,\xi^2(x)$$

and hence by Ward's identity (10)

$$\frac{\partial}{\partial x}[\xi(x)\,q(x,\ y)] = 1. \tag{26}$$

The constant λ_1 does not occur in this equation. Using the boundary condition (24a), we obtain from Eq. (26)

$$\xi(x)\,q(x,y) = x - y. \tag{27}$$

The boundary condition (25a) now gives

$$q(L,y) = L - y$$

and hence by Eq. (22)

$$d_t^{-1} = 1 + {}^4/_2\,\varepsilon_1\,(L - y). \tag{28}$$

After renormalizing by the rules described in references 3 and 4, Eq. (28) is transformed into Eq. (1), and this completes the proof of the assertion made at the beginning of this paper.

6. The results which have been obtained can be extended to the case of interaction with a scalar neutral field. In this case the interaction operator takes the form $U_1 = g_1\bar{\psi}\psi\,\varphi$, i.e. it differs from the pseudoscalar interaction by changing $i\gamma_5$ into 1. But we have seen that the matrices γ_5 appeared in our equations in pairs, separated by an odd number of matrices γ_α. Therefore

$$i\gamma_5(\ldots)\,i\gamma_5 = 1\,(\ldots),$$

i.e., the equations for both cases are identical.

The same is true for the asymptotic forms of the functions $a(x)$, $b(x)$ and $c(x)$ obtained in references 2 and 3. They also remain unchanged for a scalar field. In general, the difference between scalar and pseudoscalar fields cannot appear in effects for which the nucleon mass is negligible. This follows from the fact that there exists only one type of Dirac particle with $m = 0$, whereas with finite m there are two types of particles with different parity properties.

3. INTERACTION WITH A SYMMETRIC MESON FIELD

1. In the case of a symmetric meson field, the interaction operators are U_1 between nucleon and meson, U_2 between nucleon and photon, and U_3 between meson and photon, as follows:

$$\begin{aligned} U_1 &= ig_1\bar{\psi}\gamma_5\tau_j\psi\varphi_j, \\ U_2 &= ie_1\bar{\psi}\gamma_\sigma\frac{1-\tau_3}{2}\psi A_\sigma, \\ U_3 &= ie_1\varphi_j T_{3,jk}\frac{\partial}{\partial x_\alpha}\varphi_k A_\alpha. \end{aligned} \tag{29}$$

Here τ_j is the isotopic spin operator for the nucleon (a Pauli matrix), φ_j is the meson field (a vector in 3-dimensional isotopic space), and T_i is the isotopic spin operator for the meson,

$$T_{i,jk} = -ie_{ijk}$$

where e_{ijk} is the unit antisymmetric tensor.

As before we assume $e_1^2 \ll g_1^2$ and suppose that the nucleon and meson propagation functions and the meson-nucleon vertex function depend only on the interaction U_1. They have the same form (2) as in the case of the neutral field, but with different functions $a(x)$, $b(x)$ and $c(x)$ (see references 2 and 3). We keep the same notations for these functions, since we do not use their explicit forms. But the vertex functions of the nucleon-photon and meson-photon interactions must be investigated afresh. Denoting them by Γ_σ and V_σ respectively, we may write the vacuum-polarization tensor in the following general form

$$\begin{aligned} P_{\alpha\beta}(k) &= \frac{ie_1^2}{(2\pi)^4}\int \mathrm{Sp}\,\gamma_\alpha\frac{1-\tau_3}{2}G(p)\Gamma_\beta(p;k)G(p-k)\,d^4p \\ &- \frac{ie_1^2}{(2\pi)^4}\int \mathrm{Sp}\,i(2p_\alpha-k_\alpha)T_3\Delta(p)\,V_\beta\Delta(p-k)\,d^4p. \end{aligned} \tag{30}$$

The first term in Eq. (30) comes from nucleons (Fig. 1), the second from mesons (Fig. 3).

FIG. 3

The minus sign in the second term is connected with the Bose statistics of the mesons. The operations Sp act both on the Dirac matrices (in the first term) and on the isotopic spin matrices (in both terms). The factor $i(2p_\alpha - k_\alpha)$ is the Fourier transform of the operator $(\partial/\partial x_\alpha)$.

The vertex parts Γ_σ and V_σ satisfy integral equations which are represented graphically* in Figs. 4 and 5. As before, the wiggly lines denote photons,

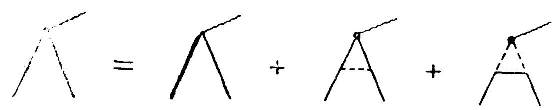

FIG. 4

FIG. 5

the dotted lines mesons, and the continuous lines nucleons. The "blob" vertices denote functions Γ_σ or V_σ, the vertices without a photon line denote $\Gamma_5\tau_\alpha$, and the ordinary vertices with a photon line denote $-\frac{1-\tau_3}{2}\gamma_\sigma$ or $i(2p_\sigma - k_\sigma)T_3$.

The graphs of Fig. 4 produce the equation

$$\begin{aligned} \Gamma_\sigma(p;k) &= \gamma_\sigma + \frac{ig_1^2}{(2\pi)^4}\int \tau_j\Gamma_5(p;p-q) \\ &\times G(q)\Gamma_\sigma(q;k)G(q-k) \\ &\times \Gamma_5(q-k;\,q-p)\tau_j\Delta(p-q)\,d^4q \\ &+ \frac{ig_1^2}{(2\pi)^4}\int \Gamma_5(p;l)G(p-l)\Gamma_5 \\ &\times (p-l;\,k-l)\Delta(l)\tau_j V_{\sigma,jk}\tau_k\Delta(l-k)\,d^4l, \end{aligned} \tag{31}$$

* In Figs. 4 and 5 we retain those graphs which in perturbation theory (i.e. when the equations are solved by iteration) give terms with asymptotic behavior $(g_1^2)^n L^n$, and we drop (considering g_1^2 to be small) those graphs which lead to asymptotic behavior $(g_1^2)^n L^m$ with $n > m$. But the graph of Fig. 5 actually belongs to the latter class. Graphs containing squares of nucleon lines (which are unrenormalizable by the usual methods) do not need to be considered in this problem.

and those of Fig. 5 produce the equation

$$V_{\sigma,jk}(q;k) = i(2q_\sigma - k_\sigma)T_{3,jk} + \frac{ig_1^2}{(2\pi)^4}\int \mathrm{Sp}\, G(f)\Gamma_\sigma(f;k)G(f-k) \times [\Gamma_5(f-k;q-k)G(f-k)\Gamma_5(f-q)\tau_j\tau_k - \Gamma_5(f-k;-q)G(f-k+q)\Gamma_5(f-k+q;q)\tau_k\tau_j]\,d^4f. \tag{32}$$

In Eq. (32) the two terms of the integrand correspond to the two directions in which the triangular loop in Fig. 5 can be described. In the case of neutral mesons, the expression corresponding to this closed loop vanishes because of the conservation of charge-parity (Furry's theorem), since a neutral spin-zero meson has even charge-parity. Hence it is clear that the isotopic spin dependence of the vertex-function V_σ must be given by

$$V_\sigma = T_3 Z_\sigma, \tag{33}$$

where Z_σ does not involve isotopic spin matrices. The vertex function V_σ will be written in the form

$$\Gamma_\sigma = {}^1/_2(X_\sigma - \tau_3 Y_\sigma), \tag{34}$$

where X_σ and Y_σ contain no isotopic spin matrices.

Substituting Eqs. (33) and (34) into (31) and (32) and using the relations

$$\tau_j\tau_j = 3;\quad \tau_j\tau_3\tau_j = -\tau_3;\quad \tau_j T_3 \tau_j = 2\tau_3;\quad \mathrm{Sp}\,\tau_j\tau_k = 2\delta_{jk};\quad \mathrm{Sp}\,T_3\tau_j\tau_k = -2T_{3,jk},$$

we eliminate the isotopic spin variables and obtain equations for X_σ, Y_σ and Z_σ:

$$X_\sigma(p,k) = \gamma_\sigma + \frac{3ig_1^2}{(2\pi)^4}\int \Gamma_5(p;p-q)G(q)X_\sigma(p,k) \times G(q-k)\Gamma_5(q-k;q-p)\Delta(p-q)\,d^4q; \tag{35}$$

$$Y_\sigma(p,k) = \gamma_\sigma - \frac{ig_1^2}{(2\pi)^4}\int \Gamma_5(p;p-q)G(q)Y_\sigma(q,k)G(q-k) \times \Gamma_5(q-k;q-p)\Delta(p-q)\,d^4q + 4i\frac{g_1^2}{(2\pi)^4}\int \Gamma_5(p;l)G(p-l)\Gamma_5(p-l;k-l) \times \Delta(l)Z_\sigma(l,k)\Delta(l-k)\,d^4l; \tag{36}$$

$$Z_\sigma(q,k) = i(2q_\sigma - k_\sigma) - \frac{ig_1^2}{(2\pi)^4}\int \mathrm{Sp}\,G(f)Y_\sigma(f,k)G(f-k) \times [\Gamma_5(f-k;q-k)G(f-k) \times \Gamma_5(f-q;-q) - \Gamma_5(f-k;-q) \times G(f-k+q)\Gamma_5(f-k+q;q)]\,d^4f. \tag{37}$$

This system separates into Eq. (35) determining X_σ, and the simultaneous Eqs. (36) and (37) determining Y_σ and Z_σ.

2. Eq. (35) differs from Eq. (6) only by a numerical coefficient, and can be solved by precisely the same method. Expanding X_σ by powers of k

$$X_\sigma(p,k) = X_\sigma^{(0)}(p) + X_\sigma^{(1)}(p,k) + X_\sigma^{(2)}(p,k),$$

we obtain

$$X_\sigma^{(0)} = \xi(x)\gamma_\sigma;\quad \xi(x) = \frac{1}{b(x)}\left(x = \ln\frac{p^2}{m^2}\right),\quad X_\sigma^{(1)} = 0,$$
$$X_\sigma^{(2)} = \frac{k^2\gamma_\sigma - \hat{k}k_\sigma}{p^2}s(x,y) = -\lambda a^2(x)c(x)q(x,y), \tag{38}$$

where $\xi(x)$ and $q(x,y)$ satisfy

$$\xi(x) = {}^3/_2\lambda_1\int_x^L a^2(z)b^2(z)c(z)\xi(z)\,dz + 1,$$
$$q(x,y) = \int_y^x b^2(z)[\xi(z) + {}^3/_2\lambda_1 a^2(z)c(z)q(z,y)]\,dz. \tag{39}$$

Transforming Eq. (37) into a differential equation, and using Ward's identity, we obtain the analog of Eq. (26)

$$\frac{\partial}{\partial x}[\xi(x)q(x,y)] = 1,\quad \xi(x)q(x,y) = x - y. \tag{40}$$

3. We determine Y_σ and Z_σ in a similar way by first expanding them in powers of k,

$$Y_\sigma = Y_\sigma^{(0)} + Y_\sigma^{(1)} + Y_\sigma^{(2)},\quad Z_\sigma = Z_\sigma^{(0)} + Z_\sigma^{(1)} + Z_\sigma^{(2)}, \tag{41}$$

By virtue of Ward's identity and Eq. (2), $Y_\sigma^{(0)}$ and $Z_\sigma^{(0)}$ are given by

$$Y_\sigma^{(0)} = i\frac{\partial G^{-1}(p)}{\partial p_\sigma} = \eta(x)\gamma_\sigma; \tag{42}$$

$$\eta(x)=\xi(x)=1/b(x); \tag{42a}$$

$$Z_\sigma^{(1)}=i\frac{\partial\Delta^{-1}(p)}{\partial p_\sigma}=2ip_\sigma\zeta(x); \tag{43}$$

$$\zeta(x)=1/c(x). \tag{43a}$$

The structure of the quantities $Y_\sigma^{(1)}$, $Y_\sigma^{(2)}$, $Z_\sigma^{(1)}$ $Z_\sigma^{(2)}$ can be determined (as in Section 2) by an explicit calculation of the inhomogeneous terms in the equations which are obtained by substituting Eq. (41) into (36) and (37). When this is done, the factor $Y_\sigma(f,k)G(f-k)$ in the integrand of Eq. (37) must be expanded as far as terms of order k^2/p^2. In the first integral of Eq. (36) the product $Y_\sigma(q,k)\,G(q-k)$ must be expanded to order (k^2/q^2); in the second integral the factors $Z_\sigma(l-k)\,\Delta(l-k)$ must be expanded to order (k^2/l^2) and the factor $G(p-l)$ to order (l/p). The last remark is connected with the fact that if we retained only $G^{(0)}(p-l)=G(p)\sim(1/p)$, we should obtain an integrand proportional to an odd power of the vector l_σ and the integral would vanish. Unlike the nucleon vertex operator Γ_σ, the meson vertex operator V_σ depends linearly on l_σ in the zero-order approximation. The next term in the expansion is $G^{(1)}(p,l)\sim(l/p^2)$ and leads to a logarithmic integral $[\int\ldots d^4l/l^4]$ with a coefficient proportional to (k^2/p^2). When this program is carried through, we obtain

$$Z_\sigma^{(1)}(p,k)=-ik_\sigma,$$

$$Y_\sigma^{(1)}(p,k)=\frac{\hat{p}\,k_\sigma}{p^2}r(x,y);$$

$$Y_\sigma^{(2)}(p,k)=\frac{k^2}{p^2}\gamma_\sigma u_1(x,y)+\frac{k^2\,\hat{p}\gamma_\sigma\hat{p}}{p^4}v_1(x,y)$$
$$+\frac{\hat{k}\,k_\sigma}{p^2}u_2(x,y)+\frac{\hat{p}\,\hat{k}\,\hat{p}\,k_\sigma}{p^4}v_2(x,y); \tag{44}$$

$$Z_\sigma^{(2)}(p,k)=\frac{ik^2p_\sigma}{p^2}w_1(x,y)+\frac{i(kp)\,k_\sigma}{p^2}w_2(x,y), \tag{45}$$

where we have separated the factors which depend only on the logarithmic variables $x=\ln(p^2/m^2)$ and $y=\ln(k^2/m^2)$.

We notice that $Z_\sigma^{(1)}$ and $Y_\sigma^{(1)}$ contain only longitudinal components (proportional to k_σ). The expressions for $Y_\sigma^{(2)}$ and $Z_\sigma^{(2)}$ also contain some purely longitudinal terms (those involving the functions u_2, v_2, and w_2). We can save time by ignoring the longitudinal terms completely. After substitution into Eq. (30), these terms give longitudinal components of the polarization tensor $P_{\alpha\beta}$. But this tensor satisfies the transversality condition $P_{\alpha\beta}k_\alpha=0$. Therefore these terms must either vanish or else cancel each other out exactly.

We thus neglect $Z_\sigma^{(1)}$ and $Y_\sigma^{(1)}$, and project the expressions (44) and (45) for $Y_\sigma^{(2)}$ and $Z_\sigma^{(2)}$ onto an arbitrary transverse vector e_σ with $e_\sigma k_\sigma=0$. Then the substitution into Eqs. (36) and (37) gives the following equations relating terms of second order in k:

$$\begin{aligned} u_1&=\frac{\lambda_1}{3}a^2(x)\,c(x)\,q_1(x,y),\\ v_1&=\frac{2}{3}\lambda_1a^2(x)\,b(x)\,q_2(x,y),\\ w_1&=-\frac{8}{3}\lambda_1a^2(x)\,b(x)\,q_1(x,y),\end{aligned} \tag{46}$$

where q_1 and q_2 satisfy the equations

$$\begin{aligned} &q_1(x,y)\\ &=\int_x^x b^2(z)\Big[\eta(z)-\frac{\lambda_1}{2}a^2(z)\,c(z)\,(q_1(z,y)\\ &+2\lambda_1a^2(z)\,b(z)\,q_2(z,p)\Big]dz,\\ &q_2(x,y)\\ &=\int_x^x c^2(z)\,[\zeta(z)+4\lambda_1a^2(z)\,c(z)\,q_1(z,y)]\,dz.\end{aligned} \tag{47}$$

The zero-order equations are the following

$$\begin{aligned} \eta(x)&=1-\frac{\lambda_1}{2}\int_x^L a^2(z)\,b^2(z)\,c(z)\,\eta(z)\,dz\\ &+2\lambda_1\int_x^L a^2(z)\,b(z)\,c^2(z)\,\zeta(z)\,dz,\\ \zeta(x)&=4\lambda_1\int_x^L a^2(z)\,b^3(z)\,\eta(z)\,dz.\end{aligned} \tag{48}$$

Eqs. (47) and (48) may be easily transformed into a system of differential equations

$$\begin{aligned} &\frac{\partial}{\partial x}[\eta(x)\,q_1(x,y)]=b^2(x)\,\eta^2(x)\\ &\quad-2\lambda_1a^2(x)\,b^2(x)\,c(x)\,[\eta(x)\,q_1(x,y)\\ &\qquad-\zeta(x)\,q_2(x,y)],\\ &\frac{\partial}{\partial x}[\zeta(x)\,q_2(x,y)]=c^2(x)\,\zeta(x)\\ &\quad+4\lambda_1a^2(x)\,b^2(x)\,c(x)\,[\eta(x)\,q_1(x,y)\\ &\qquad-\zeta(x)\,q_2(x,y)]\end{aligned} \tag{49}$$

with the boundary conditions

$$q_1(xx) = q_2(xx) = 0, \quad \eta(L) = \zeta(L) = 1. \tag{49a}$$

Since by Eq. (42)

$$b(x)\eta(x) = c(x)\zeta(x) = 1,$$

we may subtract the second Eq. (49) from the first, and with the notations

$$\eta q_1 - \zeta q_2 = R; \; 6\lambda_1 a^2 b^2 c = f,$$

we obtain $\dfrac{\partial R(x, y)}{\partial x} = -f(x) R(x, y),$

which gives

$$R(x, y) = A(y) \exp\left\{-\int^x f(z)\,dz\right\}.$$

But the boundary condition $R(x, x) = 0$ implies $A(x) = 0$, i.e. $R(x,y) = 0$ or

$$\eta(x) q_1(x, y) = \zeta(x) q_2(x, y). \tag{50}$$

Substituting Eq. (50) into (49), we obtain

$$\frac{\partial}{\partial x}[\eta(x) q_1(x, y)] = \frac{\partial}{\partial x}[\zeta(x) q_2(x, y)] = 1,$$

and with the boundary conditions this implies

$$\eta(x) q_1(x, y) = \zeta(x) q_2(x, y) = x - y. \tag{51}$$

4. We now return to the vacuum-polarization tensor. First of all, substituting Eqs. (33) and (34) into (30), we have

$$P_{\alpha\beta}(k) = \frac{ie_1^2}{(2\pi)^4}\left\{\frac{1}{2}\int \mathrm{Sp}\,\gamma_\alpha G(p)[X_\beta(p, k) + Y_\beta(p, k)]\,G(p-k)\,d^4p - 2\int i(2p_\alpha - k_\alpha)\Delta(p) Z_\beta(p, k)\Delta(p-k)\,d^4p\right\}. \tag{52}$$

Since the tensor $P_{\alpha\beta}$ is transverse, it may be written in the form

$$P_{\alpha\beta} = (\delta_{\alpha\beta} k^2 - k_\alpha k_\beta)\Pi. \tag{53}$$

To determine Π, it is enough to substitute into Eq. (52) instead of the vectors X_β, Y_β and Z_β the transverse components

$$X = X_\beta e_\beta; \; Y = Y_\beta e_\beta; \; Z = Z_\beta e_\beta \quad (e_\beta k_\beta = 0; \; e_\beta e_\beta = 1).$$

For $G(p - k)$ we substitute the expansion (7), (8), and similarly for $\Delta(p - k)$. From Eq. (2) it follows that

$$\Delta^{(0)}(p, k) = \Delta(p) = c(x)/p^2,$$
$$\Delta^{(2)}(p, k) = \frac{c(x)}{p^2}\left[\frac{4(pk)^2}{p^4} - \frac{k^2}{p^2}\right].$$

For X_σ, Y_σ and Z_σ we use Eqs. (38), (42), (43), (44) and (45). Making the transformation to Euclidean space and integrating over angles, we obtain

$$\Pi(y) = {}^2/_3\,\varepsilon_1 \int_y^L \{b^2(x)[\xi(x) + \eta(x) - {}^3/_2\, s(x, y) - {}^3/_2\, u_1(x, y) + 3v_1(x, y)] + c^2(x)[\zeta(x) - {}^3/_2\, w_1(x, y)]\}\,dx \tag{54}$$

or using also Eqs. (39), (46) and (47)

$$\Pi(y) = {}^2/_3\,\varepsilon_1\{q(L, y) + q_1(L, y) + q_2(L, y)\}. \tag{55}$$

But Eqs. (40) and (51) give

$$q(L, y) = q_1(L, y) = q_2(L, y) = L - y,$$

i.e.,

$$\Pi(y) = 2\varepsilon_1(L - y).$$

Therefore

$$d_t^{-1}(y) = 1 + \Pi(y) = 1 + 2\varepsilon_1(L - y). \tag{56}$$

After renormalization$^+$ by means of Eqs. (3) and (4), we obtain

$$\tilde{d}_t^{-1}(y) = 1 - 2\varepsilon y. \tag{57}$$

We see that the renormalized expression for the vacuum polarization

$$\tilde{\Pi}(y) = -2\varepsilon y \tag{58}$$

is independent of the meson coupling constant λ. $\tilde{\Pi}(y)$ is simply the sum of the proton polarization $\frac{4}{3}\epsilon y$ and the meson polarization $-\frac{2}{3}\epsilon y$ which are obtained from perturbation theory[6] in the limit when $y \gg 1$ and $\epsilon y \ll 1$.

4. CONCLUDING REMARKS

1. The result we have obtained, that the photon propagation function is independent of meson interactions for $e^2 \ll g^2 \ll 1$, is closely connected with the renormalizability of the theory with scalar coupling. The property of renormalizability already implies a strong restriction on the possible

$^+$ Since $\epsilon y \ll 1$, we have $d_t^{-1} = 1 - \tilde{d}_t$.

[6] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

behavior of the propagation functions. Consider first the purely electrodynamic problem. The function $d_t(y)$ depends parametrically on ϵ_1 and L

$$d_t(y) = f_1(y; \varepsilon_1, L).$$

The renormalizability means that after the transformations (3) and (4) the function $d_t(y)$ becomes independent of L. From the form of the original equations defining d_t (see reference 1), it is easy to see that y, ϵ_1 and L can occur in the solution only in the combination $\epsilon_1(L - y)$. In fact, the equations contain ϵ_1 linearly in front of the integral signs, and the limits of integration are L and y. Hence, after introducing the new variable $t = \epsilon_1(L - y)$, the equations no longer contain parameters in the coefficients or in the limits of integration. Thus

$$d_t^{-1}(y) = f[\varepsilon_1(L - y)].$$

By Eqs. (3) and (4), the function

$$\tilde{d}_t(y) = \frac{f[\varepsilon_1(L - y)]}{f(\varepsilon_1 L)}$$

with

$$\varepsilon_1 - \varepsilon f(\varepsilon_1 L) = 0$$

is independent of L. This is possible if f is a linear function

$$f(z) = 1 + \varkappa z, \tag{59}$$

in agreement with the result of reference 1 (actually $\varkappa = 4/3$).

Similarly, in the purely mesonic problem the solutions can depend on the parameters $\lambda_1 L$ only in the combination $\lambda_1(L - y)$. Consider the function

$$\begin{aligned}\theta^{-1}(x) &= a^2(x)\, b^2(x)\, c(x) \\ &= F^{-1}[\lambda_1(L - y)].\end{aligned} \tag{60}$$

By Eqs. (4) and (4a), the renormalized function

$$\tilde{\theta}(x) = \frac{F[\lambda_1(L - x)]}{F(\lambda_1 L)}$$

with

$$\lambda_1 - \lambda F(\lambda_1 L) = 0$$

is independent of L, which is possible if F is a linear function

$$F(z) = 1 + \nu z. \tag{61}$$

This agrees with the results of references 2, 3, both for neutral and symmetric meson fields (in the latter case $\nu = 5$).

2. In our problem d_t contains three parameters ϵ_1, λ_1 and L. Since we use perturbation theory with respect to ϵ_1, the function d_t has the following structure:

$$d_t^{-1}(y) = 1 + \varepsilon_1 \int_y^L \varphi[\lambda_1(L - y); \lambda_1(L - x)]\, dx = 1 + \frac{\varepsilon_1}{\lambda_1}\Phi[\lambda_1(L - y)]. \tag{62}$$

We renormalize the charge by Eqs. (4) and (62),

$$\varepsilon_1 = \frac{\varepsilon}{1 + \frac{\varepsilon}{\lambda_1}\Phi(\lambda_1 L)}$$

and the meson-nucleon coupling constant by Eqs. (4a), (60) and (61)

$$\lambda_1 = \lambda\theta(x) = \lambda F(\lambda_1 L); \quad \lambda_1 = \frac{\lambda}{1 - \nu\lambda L}.$$

then the function $\tilde{d}_t^{-1}(y)$, renormalized according to Eq. (3), takes the form

$$\tilde{d}_t^{-1} = \frac{1 + \frac{\varepsilon_1}{\lambda_1}\Phi[\lambda_1(L - y)]}{1 + \frac{\varepsilon_1}{\lambda_1}\Phi(\lambda_1 L)} \tag{63}$$

$$= 1 + \frac{\varepsilon(1 - \nu\lambda L)}{\lambda}\left\{\Phi\left[\frac{\lambda(L - y)}{1 - \nu\lambda L}\right] - \Phi\left(\frac{\lambda L}{1 - \nu\lambda L}\right)\right\}.$$

Eq. (63) is independent of L only when $\Phi(z)$ is a linear function of z. But in that case, it is easy to see that $\tilde{d}_t$ is also independent of λ.

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