The Theory of Strong Coupling for Meson Fields I

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Scalar and pseudoscalar meson fields which interact with infinitely heavy extended nucleons are considered in the strong coupling approximation. A theory of excitation is developed in the reciprocal coupling constant of the nucleons with the meson field.

A series of experimental facts and, in particular, the scattering of π-mesons by protons bear witness to the inapplicability of the theory of excitation (weak coupling) for the description of the interaction of the meson field with nucleons. In view of this, considerable interest attaches to the study of another limiting case for the meson field – that of strong coupling, i.e., expansion not in direct, but in reciprocal powers of the coupling constant of the nucleons with the meson field $g^2/\hbar c$.

In a series of published works, the meson field interacting with nucleons in the approximation of strong coupling has been studied. In the present paper I and in the following paper II, the nucleons are assumed to be extended and to be infinitely heavy, and the nucleonic vacuum is not considered. In a third paper, a field interacting with moving, extended nucleons of finite mass will be considered, and an attempt will be made to set up a relativistic theory with account taken of the nucleonic vacuum. The extension of the nucleon (smeared out interaction) is introduced temporarily. After passing to renormalization, the "radius" a of the nucleon must be equated to zero. In certain cases (nonguage coupling) such a transition to $a = 0$ can be made even before renormalization (see II).

1. We consider first the meson field corresponding to a spin of zero which interacts with a single quiescent, infinitely massive, extended nucleon. The Hamiltonian in this case has the form[1-3]:

$$H = \frac{1}{2}\int \sum_{\alpha=1}^{3} (\Pi_\alpha^2 + c^2(\nabla\Phi_\alpha)^2 + c^2\varkappa^2\Phi_\alpha^2 - 2\sqrt{4\pi}\, gcO_\alpha\Phi_\alpha)\, d\mathbf{r}, \tag{1}$$

$$[\Pi_\alpha(\mathbf{r}), \Phi_\beta(\mathbf{r}')] = -i\hbar\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}'); \quad [\Pi_\alpha(\mathbf{r}), \Pi_\beta(\mathbf{r}')] = [\Phi_\alpha(\mathbf{r}), \Phi_\beta(\mathbf{r}')] = 0.$$

Here $O_\alpha = \tau_\alpha U(\mathbf{r}-\mathbf{r}_1)$ for the scalar field and $O_\alpha = (\tau_\alpha/\varkappa)(\vec{\sigma}, \nabla U(\mathbf{r}-\mathbf{r}_1))$ for the pseudoscalar field; in the case of a neutral field, α has only a single value and τ_α must be replaced by unity. $\tau_1 = \tau_x$; $\tau_2 = \tau_y$; $\tau_3 = \gamma_1 + \gamma_2\tau_z$; $\varkappa = \mu c/\hbar$. $U(\mathbf{r}-\mathbf{r}_1)$ is the source function; $\int U(\mathbf{r})\, r d\mathbf{r} \approx a$ is the "radius" of the nucleon; $a \ll \varkappa^{-1}$. In the transition to a point source, $U(\mathbf{r}-\mathbf{r}_1) \to \delta(\mathbf{r}-\mathbf{r}_1)$. For $U(\mathbf{r}-\mathbf{r}_1)$, we can choose the formally covariant expression (four-dimensional representation; see reference 4):

$$U(\mathbf{r}-\mathbf{r}_1) = \int_{-\infty}^{\infty} F\left((\mathbf{r}-\mathbf{r}_1)^2 - c^2(t-t_1)^2\right) dt_1,$$

where F is some function of the invariant $(\mathbf{r}-\mathbf{r}_1)^2 - c^2(t-t_1)^2$.

From the Hamiltonian (1) we obtain the field equation:

$$\Box\Phi_\alpha - \varkappa^2\Phi_\alpha = -(g/c)\sqrt{4\pi}\, O_\alpha.$$

We expand Π_α and Φ_α in plane waves:

[1] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); **14**, 633 (1941).
[2] W. Pauli and S. Dancoff, Phys. Rev. **62**, 85 (1942).
[3] R. Server and S. Dancoff, Phys. Rev. **63**, 143 (1943).
[4] H. McManus, Proc. Roy. Soc. (London) **195A**, 323 (1948).

$$\Pi_\alpha = \sum p_{k\alpha} e^{i(\mathbf{k},\mathbf{r})}/\sqrt{V}; \quad \Phi_\alpha = \sum q_{k\alpha} e^{i(\mathbf{k},\mathbf{r})}/\sqrt{V}.$$

Then, from Eq. (1) we obtain

$$H = \frac{1}{2}\sum_{k,\alpha} (p^*_{k\alpha} p_{k\alpha} + \omega_k^2 q^*_{k\alpha} q_{k\alpha} - gcO_{\alpha k} q_{k\alpha}); \quad (2)$$

$$\omega_k^2 = c^2(\varkappa^2 + k^2);$$

$$O_{\alpha k} = \frac{2\sqrt{4\pi}}{\sqrt{V}} \int e^{i(\mathbf{k},\mathbf{r})} O_\alpha(\mathbf{r})\, d\mathbf{r}.$$

For $g = 0$ the positions of equilibrium $q^0_{k\alpha}$ of the field are equal to zero; for $g \neq 0$, the $q^0_{k\alpha}$ differ from zero, thanks to an energy of interaction of the meson field linear relative to $q_{k\alpha}$. We can find $q^0_{k\alpha}$ by minimizing the potential energy of the oscillators of the field with respect to $q_{k\alpha}$. In the case of a neutral scalar field, when $O_{\alpha k}$ is a number, $q^0_{k\alpha}$ is determined from the equation:

$$\omega_k^2 q_{k\alpha} - (gc/2) O_{\alpha k} = 0.$$

In the general case $O_{\alpha k}$ and, consequently, the potential energy V_f of the meson field are spin operators (by spin we imply both ordinary and isotopic spin):

$$V_f = {}^1/_2 \sum_{k,\alpha} (\omega_k^2 q^*_{k\alpha} q_{k\alpha} - gc\, q_{k\alpha} O_{\alpha k}).$$

Therefore, in order to find the equilibrium positions $q^0_{k\alpha}$ in this case, we must first diagonalize V_f in the spin variable and then minimize the eigenvalues $V_{f\lambda}$ with respect to $q_{k\alpha}$. Here the $q^0_{k\alpha}$ will in general be different for each of the four spin states λ. Inasmuch as the $q^0_{k\alpha}$ are evidently proportional to g, as in the case of a neutral scalar field, the potential energy of the equilibrium position will be proportional to g^2. If we now take into account the kinetic energy of the meson field, $T_f = \frac{1}{2}\sum_{k,\alpha} p^*_{k\alpha} p_{k\alpha}$, and consider vibrations about the position of equilibrium $q^0_{k\alpha}$ for each of the spin states, then the energy of vibration will be determined by the usual expression: $\Sigma(n_{k\alpha} + \frac{1}{2})\hbar\omega_k$. Then the energy of vibration is proportional to g^0. For the neutral scalar field, when O_α is not an operator, preliminary separation of the potential energy and minimization of it represents a rigorous operation which leads to the same results as in the well-known canonical transformation, which diagonalizes the Hamiltonian [Eq. (1)][5]. In the general case, when O_α is a spin operator, initial neglect of the kinetic energy T_f and diagonalization of the single potential energy V_f according to the spin variables is sensible only if T_f is significantly less than V_f for $q_{k\alpha} = q^0_{k\alpha}$, i.e., as follows from the estimates made above, if $g^2/\hbar c$ is sufficiently large. For a sufficiently large value of $g^2/\hbar c$ (a more precise criterion for the applicability of the expansion for different variants of coupling will be determined later), the scheme of reasoning mentioned above can be assumed on the basis of the simple method of diagonalization of the total Hamiltonian (1) or (2). We first neglect the kinetic energy of the meson field T_f in the Hamiltonian; we then diagonalize the remaining potential energy in the spin variables and find the eigenfunctions which depend on the spin variables, and the eigenvalues. The eigenfunctions and eigenvalues, like the parameters, depend on the $q_{k\alpha}$. If we minimize the eigenvalues with respect to q for each of the spin states, we find the positions of equilibrium of the oscillators. In the following approximation, it is possible to take into account the kinetic energy neglected earlier and consider the vibrations of the oscillators about the known equilibrium positions. In contrast to the neutral scalar field, the potential energy of the vibrations, in view of the operator character of $O_{\alpha k}$, apart from terms quadratic in $q_{k\alpha} - q^0_{k\alpha}$ (which are proportional to g^0), will contain anharmonic terms which are proportional to g^{-2}, g^{-4}, etc. For practical calculations it is more convenient to consider the Hamiltonian (1) directly, not expanding Π_α and Φ_α in plane waves, as was done in Eq. (2).

The suggested method of solution of the Schrödinger equation is similar to the method which is employed in the theory of molecules. In the Hamiltonian which contains the coordinates of the electrons $\mathbf{r}_i$ and of the nucleus $\mathbf{R}_i$:

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[5] G. Wentzel, *Quantum Theory in Wave Fields*, p. 61.

$$H = \sum_i \frac{p_i^2}{2m} + \sum_i \frac{P_i^2}{2M} + V(\mathbf{r}_i, \mathbf{R}_i)$$

$$\equiv T_e(\mathbf{p}_i) + Tn(\mathbf{P}_i) + V(\mathbf{r}_i, \mathbf{R}_i),$$

we first neglect the kinetic energy of the nucleus, which is proportional to a small parameter m/M (in our case we discard the kinetic energy of the meson field), and the remaining Hamiltonian is diagonalized in terms of the electronic variables (in our case, in the spin variables). For each electronic state the energy is minimized with respect to the coordinates of the nucleus (in our case, with respect to the coordinates of the oscillator field). In the next approximation, the kinetic energy of the nucleus is taken into account and, for each electronic state m, the vibrations of the nucleus about its known equilibrium position and the rotation of the molecule as a whole are considered.

In the theory of molecules two variants of this method are applied. In the first case we select as the unperturbed Hamiltonian $H^0 = T_e + V(\mathbf{r}_i, \mathbf{R}_i)$ for arbitrary values of the variables $\mathbf{R}_i$, and as the perturbation H', the kinetic energy of the nucleus T_n[6,7]. Here the ψ function of zeroth approximation $\psi_m^0(\mathbf{r}_i, \mathbf{R}_i)$ depends not only on $\mathbf{r}_i$ but also on $\mathbf{R}_i$, the eigenvalues $E_m^0(\mathbf{R}_i)$ also depend on the $\mathbf{R}_i$. The total Ψ function of the system is sought in the form of an expansion in the functions of zero approximation $\psi_m^0(\mathbf{r}_i, \mathbf{R}_i)$:

$$\Psi_l(\mathbf{r}_i, \mathbf{R}_i) = \sum_m \chi_{lm}(\mathbf{R}_i)\, \psi_m^0(\mathbf{r}_i, \mathbf{R}_i);$$

$$\chi_{lm} = \chi_l^0 \delta_{lm} + \chi'_{lm};$$

As always in the theory of perturbation, it is assumed that $\chi'_{lm} \ll \chi_l^0$. Substituting $\Psi_l(\mathbf{r}_i, \mathbf{R}_i)$ in the equation $(H^0 + H')\Psi = E\Psi$, we must take it into account that the momenta of the nuclei, $\mathbf{P}_i$, do not commute with the $\mathbf{R}_i$ which enter into $\psi_m^0(\mathbf{r}_i, \mathbf{R}_i)$. However, in actuality, the nuclei in the molecule contain vibrations close to the minimum energy $E_l^0(\mathbf{R}_i)$ which plays the role of potential energy for the nucleus. Therefore, $E_l^0(\mathbf{R}_i)$ and $\psi_l^0(\mathbf{r}_i, \mathbf{R}_i)$ must be expanded in series of $\mathbf{R}_i - \mathbf{R}_{li}^0$ ($\mathbf{R}_{li}^0$ is the value of $\mathbf{R}_i$ which corresponds to minimum E_l^0) and limit ourselves to the first two or three terms of the series.

In view of this, we must go by a different path in the consideration of the bound states of the nuclei in the molecule; initially, we choose as the unperturbed Hamiltonian $H^0 = T_e + V(\mathbf{r}_i, \mathbf{R}_{li})$ where the $\mathbf{R}_{li}$ are fixed values of $\mathbf{R}_i$, which must subsequently be so chosen that they correspond to the minimum potential energy of the nuclei (reference 7, Sec. 3). The perturbation in this case is evidently

$$H' = Tn + V(\mathbf{r}_i, \mathbf{R}_i) - V(\mathbf{r}_i, \mathbf{R}_{li}).$$

We seek a Ψ function of the system in the form

$$\Psi_l(\mathbf{r}_i, \mathbf{R}_i) = \sum_m \chi_{lm}(\mathbf{R}_i)\, \psi_m^0(\mathbf{r}_i, \mathbf{R}_i).$$

It was shown by Born and Oppenheimer[7] that the $\mathbf{R}_{li}$ are determined from the equation

$$\int \psi_l^{0*}(\mathbf{r}_i, \mathbf{R}_{li}) \left[\frac{\partial V}{\partial \mathbf{R}_i}(\mathbf{r}_i, \mathbf{R}_i)\right]_{\mathbf{R}_i = \mathbf{R}_{li}} \psi_l^0(\mathbf{r}_i, \mathbf{R}_{li}) \prod_i d\mathbf{r}_i$$

$$= \left|\frac{\partial V}{\partial \mathbf{R}_{li}}\right|_{ll} = 0, \tag{4}$$

which represents the condition of solvability for the ψ function of first approximation. The condition (4) corresponds to the minimum potential energy of the nuclei in the lth state.

2. Two variants are also possible in the case of the meson field which interacts with the nucleon.

In the first variant:

$$H^0 = \frac{c^2}{2} \int \sum_\alpha \left[(\nabla \Phi_\alpha)^2 + \varkappa^2 \Phi_\alpha^2 - 2\sqrt{4\pi} \frac{g}{c} O_\alpha \Phi_\alpha\right] d\mathbf{r}$$

$$H' = {}^1/_2 \int \sum_\alpha \Pi_\alpha^2 d\mathbf{r};$$

[6] W. Pauli, *General Principles of Wave Mechanics.*

[7] M. Born and R. Oppenheimer, Ann. d. Physik **84**, 457 (1927).

we find the ψ function of zeroth approximation, which depends on the variable of ordinary and isotopic spin s, from the equation $H^0\psi^0_\lambda(s, \Phi_\alpha) = E^0_\lambda(\Phi_\alpha)\psi^0_\lambda(s, \Phi_\alpha)$. In this case it is expedient to construct Φ_α (and Π_α) from plane waves. The functionals ψ^0_λ and E^0_λ depend on the Φ_α as parameters.

We seek the Ψ function of the system in the form:

$$\Psi_\lambda = \sum_\mu \chi_{\lambda\mu}(\Phi_\alpha)\,\psi^0_\mu(s, \Phi_\alpha). \tag{5}$$

In the case of a charged pseudoscalar field, $\mu = 1, \ldots, 4$; for a charged scalar and neutral pseudoscalar field, $\mu = 1, 2$. If the eigenvalues E^0_λ are degenerate, than, in seeking the correct functions of zeroth approximation, i.e., $\chi_{\lambda\mu}$, the secular equation must be solved. This case will be considered in III.

If all the E^0_λ for a given Φ_α are not degenerate, it is immediately possible to assume that $\chi_{\lambda\mu} = \chi^0_\lambda\delta_{\lambda\mu} + \chi'_{\lambda\mu}$; $\chi'_{\lambda\mu} \ll \chi^0_\lambda$. Then, substituting (5) in (1) and multiplying by ψ^{0*}_μ we get, for $\mu = \lambda$:

$$|H'\chi^0_\lambda|_{\lambda\lambda} + \sum |H'\chi'_{\lambda\nu}|_{\lambda\nu} = (E_\lambda - E^0_\lambda)\chi^0_\lambda, \tag{6}$$

and for $\mu \neq \lambda$,

$$|H'\chi^0_\lambda|_{\mu\lambda} + \sum_\nu |H'\chi'_{\lambda\nu}|_{\mu\nu} = (E_\lambda - E^0_\mu)\chi'_{\lambda\mu}. \tag{7}$$

Here $|H'\chi'|_{\mu\nu} = (\psi^{0*}_\mu, H'\chi_{\lambda\nu}\psi^0_\nu)$ are the matrix elements for the spin functions ψ^0_μ. The H' acts on the parameter Φ_α which enters into ψ^0_μ. Limiting ourselves to terms of first order in H', we get from Eq. (7) an expression for $\chi'_{\lambda\mu}(\lambda \neq \mu)$:

$$\chi'_{\lambda\mu} = (E^0_\lambda - E^0_u)^{-1}|H'\chi^0_\lambda|_{\mu\lambda} \tag{8}$$

and substituting Eq. (8) in Eq. (6), the equation for χ^0_λ ($\chi'_{\lambda\lambda}$ can be set equal to zero; see Landau and Lifshitz[8]):

$$|H'\chi^0_\lambda|_{\lambda\lambda} + \sum_{\nu\neq\lambda}\{H'(E^0_\lambda - E^0_\nu)^{-1}|H'\chi^0_\lambda|_{\nu\lambda}\}_{\lambda\nu} + E^0_\lambda\chi^0_\lambda \equiv H_\lambda\chi^0_\lambda \tag{9}$$

$$\equiv \int \tilde{H}_\lambda d\mathbf{r}\,\chi^0_\lambda = E_\lambda\chi^0_\lambda.$$

In the Hamiltonian H_λ we must set $\Phi_\alpha = \varphi^0_\alpha + \varphi_\alpha$. If φ^0_α is determined from the condition of the variation minimum of H_λ (analogous to the condition $\partial E^0_l/\partial R_i = 0$ in the theory of molecules):

$$\frac{\delta H_\lambda}{\delta\varphi^0_\alpha} = \frac{\partial\tilde{H}}{\partial\varphi^0_\alpha} - \sum_k^3 \frac{\partial}{\partial x_k}\left(\frac{\partial\tilde{H}_\lambda}{\partial(\partial\varphi^0_\alpha/\partial x_k)}\right) = 0,$$

then the terms $H^{(1)}_\lambda$, linear in φ_α, will be absent in the functional expansion of $\tilde{H}_\lambda$ in powers of φ_α:

$$\tilde{H}_\lambda = \tilde{H}^0_\lambda + \tilde{H}^{(1)}_\lambda + H^{(2)}_\lambda + \cdots$$

It is not difficult to show that $\tilde{H}_\lambda$ from the beginning contains only quadratic terms in Π_α.

Concrete calculations according to the first method are usually rather difficult in the case of the meson field. Computations according to the second variant, which we shall consider in more detail[9], are much simpler.

In this case we immediately assume $\Phi_\alpha = \varphi^0_{\alpha\lambda} + \varphi_\alpha$. The field $\varphi^0_{\alpha\lambda}$ will be determined precisely later. In view of the fact that $\varphi^0_{\alpha\lambda}$ is not an operator, the commutator relations for φ_α and Π_α have the same form as for Φ_α and Π_α:

$$[\Pi_\alpha(\mathbf{r}), \varphi_\beta(\mathbf{r}')] = -i\hbar\delta_{\alpha\beta}\delta(\mathbf{r} - \mathbf{r}')$$

The unperturbed Hamiltonian is

$$H^0 = \frac{c^2}{2}\int\sum_\alpha^3\Big[(\nabla\varphi^0_{\alpha\lambda})^2 + \varkappa^2(\varphi^0_{\alpha\lambda})^2 - 2\sqrt{4\pi}\,\frac{g}{c}\,O_\alpha\varphi^0_{\alpha\lambda}\Big]d\mathbf{r},$$

[8] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, I, GITTL, 1948, pp. 162, 182, 498.

[9] B. Geilikman, Dokl. Akad. Nauk SSSR **90**, 359 (1953).

with the perturbation

$$H' = H - H^0 = H^{(1)} + H^{(2)};$$

$$H^{(1)} = c^2 \int \sum_\alpha \left[(\varkappa^2 - \Delta)\, \varphi^0_{\alpha\lambda} - \frac{g}{c} \sqrt{4\pi} O_\alpha \right] \varphi_\alpha d\mathbf{r}; \tag{10}$$

$$H^{(2)} = {}^1/_2 \int \sum_\alpha \left(\Pi_\alpha^2 + c^2 \varphi_\alpha (\varkappa^2 - \Delta)\, \varphi_\alpha \right) d\mathbf{r};$$

$H^{(1)}$ is first order in φ_α, $H^{(2)}$ is second order.

The eigenfunctions of the zeroth approximation, which depend on the variable s, satisfy the Schrödinger equation:

$$H^0 \psi^0_{\lambda\mu}(s, \varphi^0_{\alpha\lambda}) = E^0_{\lambda\mu} \psi^0_{\lambda\mu}(s, \varphi^0_{\alpha\lambda}). \tag{11}$$

We seek the complete Ψ function of the system in the form:

$$\Psi_\lambda(s, \varphi_\alpha) = \sum_\mu \chi_{\lambda\mu}(\varphi_\alpha)\, \psi^0_\mu(s, \varphi^0_{\alpha\lambda}) \qquad (\psi^0_\mu \equiv \psi^0_{\lambda\mu}),$$

where we write $\Psi_\lambda(\chi_{\lambda\mu})$ and the eigenvalue E_λ in the absence of degeneracy for $E^0_{\lambda\lambda}(E^0_{\lambda\lambda} \neq E^0_{\lambda\mu}$ for $\mu \neq \lambda)$ in the form of series in the expansion parameter:

$$\Psi_\lambda = \Psi^0_\lambda + \Psi^{(1)}_\lambda + \Psi^{(2)}_\lambda + \cdots;$$

$$\chi_{\lambda\mu} = \chi^0_\lambda \delta_{\lambda\mu} + \chi^{(1)}_{\lambda\mu} + \chi^{(2)}_{\lambda\mu} + \cdots;$$

$$E_\lambda = E^0_\lambda + E^{(1)}_\lambda + E^{(2)}_\lambda + \cdots; \quad E^0_\lambda \equiv E^0_{\lambda\lambda}.$$

We then get for $\Psi^{(1)}_\lambda$, according to perturbation theory,

$$(H^0 - E^0_\lambda)\, \Psi^{(1)}_\lambda = (E^{(1)}_\lambda - H^{(1)})\, \Psi^0_\lambda;$$

$$\Psi^0_\lambda = \chi^0_\lambda \psi^0_\lambda(s).$$

This equation has a solution if the right side is orthogonal (over spin coordinates) to $\psi^0_\lambda(s)$, i.e.,

$$(H^{(1)}_{\lambda\lambda} - E^{(1)}_\lambda)\, \chi^0_\lambda = 0;$$

Here

$$H^{(1)}_{\lambda\lambda} = (\psi^{0*}_\lambda, H^{(1)} \psi^0_\lambda)/(\psi^{0*}_\lambda, \psi^0_\lambda);$$

since $E^{(1)}_\lambda$ does not depend on the φ_α, this is possible for $\chi^0_\lambda \neq 0$ only under the condition $H^{(1)}_{\lambda\lambda} = 0$, i.e., as is evident from Eq. (10),

$$(\Delta - \varkappa^2)\, \varphi^0_{\alpha\lambda} = -\frac{g}{c} \sqrt{4\pi}\, |O_\alpha(\mathbf{r})|_{\lambda\lambda}; \tag{12}$$

$$|O_\alpha|_{\lambda\lambda} = (\psi^{0*}_\lambda, O_\alpha \psi^0_\lambda)/(\psi^{0*}_\lambda, \psi^0_\lambda).$$

Equation (12) is equivalent to the condition $\delta H_\lambda / \delta \varphi^0_\alpha = 0$ in the first variant. It also follows from the requirement of a minimum $\delta E^0_\lambda = 0$ for the energy $E^0_\lambda = (\psi^{0*}, H^0 \psi^0)/(\psi^{0*}, \psi^0)$ (see reference 9). For such a choice of $\varphi^0_{\alpha\lambda}$,

$$H^{(1)} = gc \sqrt{4\pi} \int \sum_\alpha (|O_\alpha|_{\lambda\lambda} - O_\alpha)\, \varphi_\alpha\, d\mathbf{r}.$$

The solution of Eq. (12) has the form

$$\varphi^0_\alpha = \frac{g}{c\sqrt{4\pi}} \int G(\mathbf{r}, \mathbf{r}')\, |O_\alpha(\mathbf{r}')|_{\lambda\lambda}\, d\mathbf{r}'; \tag{13}$$

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-\varkappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}.$$

We substitute Eq. (13) in Eq. (11):

$$\left\{ \frac{g^2}{2} \int \sum_\alpha [|O_\alpha(\mathbf{r})|^2_{\lambda\lambda} - 2O_\alpha(\mathbf{r})]\, |O_\alpha(\mathbf{r}')|_{\lambda\lambda} \right. \tag{14}$$

$$\left. \times\, G(\mathbf{r}, \mathbf{r}') d\mathbf{r} d\mathbf{r}' \right\} \psi^0_{\mu_1} = E^0_\mu \psi^0_\mu.$$

Equation (14) represents formally a nonlinear system of four algebraic equations for the four components of ψ^0_μ (in the case of a charged pseudoscalar field). However, the system is in reality easily solved in the following way[10]. We replace the mean values of the spin operators τ_α and $\tau_\alpha \sigma_k$ by undetermined numerical parameters: $|\tau_\alpha|_{\lambda\lambda} = \eta_\alpha$; $|\tau_\alpha \sigma_k|_{\lambda\lambda} = \xi_{\alpha k}$. Then the system is linear and can be solved without difficulty. The

[10] B. Geilikman, Dokl. Akad. Nauk SSSR 91, 39 (1953).

eigenfunctions and eigenvalues depend on these parameters. Evidently the numerical values of the parameters $\xi_{\alpha k}(\eta_\alpha)$ can be found from the equations:

$$\xi_{\alpha k} = (\overset{0*}{\psi}_{\lambda\lambda}(s, \xi_{\alpha k}), \tau_\alpha \sigma_k \overset{0}{\psi}_{\lambda\lambda}(s, \xi_{\alpha k})); \qquad (15)$$

$$\eta_\alpha = (\overset{0*}{\psi}_{\lambda\lambda}(s, \eta_\alpha), \tau_\alpha \overset{0}{\psi}_{\lambda\lambda}(s, \eta_\alpha)) \quad (\overset{0*}{\psi}, \overset{0}{\psi}) = 1.$$

It is simplest to find $\xi_{\alpha k}$ and η_α by comparing the expression for the eigenvalue $E^0_{\lambda\lambda}$ which is obtained by the solution of the system (14), in which $|\tau_\alpha \sigma_k|$ and $|\tau_\alpha|_{\lambda\lambda}$ are replaced by the parameters $\xi_{\alpha k}$ and η_α, with the general expression for $E^0_{\lambda\lambda}$:

$$E^0_{\lambda\lambda} = |H^0_\lambda|_{\lambda\lambda}$$

$$= -(g^2/2) \int \sum_\alpha |O_\alpha(\mathbf{r})|_{\lambda\lambda} |O_\alpha|(\mathbf{r}')|_{\lambda\lambda} \times G(\mathbf{r}, \mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}'.$$

Usually, as is easy to show, the equations (15) are equivalent to the conditions of a minimum $E^0_{\lambda\lambda}$ with respect to $\xi_{\alpha k}(\eta_\alpha)$. Evidently, in the case of the pseudoscalar field, $E^0_{\lambda\mu} \sim g^2/a^3\varkappa^2$ and in the case of the scalar field, $E^0_{\lambda\mu} \sim g^2/a$ (see II[11]). The eigenfunctions of the Hamiltonian $H^0 = H^0_\lambda$ in which are substituted the parameters $\xi^\lambda_{\alpha k}(\eta^\lambda_\alpha)$ found from Eq. (15), form a complete orthogonal set. Here it is essential that for each state λ the sets of numbers $\xi^\lambda_{\alpha k}(\eta^\lambda_\alpha)$ (and consequently the field $\varphi^0_{\alpha\lambda}$) are different, as was shown in reference 11. We assume that for the given Hamiltonian H^0_λ the eigenvalues $E^0_{\lambda\lambda}$ are not degenerate, although there can be similar $E^0_{\lambda\lambda}$ among the different λ ($E^0_{\lambda\lambda} = E^0_{\lambda'\lambda'}$), as is the actual case (see reference 11). The generalization to the case of the presence of degeneracy will be considered in III.

[11] B. Geilikmann, J. Exper. Theoret. Phys. USSR 29, 430 (1955).

We now substitute the expression for the complete ψ function in the Schrödinger equation with the total Hamiltonian (1) $H = H^0_\lambda + H'$. It is appropritae here, as in the first variant, to write down the equations for $\chi^{(1)}_{\lambda\mu}$, $\chi^{(2)}_{\lambda\mu}$ etc., separately, and to obtain one equation for $\chi'_{\lambda\mu} = \chi^{(1)}_{\lambda\mu} + \chi^{(2)}_{\lambda\mu} + \ldots$ in first, second, etc., approximations. Multiplying the Schrodinger equation by ψ^{0*}_μ we get for $\lambda = \mu$

$$H'_{\lambda\lambda}\chi^0_\lambda + \sum_\nu H'_{\lambda\nu}\chi'_{\lambda\nu} = E'_\lambda(\chi^0_\lambda + \chi'_{\lambda\lambda}) \qquad (16)$$

and for $\lambda \neq \mu$

$$H'_{\mu\lambda}\chi^0_\lambda + \sum_\nu H'_{\nu\nu}\chi'_{\lambda\nu} = (E^0_\lambda - E^0_{\lambda\mu} + E'_\lambda)\chi'_{\lambda\mu}; \qquad (17)$$

$$H'_{\mu\nu} = (\overset{0*}{\psi}_\mu H' \overset{0}{\psi}_\nu);$$

$$E'_\lambda = E_\lambda - E^0_\lambda = E^{(1)}_\lambda + E^{(2)}_\lambda + \ldots$$

Now, assuming that $\chi'_{\lambda\mu} \ll \chi^0_\lambda$ we find $\chi'_{\lambda\mu}$ from Eq. (17) (by the method of successive approximations) with accuracy to terms of second order in φ_α:

$$\chi'_{\lambda\mu} = \Big\{ H'_{\mu\lambda}(E^0_\lambda - E^0_{\lambda\mu} + E'_\lambda)^{-1} + \sum_{\nu\neq\lambda} H^{(1)}_{\mu\nu} H^{(1)}_{\nu\lambda} (E^0_\lambda - E^0_{\lambda\nu})^{-1} (E^0_\lambda - E^0_{\lambda\mu})^{-1} \Big\} \chi^0_\lambda; \qquad (18)$$

substituting Eq. (18) in (16), we find the equation χ^0_λ with accuracy to terms of third order in φ_α (we assume $\chi'_{\lambda\lambda}$ equal to zero):

$$(H_{\lambda 2} + H'_{\lambda 3} + H'_{\lambda 3})\chi^0_\lambda = E'_\lambda \chi^0_\lambda, \qquad (19)$$

$$H_{\lambda 2} = H'_{\lambda\lambda} + \sum_{\mu\neq\lambda} H^{(1)}_{\lambda\mu} H^{(1)}_{\mu\lambda} (E^0_\lambda - E^0_{\lambda\mu})^{-1}$$

$$= {}^1/_2 \int \sum_\alpha^3 (\Pi^2_\alpha + c^2(\nabla\varphi_\alpha)^2 + c^2\varkappa^2\varphi^2_\alpha)\, d\mathbf{r}$$

$$+ c^2 \int \sum_{\alpha,\beta}^{3,3} K_{\alpha\beta}(\mathbf{r}, \mathbf{r}')\varphi_\alpha(\mathbf{r})\varphi_\beta(\mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}';$$

$$K_{\alpha\beta}(\mathbf{r},\mathbf{r}') = 4\pi g^2 \sum_{\mu\neq\lambda} \frac{|O_\alpha(\mathbf{r})|_{\lambda\mu}|O_\beta(\mathbf{r}')|_{\mu\lambda}}{E^0_\lambda - E^0_{\lambda\mu}};$$

$$H'_{\lambda 3} = \sum_{\mu\neq\lambda} H^{(1)}_{\lambda\mu}H^{(1)}_{\mu\lambda}[(E^0_\lambda - E^0_{\lambda\mu}+E'_\lambda)^{-1} - (E^0_\lambda - E^0_{\lambda\mu})^{-1}];$$

$$H''_{\lambda 3} = \sum_{\mu\neq\lambda}\sum_{\nu\neq\lambda} H^{(1)}_{\lambda\mu}H^{(1)}_{\mu\nu}H^{(1)}_{\nu\lambda}(E^0_\lambda - E^0_{\lambda\mu})^{-1}(E^0_\lambda - E^0_{\lambda\nu})^{-1};$$

$$H^{(2)}_{\mu\nu} = H^{(2)}\delta_{\mu\nu}.$$

It is essential that $H^{(1)}_{\lambda\lambda} = 0$, whereupon the smallest terms in Eq. (10) are second order in φ_α.

3. At first we neglect in Eq. (19) the terms $H'_{\lambda 3}$ and $H''_{\lambda 3}$. By a variation of the Hamiltonian or with the aid of the quantum Poisson brackets, $\left(\dot{\varphi}_\alpha = \frac{i}{\hbar}[H_{\lambda 2}, \varphi_\alpha]; \dot{\Pi}_\alpha = \frac{i}{\hbar}[H_{\lambda 2}, \Pi_\alpha]\right)$, it is possible to obtain the equation of the field:

$$\Box\varphi_\alpha - \varkappa^2\varphi_\alpha = \int \sum_\beta \tilde{K}_{\alpha\beta}(\mathbf{r},\mathbf{r}')\varphi_\beta(\mathbf{r}')\,d\mathbf{r}':$$

$$\tilde{K}_{\alpha\beta} = K_{\alpha\beta}(\mathbf{r},\mathbf{r}') + K_{\beta\alpha}(\mathbf{r}',\mathbf{r});$$

if we set $\varphi_\alpha(\mathbf{r},t) = \varphi_\alpha(\mathbf{r})e^{-i\epsilon t/\hbar}$, then

$$(\varkappa^2 - \Delta)\varphi_{\alpha k} + \int \sum_\beta \tilde{K}_{\alpha\beta}(\mathbf{r},\mathbf{r}')\varphi_{\beta k}(\mathbf{r}')\,d\mathbf{r}' = \frac{\omega_k^2}{c^2}\varphi_{\alpha k}; \qquad (20)$$

$$\varepsilon_k = \hbar\omega_k.$$

Each solution of the system (20) can be represented in the form of a three component function $\varphi^\rho_k = \{\varphi^\rho_{k1}, \varphi^\rho_{k2}, \varphi^\rho_{k3}\}$. For a given quantum number k, which characterizes the motion of the meson in the space coordinates, $\rho = 1, 2, 3$. The eigenfunctions of Eq. (20), φ^ρ_k, form a complete orthogonal set: $\int (\varphi^{\rho*}_k, \varphi^{\rho'}_{k'})\,d\mathbf{r} = \delta_{\rho\rho'}\delta_{kk'}$, ω_k^2/c^2 are the eigenvalues of Eq. (20).

We expand Π and φ in the functions φ^ρ_k:

$$\varphi = \{\varphi_1, \varphi_2, \varphi_3\} = \sum_{k,\rho} q_{k,\rho}\varphi^\rho_k; \quad \Pi = \sum_{k,\rho} p_{k\rho}\varphi^\rho_k$$

and substitute in the equation $H_{\lambda 2}\chi^0_\lambda = E_{\lambda 2}\chi^0_\lambda$. Taking into account Eq. (20) we obtain

$$\tfrac{1}{2}\sum_{k,\rho}(p^*_{k\rho}p_{k\rho} + \omega_k^2 q^*_{k\rho}q_{k\rho})\chi^0_\lambda(q_{k\rho}) = E_{\lambda 2}\chi^0_\lambda(q_{k\rho}); \qquad (21)$$

$$E_{\lambda 2} = \sum_{k,\rho}(n_{k\rho} + \tfrac{1}{2})\hbar\omega_k.$$

Equation (21) corresponds to a set of mutually noninteracting meson oscillators. The eigenfunctions φ^ρ_k of Eq. (20) thus describe the motion of the meson which interacts with the nucleon. The energy of interaction

$$(\hbar^2/2\mu)\int \tilde{K}_{\alpha\beta}(\mathbf{r},\mathbf{r}')\ldots,$$

is of zeroth order relative to g, since $E^0_\lambda - E^0_{\lambda\mu} \sim g^2$. As $a \to 0$, the interaction undergoes transition to point interaction.

We consider first the solution of Eq. (20) for $\omega \geq c\varkappa$ ($\epsilon \geq \mu c^2$; free motion of the meson). In this case we get from Eq. (20)

$$\varphi_\alpha = a_\alpha e^{i(\mathbf{k},\mathbf{r})} - \sum_{\beta\mu}\int [L'_{\alpha\mu}(\mathbf{r})|O_\beta(\mathbf{r}')|_{\mu\lambda} + L''_{\alpha\mu}(\mathbf{r})|O_\beta(\mathbf{r}')|_{\lambda\mu}]\varphi_\beta(\mathbf{r}')\,d\mathbf{r}'. \qquad (22)$$

Here

$$k^2 = \frac{\omega_k^2}{c^2} - \varkappa^2; \quad L'_{\alpha\mu}(\mathbf{r}) = g^2(E^0_\lambda - E^0_{\lambda\mu})^{-1}\int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}|O_\alpha(\mathbf{r}')|_{\lambda\mu}\,d\mathbf{r}',$$

$$L''_{\alpha\mu}(\mathbf{r}) = g^2(E^0_\lambda - E^0_{\lambda\mu})^{-1}\int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}|O_\alpha(\mathbf{r}')|_{\mu\lambda}\,d\mathbf{r}'.$$

The solution of the system of integral equations (22) with degenerate nuclei can easily be found[12]

$$\varphi_\alpha = a_\alpha e^{i(\mathbf{k},\mathbf{r})} + \sum_{\mu,\beta} (c'_{\alpha\beta\mu}(\mathbf{k}) L'_{\alpha\mu}(\mathbf{r}) + c''_{\alpha\beta\mu}(\mathbf{k}) L''_{\alpha\mu}(\mathbf{r})). \quad (23)$$

We get a set of equations for the coefficients $c'_{\alpha\beta\mu}$ and $c''_{\alpha\beta\mu}$ by substituting Eq. (23) in Eq. (22):

$$c_{\alpha\beta\mu} + \sum_{\nu,\gamma}\Big[c'_{\beta\gamma\nu}\int |O_\beta(\mathbf{r})|_{\mu\lambda} L'_{\beta\nu}(\mathbf{r})d\mathbf{r} + c''_{\beta\gamma\nu}\int |O_\beta|_{\mu\lambda} L''_{\beta\nu} d\mathbf{r}\Big] + a_\beta\int |O_\beta|_{\mu\lambda} e^{i(\mathbf{k},\mathbf{r})}d\mathbf{r} = 0; \quad (24)$$

$$c''_{\alpha\beta\mu} + \sum_{\nu,\gamma}\Big[c'_{\beta\gamma\nu}\int |O_\beta|_{\lambda\rho} L'_{\beta\nu}\, d\mathbf{r} + c''_{\beta\gamma\nu}\int |O_\beta|_{\lambda\mu} L''_{\beta\nu} d\mathbf{r}\Big] + a_\beta\int |O_\beta|_{\lambda\mu} e^{i(\mathbf{k},\mathbf{r})}d\mathbf{r} = 0.$$

Equation (20) can also have solutions which correspond to the coupled state of the meson ($\epsilon < \mu c^2$). For example, for a scalar, nonrelativistic meson, for $\lambda = 1$, Eq. (20) has the form (we have assumed that $E^0_1 - E^0_{12} = \gamma g^2/a$):

$$-\frac{\hbar^2}{2\mu}\Delta\varphi_\alpha + \frac{4\pi\hbar^2 a}{\mu\gamma}\sum_{\beta=1}^{3}(|\tau_\beta|_{12}|\tau_\alpha|_{21} + |\tau_\alpha|_{12}|\tau_\beta|_{21}) U(\mathbf{r}-\mathbf{r}_1)\int U(\mathbf{r}')\varphi_\beta(\mathbf{r}')\,d\mathbf{r}' = \varepsilon'\varphi_\alpha; \quad \varepsilon' = \varepsilon - \mu c^2 \ll \mu c^2.$$

In the limit as $a \to 0$;

$$-\frac{\hbar^2}{2\mu}\Delta\varphi_\alpha + \frac{4\pi\hbar^2}{\mu\gamma}\sum_{\beta}(|\tau_\beta|_{12}|\tau_\alpha|_{21} + |\tau_\alpha|_{12}|\tau_\beta|_{21})\lim_{a\to 0} aU(\mathbf{r}-\mathbf{r}_1)\varphi_\beta(\mathbf{r}) = \varepsilon'\varphi_\alpha.$$

Negative levels are not possible if the interaction is not attractive, and in the case of attraction also if $U_0 \ll \hbar^2/\mu a^2$ (U_0 is the depth of the potential well)[8]. If this condition is not satisfied, coupled states of the mesons are possible. These states correspond to so-called isobars. The function of the coupled s-state for $r > a$ has the form $e^{-\alpha r}/r$; $\alpha = \sqrt{2\mu|\epsilon'|}/\hbar$. The energy of the coupled state, as well as the energy of the free state, is of zeroth order in g.

We consider the terms $H'_{\lambda 3}$ and $H''_{\lambda 3}$. The terms in $H'_{\lambda 3}$ quadratic in φ can easily be computed. The Hamiltonian $H_{\lambda 2} + H'_{\lambda 3}$ is decomposed into a sum of the Hamiltonians of the oscillators if we decompose Π and φ in the eigenfunctions $\tilde{\varphi}^\rho_k$ of Eq. (20). Here, however, the quantities $E^0_\lambda - E^0_{\lambda\mu}$ are replaced by the quantities $E^0_\lambda - E^0_{\lambda\mu} + E_{\lambda 2}(n_{k\rho})$ [with numbers $n_{k\rho}$ which are defined by the zero approximation, i.e., by Eq. (21)]. In such a substitution, only the frequencies ω_i of the discrete spectrum ($\hbar\omega_i < \mu c^2$) are changed. For $\epsilon \geqslant \mu c^2$, the functions $\tilde{\varphi}^\rho_k$ have the form of Eq. (23), but in the expression for $L'_{\alpha\mu}$ and $L''_{\alpha\mu}$ (in Eqs. (23) and (24), we now have $E^0_\lambda - E^0_{\lambda\mu} + E_{\lambda 2}(n_{k\rho})$ instead of $E^0_\lambda - E^0_{\lambda\mu}$. The values $L'_{\alpha\mu}(\mathbf{r})$ and $L''_{\alpha\mu}(\mathbf{r})$ represent diverging waves; actually, as $r \to \infty$, $L_{\alpha\mu} \sim f(\theta)\, e^{ikr}/r$. It is therefore easy to find the scattering cross sections of the meson by the nucleon, φ^ρ_k, from the expression (23), for $d\sigma$. If we assume, in the factor $E^0_\lambda - E^0_{\lambda\mu} + E_{\lambda 2}$ which enters into Eq. (23), $n_{k\rho} = 1$, $n_{k'\rho'} = 0$; $k' \neq k$, $\rho' \neq \rho$, we obtain the amplitude of the scattering of the meson in the state ρ (in the actual scattering problem the initial and final ψ functions of the state are linear combinations of functions with different $\rho = 1, 2, 3$; the degeneracy for different λ must also be considered, see below):

[12] V. I. Smirnov, *Course in Higher Mathematics* 4, GITTL, 1951, p. 50.

$$f_{\alpha\rho}(\vartheta) = \frac{1}{k}\sum_{\beta,\mu}\frac{\Gamma^{\rho}_{\alpha\beta\mu}}{\varepsilon(k) - (E^0_{\lambda\mu} - E^0_{\lambda})}; \qquad (25)$$

$$\Gamma^{\rho}_{\alpha\beta\mu} = kg^2\frac{1}{a^{\rho}_{\alpha}}\Big[c'_{\alpha\beta\mu}(\mathbf{k})\int e^{-i(\mathbf{k}',\mathbf{r})}\,|\,O_{\alpha}(\mathbf{r})\,|_{\lambda\mu}d\mathbf{r}'$$

$$+ c''_{\alpha\beta\mu}(\mathbf{k})\int e^{-i(\mathbf{k}',\mathbf{r})}\,|\,O_{\alpha}(\mathbf{r})\,|_{\mu\lambda}\,d\mathbf{r}\Big];$$

$$\mathbf{k}' = k\mathbf{r}/r.$$

Damping has been introduced in Eq. (25) in the usual fashion (see reference 8). It is essential that the quantities $E^0_{\lambda} - E^0_{\lambda\mu}$ which enter into Eq. (25) as well as the eigenvalue of the energy of the nucleon be of the order g^2.

The term $H''_{\lambda 3}$, cubical in φ_{α}, defines the interaction of the mesons (in addition to the analogous interaction between the nucleonic vacuum); it forms a correction to the energy of order g^{-2} (in the same way as the term of fourth order in $\varphi - H_{\lambda_4}$).

Thus in the order g^2 (zeroth approximation) we obtain the classical field φ^0_{α} (pseudomeson) and the eigenvalue of the energy of the nucleon; in order g^0 (first approximation) – the second quantized field--real mesons which do not interact among themselves, free and in a bound state (isobars), and in order g^{-2} (second approximation) – the anharmonicity of the field of real mesons (and corrections to the eigenvalue of the energy of the nucleon).

Knowing the field $\Phi_{\alpha} = \varphi^0_{\alpha} + \varphi_{\alpha}$ we can find the magnetic moment of the nucleon M in the state λ in the absence of mesons (see reference 2):

$$\mathbf{M}_{\lambda} = \frac{e\hbar}{4mc}(\vec{\sigma}_{\lambda\lambda} + \{\vec{\tau_z\sigma}\}_{\lambda\lambda}) + \frac{ec}{2\hbar}\int[\mathbf{r}, \mathbf{S}]\,d\mathbf{r};$$

for the pseudoscalar field

$$\mathbf{S} = \varphi_{2\lambda}\nabla\varphi_{1\lambda} - \varphi_{1\lambda}\nabla\varphi_{2\lambda}$$

$$-2(g\sqrt{\pi}/\varkappa c)(\varphi_{2\lambda}\{\vec{\tau_1\sigma}\}_{\lambda\lambda} - \varphi_{1\lambda}\{\vec{\tau_2\sigma}\}_{\lambda\lambda})U(\mathbf{r}-\mathbf{r}_1).$$

In the zeroth approximation $\Phi_{\alpha} \approx \varphi^0_{\alpha}$ and in accord with Eq. (13)

$$\mathbf{M}_{\lambda} = \frac{e\hbar}{4mc}(\vec{\sigma}_{\lambda\lambda} + \{\vec{\tau_z\sigma}\}_{\lambda\lambda}) \qquad (26)$$

$$+\left(\frac{g^2\alpha}{\hbar c}\right)\left(\frac{e}{\varkappa^2 a}\right)[\{\vec{\tau_2\sigma}\}_{\lambda\lambda}, \{\vec{\tau_1\sigma}\}_{\lambda\lambda}];$$

$$\alpha = -a\int d\mathbf{r}\,d\mathbf{r}'x\frac{\partial}{\partial x}\left(\frac{1}{|\mathbf{r}-\mathbf{r}'|}\right)U(\mathbf{r}')\Big[U(\mathbf{r})$$

$$+\frac{1}{4\pi}\int\frac{U(\mathbf{r}'')\,d\mathbf{r}''}{|\mathbf{r}-\mathbf{r}''|^3}\Big]; \quad \alpha \sim 1.$$

The calculated field φ_{α} gives the corrections of order g^0.

4. Let us consider the conditions of applicability of our theories. We have assumed that $\chi'_{\lambda\mu} \ll \chi^0_{\lambda}$; $H' \ll H^0(\varphi_{\alpha} \ll \varphi^0_{\alpha})$. In the absence of mesons, $\varphi^2_{av} \sim \frac{\omega^3_{max}}{c^3}(q^2_k)_{av} \sim \frac{\hbar}{ca^2}(\omega_{max} \sim \frac{c}{a})$, and $(\varphi^0)_{av} \sim \frac{g}{ca}$; $(\varphi^0)_{ps} \sim \frac{g}{c\varkappa a^2}$; therefore $H^{(1)} \ll H^0$ and $\chi'_{\lambda\mu} \ll \chi^0_{\lambda}$ ($H^{(2)}$ does not in general have nondiagonal matrix elements), if $g^2/\hbar c \gg 1$ in the case of a scalar field, and $g^2/\hbar c \gg \varkappa^2 a^2$ in the case of a pseudoscalar field. In the presence of mesons the conditions below ought to be satisfied: $\Sigma n_{k\rho}\hbar\omega_k \ll g^2/a$ (scalar field) and $\Sigma n_{k\rho}\hbar\omega_k \ll g^2/\varkappa^2 a^3$ (pseudoscalar field).

In the method just set forth, use is made of a system of spin functions $\psi^0_{\lambda\mu}$ and eigenvalues $E^0_{\lambda\mu}$ for each state, more precisely, one system ψ^0_{μ} and E^0_{μ}, but with different numerical values of the parameters $\xi^{\lambda}_{\alpha k}$ (η^{λ}_{α}).

It was shown in reference 11 that in the case of a symmetric field all $E^0_{\lambda\lambda}$ are equal: $E^0_{\lambda'\lambda'} = E^0_{\lambda\lambda'}$. It is easy to see that this degeneracy cannot change the energy spectra found above.

For simplicity, let us consider a scalar field, and let us assume that for each λ the energy and the Ψ function $\Psi_{\lambda} = \sum_{\mu}\chi_{\lambda\mu}\psi^0_{\lambda\mu}(s)$ have been to terms of order g^{-2}.

If we have two Ψ functions Ψ_1 and Ψ_2 with similar energies, then we can assume that the correct Ψ function of the system has the form $\Psi = c_1\Psi_1 + c_2\Psi_2$. This problem is similar to the problem of the particle which can move in two

potential wells located at a certain distance from each other. If the ψ functions for the motion of the particle in each well are known, the symmetric ψ function is a linear combination of these functions[13].

Substituting $\psi = c_1\Psi_1 + c_2\Psi_2$ in the Schröndinger equation, we get the secular equation for c_1 and c_2:

$$(V_{11}+\varepsilon)c_1 + (S\varepsilon + V_{12})c_2 = 0;$$

$$(S^*\varepsilon + V_{21})c_1 + (V_{22}+\varepsilon)c_2 = 0,$$

where

$$V_{11} = -\int(\Psi_1^*, (H-H_1)\Psi_1)\,d\tau;$$

$$V_{22} = E_1 - E_2 - \int(\Psi_2^*, (H-H_2)\Psi_2)\,d\tau; \quad (27)$$

$$V_{12} = S(E_1-E_2) - \int(\Psi_1^*, (H-H_2)\Psi_2)\,d\tau;$$

$$V_{21} = -\int(\Psi_2^*, (H-H_1)\Psi_1)\,d\tau;$$

$$\varepsilon = E - E_1;\ d\tau = \prod_{k,\rho} dq_{k\rho}^{(1)} = \prod_{k,\rho} dq_{k\rho}^{(2)};$$

$$S = \int(\Psi_1^*, \Psi_2)\,d\tau;$$

$$\Phi - \varphi_\lambda^0 = \sum_{k,\rho} q_{k\rho}^{(\lambda)}\varphi_{k\lambda}^{\rho},$$

whence $H_1\Psi_1 = E_1\Psi_1$ and $H_2\Psi_2 = E_2\Psi_2$.

Inasmuch as we have assumed that E_1 and E_2 have been computed with accuracy $\sim g^{-2}$, then $V_{11} \sim g^{-4}$, and $V_{22} = E_1 - E_2 + O(g^{-4})$. We estimate $S_1 V_{21}$ and V_{12}:

$$S = \sum_{\mu,\nu}(\psi_{1\mu}^{0*}, \psi_{2\nu}^0)\int\chi_{1\mu}^*\chi_{2\nu}\,d\tau;\quad \chi_\lambda^0 = \prod_{k,\rho} F_{n_{k\rho}}(q_{k\rho}^{\lambda});$$

$F_{n_{k\rho}}$ is the Hermite polynomial. It follows from Eq. (27) that

$$q_{k\rho}^{(2)} = \int\Big(\varphi_1^0 - \varphi_2^0 + \sum_{k',\rho'} q_{k'\rho'}^{(1)}\varphi_{k'1}^{\rho'}\Big)\varphi_{k2}^{\rho*}\,d\mathbf{r};$$

It was shown in reference 11 that $\varphi_{\alpha k\lambda}^{\rho} = \varphi_{\alpha k}\delta_{\rho\alpha}$ and $\varphi_{\alpha k1} = \varphi_{\alpha k2}$; then $q_{\alpha k}^{(2)} = q_{\alpha k}^0 + q_{\alpha k}^{(1)}$; $q_{\alpha k}^0 = 0$ for $\alpha = 1.2$; $q_{3k}^0 = \frac{g}{2c\sqrt{\pi V}\pi^2}\frac{1}{k^2+\varkappa^2}$; $S \approx \sqrt{\frac{\hbar c}{\rho^2}}\,e^{-(g^2/\hbar c)\ln(\varkappa a)^{-1}}$ and $V_{21} \approx \frac{g\sqrt{\hbar c}}{a}e^{-(g^2/\hbar c)\ln(\varkappa a)^{-1}}$; for the pseudoscalar field $S \approx \frac{\varkappa a\sqrt{\hbar c}}{g}e^{-g^2/(\hbar c\varkappa^2 a^2)}$,

$$\varepsilon = \frac{1}{2(1-|S|^2)}\Big[V_{11}+V_{22}-V_{12}S^*-V_{21}S$$

$$\mp\sqrt{(V_{11}+V_{22}-V_{12}S^*-V_{21}S)^2 + 4(1-|S|^2)(V_{12}V_{21}-V_{11}V_{22})}\Big] \quad (28)$$

for $V_{12} = V_{21} = S = 0$ the correction ϵ to E_1 is the same as for the nonsymmetrized function Ψ_χ. This correction, connected with V_{21}, V_{12}, S, i.e., with the symmetrization of the function ψ_χ is, according to Eq. (28), of the order of $e^{-g^2/\hbar c}(e^{-g^2/(\hbar c\varkappa^2 a^2)})$ and in view of this is negligible.

If we do not consider ϵ but the Ψ function of the system, then it is evident that, in the absence of degeneracy, for different λ we can use linear combinations of the functions χ_λ.

The results we have obtained are quite intelligible, since in the decomposition in a complete orthogonal set of functions $\psi_{\lambda\mu}^0$ the effect of the field corresponding to other λ was automatically considered (but with power, not exponential accuracy).

The same results are obtained if symmetrization of the functions of zeroth approximation is carried out initially: $\psi^0(s) = \Sigma c_\chi\psi_{\lambda\lambda}^0$, but the computations in this case are much more complicated.

The theory is easily generalized to the case of several nucleons which interact with the meson field[10, 14]. The Hamiltonian for n nucleons has the form

$$H = \frac{1}{2}\int\sum_\alpha(\Pi_\alpha^2 + c^2(\nabla\Phi_\alpha)^2 + c^2\varkappa^2\Phi_\alpha^2 - 2\sqrt{4\pi}gc\sum_i^n O_\alpha(\mathbf{r}-\mathbf{r}_i)\Phi_\alpha)\,d\mathbf{r}. \quad (29)$$

[13] P. Gombas, *The Problem of Many Particles*, IIL, 1952, p. 219.

[14] B. Geilikman, Dokl. Akad. Nauk SSSR **90**, 991 (1953).

As the Hamiltonian of zeroth approximation we choose

$$H^0 = \frac{c^2}{2}\int \sum_\alpha \Big[(\nabla\varphi_\alpha^0)^2 + \varkappa^2(\varphi_\alpha^0)^2 - 2\sqrt{4\pi}gc\sum_i^n O_{\alpha i}\varphi_\alpha^0\Big]d\mathbf{r},$$

in which the φ_α^0 satisfy the equation

$$(\varkappa^2 - \Delta)\varphi_\alpha^0 = \frac{g}{c}\sqrt{4\pi}\sum_i^n |O_{\alpha i}|_{\lambda\lambda}.$$

Then

$$H^0 = \frac{g^2}{2}\int \sum_{\alpha,i,j}^{3,n,n} [|O_{\alpha i}(\mathbf{r})|_{\lambda\lambda} - 2O_{\alpha i}(\mathbf{r})]\,|O_{\alpha j}(\mathbf{r}')|_{\lambda\lambda} \times G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}', \quad (30)$$

and the perturbation has the form $H' = H^{(1)} + H^{(2)}$;

$$H^{(1)} = gc\sqrt{4\pi}\int \sum_{\alpha,i}\varphi_\alpha(|O_{\alpha i}|_{\lambda\lambda} - O_{\alpha i})\,d\mathbf{r};$$

$$H^{(2)} = \frac{1}{2}\int \sum_\alpha (\Pi_\alpha^2 + c^2\varphi_\alpha(\varkappa^2 - \Delta)\varphi_\alpha)\,d\mathbf{r}.$$

The equation for $\chi'_{\lambda\mu}$ and the equation for χ^0_λ have the same form as in the case of a single nucleon – Eqs. (18) and (19), except that $K_{\alpha\beta}$ is now determined by the expression

$$K_{\alpha\beta} = 4\pi g^2 \sum_{\mu\neq\lambda}^{n,n}\sum_{i,j} |O_{\alpha i}(\mathbf{r})|_{\lambda\mu}\,|O_{\beta j}(\mathbf{r}')|_{\mu\lambda}(E_\lambda^0 - E_{\lambda\mu}^0)^{-1}.$$

The equation for φ_k^ρ coincides with Eq. (20), but with the new expression for $K_{\alpha\beta}$. The more general expression for the energy in the zeroth approximation has the form

$$E_\lambda^0 = H_{\lambda\lambda}^0 = -\frac{g^2}{2}\int \sum_{\alpha,i,j} |O_{\alpha i}(\mathbf{r})|_{\lambda\lambda}\,|O_{\alpha j}(\mathbf{r}')|_{\lambda\lambda}\,G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}',$$

Here E_λ^0 is the self-energy and the energy of interaction of the nucleons. If all $|\mathbf{r}_i - \mathbf{r}_j| \gg a$:

$$(E_\lambda^0)_{sc} = -\frac{g^2}{2}\sum_{i\neq j,\alpha} |\tau_{\alpha i}|_{\lambda\lambda}\,|\tau_{\alpha j}|_{\lambda\lambda}\frac{e^{-\varkappa|\mathbf{r}_i - \mathbf{r}_j|}}{|\mathbf{r}_i - \mathbf{r}_j|} + \text{const};$$

$$(E_\lambda^0)_{ps} = -\frac{g^2}{2\varkappa^2}\sum_{i\neq j,\alpha} (\{\tau_{\alpha i}\vec{\sigma}\}_{\lambda\lambda},\ \nabla_i)\,(\{\tau_{\alpha j}\vec{\sigma}\}_{\lambda\lambda},\ \nabla_j)\frac{e^{-\varkappa|\mathbf{r}_i - \mathbf{r}_j|}}{|\mathbf{r}_i - \mathbf{r}_j|} + \text{const}.$$

As is known, in the case of weak coupling, the self-energy and the energy interaction of the nucleons in the second approximation in the theory of perturbations appear as the diagonal matrix element of the spin operator V[5]:

$$V = -\frac{g^2}{2}\lim_{a\to 0}\int \sum_{\alpha,i,j} O_{\alpha i}(\mathbf{r})O_{\alpha j}(\mathbf{r}')\frac{e^{-\varkappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}\,d\mathbf{r}'. \quad (31)$$

In the presence of strong coupling, $O_{\alpha i}(\mathbf{r})O_{\alpha j}(\mathbf{r}')$ appears in Eq. (31) in place of $O_{\alpha i}(\mathbf{r})|O_{\alpha j}(\mathbf{r}')|_{\lambda\lambda}$, and $|O_{\alpha i}(\mathbf{r})O_{\alpha j}(\mathbf{r}')|_{\lambda\lambda}$ appears in $V_{\lambda\lambda}$, not $|O_{\alpha i}(\mathbf{r})|_{\lambda\lambda}|O_{\alpha j}(\mathbf{r}')|_{\lambda\lambda}$, as in E_λ^0.

Translated by R. T. Beyer
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