The Theory of the Hall Effect in Ionic Semiconductors

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The Hall coefficient is calculated for ionic semiconductors. The interaction of the electron with the polarizing vibrations of the crystal is considered both in the adiabatic approximation and the weak coupling case.

1. INTRODUCTION

THE calculation of the Hall coefficient R is usually carried out by means of the solution of the kinetic equation. However, as is pointed out in references 1 and 2, the quantization of the energy spectrum of the current carriers in crossed electric and magnetic fields $E \perp H$ remains unstudied.

The problem of the present research was the calculation of the Hall coefficient for an ionic semiconductor by the method of stationary states[2], which allows a consideration of the quantization of the energy spectrum by the fields $E \perp H (E \| OX, H \| OZ)$.

The following expression is obtained in Appendix II for the Hall coefficient R:

$$R = -j_y E_x (j_y^2 + j_x^2)^{-1} H^{-1}, \qquad (1)$$

where j_y, j_x are the currents, in an unbounded gyrotropic layer, along the axes OY and OX. As noted in Appendix II, $|j_y| > |j_x|$ usually, since the current j_x is proportional to a small parameter which characterizes the weak coupling of the current carrier with the phonons which scatter it. Therefore,

$$R \approx -\frac{1}{j_y}\frac{E}{H}. \qquad (2)$$

To compute the current j_y we need a concrete model of the semiconductor under consideration. For example, for the simplest solid, in which the current carrier is an electron with effective mass μ[2],

$$j_y^{\pm} = \mp e N_{\pm} \gamma, \qquad (3)$$

where $N_{\pm}$ is the number of current carriers [in semiconductors $N_{\pm}$ is a function of the temperature $N_{\pm} = N_{\pm}(T)$]: $\gamma = -cE/H = -\bar{v}_y$ is the mean y component of the velocity [the upper sign in (3) refers to holes, the lower to electrons].

Therefore, in this model, we obtain from Eqs. (2) and (3), for $|j_y^{\pm}| > |j_x^{\pm}|$:

$$R_+ \approx \pm 1/N_{\pm} ec. \qquad (4)$$

In the presence of combined conduction,

$$R = -\frac{j_y^+ + j_y^-}{(j_y^+ + j_y^-)^2 + (j_x^+ + j_x^-)^2}\frac{E}{H} \qquad (5)$$

$$= -\frac{c}{eH}\frac{N_- - N_+}{\left[(N_- - N_+)^2 \frac{c^2}{H^2} + (N_+ \tilde{u}_+ + N_- \tilde{u}_-)^2\right]},$$

where $\tilde{u}_-$, $\tilde{u}_+$ are the mobilities of the electrons and the holes.

The case of small $|N_- - N_+|$, i.e., semiconductors with a small amount of impurities, requires a special investigation, because, as follows from Eq. (5), it turns out that $|j_y^+ + j_y^-| < |j_x^+ + j_x^-|$ in this case, even for $|j_y^{\pm}| \gg |j_x^{\pm}|$.

2. THE HALL EFFECT IN IONIC SEMICONDUCTORS WITH WEAK COUPLING

We now consider the Hall effect in an ionic semiconductor in which the coupling between the electron and the optical vibrations is small.

As is given in reference 2, the operator of the y component of the velocity is defined by the relation

$$\hat{v}_y = \frac{1}{\mu}(P_y + \mu\omega_0 x - \Sigma \hbar f_y \xi_f^+ \xi_f), \qquad (6)$$

where $P_y = p_y + \Sigma \hbar f_y \xi_f^+ \xi_f$ is the component of the operator of the total momentum of the system; ξ_f^+, ξ_f are the operators of emission and absorption of the optical vibrations; $\omega_0 = eH/\mu c$. Carrying out a canonical transformation of Eq. (6)[2,3]

$$\xi_f \to \zeta_f = \xi_f - \alpha_f,$$

[1] S. Titeica, Ann. Physik 22, 129 (1935).

[2] M. I. Klinger, J. Exper. Theoret. Phys. USSR 26, 159 (1954).

[3] S. B. Tiablikov, J. Exper. Theoret. Phys. USSR 21, 16 (1951).

where

$$\alpha_f = -A_f\left(\hbar\omega + \frac{\hbar^2 f^2}{2\mu} + \hbar\gamma f_y - \frac{\hbar}{\mu}P_z f_z\right)^{-1},$$

$$A_f = \frac{1}{|f|}\sqrt{\frac{2\pi\hbar\omega c' e^2}{V}},$$

$$c' = \frac{1}{n_1^2} - \frac{1}{\varepsilon_1},$$

ω is the limiting frequency of the optical branch, we obtain the following expression for the y component of the velocity of the current carrier:

$$\hat{v}_y = \frac{1}{\mu}\{P_y + \mu\omega_0 x - \Sigma\hbar f_y |\alpha_f|^2 \qquad (7)$$

$$- \Sigma\hbar f_y (\alpha_f \zeta_f + \alpha_f^+ \zeta_f^+)\}.$$

Averaging $\hat{v}_y$ over the unperturbed state of the system, we get the following expression for $\bar{v}_y$ ($\alpha_f = \alpha_f^+$):

$$\bar{v}_y = \frac{1}{\mu}\{P_y + \mu\omega_0 x_0 - \Sigma\hbar f_y \alpha_f^2\}, \qquad (8)$$

or, after making the substitution [2] $x_0 = -\frac{P_y}{\mu\omega_0} - \frac{eE}{\mu\omega_0^2}$,

$$\bar{v}_y = -\gamma + J_y, \qquad (9)$$

where

$$J_y = -\frac{2\pi\hbar\omega c' e^2}{V\mu}\sum \hbar f_y / f^2$$

$$\left(\hbar\omega + \frac{\hbar^2 f^2}{2\mu} + \hbar\gamma f_y - \frac{\hbar}{\mu}P_z f_z\right)^2.$$

We calculate J_y for the special cases (1) $\hbar\omega > \mu\gamma^2/2$ and (2) $\hbar\omega < \frac{1}{2}\mu\gamma^2$.

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We change Eq. (9) from a summation to an integration and limit ourselves, in the expansion of J_y in powers of γ and P_z, to terms of order γ and P_z^2 (for $P_z^2 < 2\mu\,|\,\hbar\omega - \frac{1}{2}\mu\gamma^2\,|$); we get

$$J_y = -(\gamma/\mu)(J_1' + J_1'' P_z^2), \qquad (10)$$

where

$$J_1' \approx \frac{\varepsilon_0^2 \mu}{48\pi}\sqrt{\frac{\mu}{m}}, \quad J_1'' \approx \frac{\varepsilon_0^2}{160\hbar\omega\pi}\sqrt{\frac{\mu}{m}}.$$

$\varepsilon_0 = \left(\frac{2\pi c' e^2 \hbar\omega}{(\hbar\omega - \mu\gamma^2/2)^2}\sqrt{\frac{2m\,|\hbar\omega - \mu\gamma^2/2|}{\hbar^2}}\right)^{1/2} \ll 1$ is the small parameter in the theory of weak coupling [2,3].

2) $\hbar\omega < \frac{1}{2}\mu\gamma^2$.

In this case, in the transition in J_y from a summation to an integration, we must obtain the integral in the form of a principal value; this amounts to a neglect of the finiteness of the lifetime of the stationary state considered in the given approximation [2,3]. For the calculation we express J_y in the following form:

$$J_y = -\frac{\hbar\omega c' e^2}{(2\pi)^2\mu}\int \frac{(\hbar F_y - \mu\gamma)\,(dF)}{\left(F^2 - \frac{2\mu}{\hbar}\gamma F_y + \frac{\gamma^2\mu^2}{\hbar^2} + \frac{P_z^2}{\hbar^2} + \frac{2}{\hbar}P_z F_z\right)\left(A + \frac{\hbar^2 F^2}{2\mu}\right)^2}, \qquad (11)$$

where

$$A = \hbar\omega - \frac{\mu\gamma^2}{2} - \frac{P_z^2}{2\mu} < 0.$$

By means of an expansion of the integral in powers of γ and P_z, we obtain

$$J_y = -\frac{\gamma}{\mu}(J_2' + J_2'' P_z^2), \qquad (12)$$

where

$$J_2' = -\frac{4}{3}\mu\varepsilon^2\left(\frac{\hbar\omega}{\mu\gamma^2}\right)^{3/2}, \quad J_2'' = -28\,\frac{\varepsilon^2}{\hbar\omega}\left(\frac{\hbar\omega}{\mu\gamma^2}\right)^{5/2}.$$

Substituting the expressions for J_y from Eqs. (10) and (12) in Eq. (9), we get

1) $\hbar\omega > \mu\gamma^2/2$;

$$\bar{v}_y \approx -\gamma\left(1 - \frac{\varepsilon_0^2}{48\pi}\sqrt{\frac{\mu}{m}} + \frac{\varepsilon_0^2}{160\,\mu\hbar\omega}\sqrt{\frac{\mu}{m}}P_z^2\right); \qquad (13)$$

2) $\hbar\omega < \mu\gamma^2/2$;

$$\bar{v}_y \approx -\gamma\left[1 + \frac{4\varepsilon^2}{3}\left(\frac{\hbar\omega}{\mu\gamma^2}\right)^{3/2} + \frac{28\varepsilon^2}{\mu\hbar\omega}\left(\frac{\hbar\omega}{\mu\gamma^2}\right)^{5/2}P_z^2\right]. \qquad (14)$$

It follows from Eqs. (13) and (14) that in this model $\bar{v}_y$ depends on the state (on P_z); therefore, in the calculation of the macro-current $j_y = -eN\langle\bar{v}_y\rangle$ (the symbol $\langle\ \rangle$ denotes the statistical average of $\bar{v}_y$ over the states), we must compute $\langle P_z^2\rangle$. As is shown in Appendix I, the equilibrium distribution function over P_z for the carriers has the following form in the weak coupling case*

* A misprint appears in the expression for T_1 in our earlier paper [reference 1, Eq. (4.5)].

(for

$$T_1 \lesssim T \lesssim \frac{\hbar\omega}{k}, \quad T_1 = \frac{\hbar\omega}{2k}\ln 2 - \left[\left(\frac{\hbar\omega}{2k}\ln 2\right)^2 - 4\frac{\hbar a_0^2}{k^2\gamma^2}|P_z|^3\right]^{1/2}):$$

$$f(P_z) = (2\pi\mu_e' kT)^{-1/2}\exp\left(-\frac{P_z^2}{2\mu_e' kT}\right), \quad (15)$$

where μ_e' is the effective mass of the quasiparticles moving along OZ. We obtain the expression for μ' from the expression for μ_e^2 by substituting

$$\bar{n}_0 = \left[\exp\left(\frac{\hbar\omega_0}{kT}\right) - 1\right]^{-1} \quad \text{for}$$

n_0 (see Appendix I).

Restricting ourselves to the case $T_1 \lesssim T \lesssim \hbar\omega/k$ and assuming that $\langle P_z^2\rangle = \mu_e' kT$, we obtain the following expression for

$$j_y = -eN\langle\bar{v}_y\rangle = j_y^0 + j_y' \quad [j_y^{0\pm} = \mp ecN_\pm(E/H)]:$$

1) $\hbar\omega > \frac{1}{2}\mu\gamma^2$:

$$j_y'^{\pm} \approx \mp ecN_\pm \frac{E}{H}\left(\frac{\varepsilon_0^2}{48\pi}\sqrt{\frac{\mu}{m}} - \frac{kT}{160\pi\hbar\omega}\varepsilon_0^2\sqrt{\frac{\mu}{m}}\frac{\mu_e'^{\pm}}{\mu_\pm}\right) \quad (16)$$

2) $\hbar\omega < \frac{1}{2}\mu\gamma^2$:

$$j_y'^{\pm} \approx \mp ecN_\pm \frac{E}{H}\left[\frac{4\varepsilon^2}{3}\left(\frac{\hbar\omega}{\mu_\pm\gamma^2}\right)^{3/2} + \frac{28\,kT}{\hbar\omega}\varepsilon^2\left(\frac{\hbar\omega}{\mu_\pm\gamma^2}\right)^{5/2}\frac{\mu_e'^{\pm}}{\mu_{e\pm}}\right]. \quad (17)$$

If we substitute j_y from Eqs. (16) and (17) in (2), we get in the two cases:

1) $\hbar\omega > \frac{1}{2}\mu\gamma^2$, $|j_y| > |j_x|$:

$$R_\pm \approx \pm\frac{1}{N_\pm ec}\left[1 - \frac{\varepsilon_0^2}{2\pi}\left(\frac{1}{24} - \frac{1}{80}\frac{kT}{\hbar\omega}\frac{\mu_e'^{\pm}}{\mu_\pm}\right)\right]; \quad (18)$$

2) $\hbar\omega < \frac{1}{2}\mu\gamma^2$:

$$R_\pm \approx \pm\frac{1}{N_\pm ec}\left[1 - \varepsilon^2\left(\frac{4}{3}\left(\frac{\hbar\omega}{\mu_\pm\gamma^2}\right)^{3/2} - \frac{28\,kT}{\hbar\omega}\left(\frac{\hbar\omega}{\mu_\pm\gamma^2}\right)^{5/2}\frac{\mu_e'^{\pm}}{\mu_\pm}\right]. \quad (19)$$

It follows from these equations that:

1) $N_\pm R_\pm$ depends on T and on $\gamma = cE/H$.

2) $R_\pm$ is determined by different expressions in the two cases $\hbar\omega > \mu\gamma^2/2$ and $\hbar\omega < \mu\gamma^2/2$.

We now return to the consideration of a semiconductor with combined conduction. If $N_- \gg N_+$ or $N_+ \gg N_-$, or

$$|j_y^+ + j_y^-| > |j_x^+ + j_x^-| \text{ and } |j_y^{0+} + j_y^{0-}| > |j_y'^+ + j_y'^-|,$$

then

$$R \approx -1/ec(N_- - N_+). \quad (20)$$

We take the case of a semiconductor without impurity ($N_- = N_+$).

In this case the expression which defines R depends on the ratio of the two quantities $|j_y'^+ + j_y'^-|$ and $|j_x^+ + j_x^-|$, since $j_y^{0+} + j_y^{0-} = 0$.

For $|j_x^+ + j_x^-| > |j_y'^+ + j_y'^-|$, $N_+ = N_- = N$.

$$R \approx \frac{ecN}{H^2(\sigma_+(H) + \sigma_-(H))^2}\frac{\varepsilon_0^2}{2\pi}\left(\frac{1}{24} - \frac{kT}{80\hbar\omega}\right) \times \left(\sqrt{\frac{\mu_+}{m}} - \sqrt{\frac{\mu_-}{m}}\right) \text{ for } \hbar\omega > \frac{\mu\gamma^2}{2}, \quad (21)$$

where $\sigma_+(H)$ and $\sigma_-(H)$ are the conductivities of the hole and electron currents in the magnetic field.

In this case R is defined by the relation of the effective mass $\mu_\pm$ and the conductivities $\sigma_+(H)$ and $\sigma_-(H)$: for large H ($\hbar\omega_0 > kT$), $j_x^\pm(H)$ changes as $\exp(-\hbar\omega_0/2kT)$ with increase in H, and $R \sim \exp(\hbar\omega_0/2kT)$.

In the other case, $|j_x^+ + j_x^-| < |j_y'^+ + j_y'^-|$, $N_- = N_+ = N$,

$$R = \frac{1}{ecN}\frac{2\pi}{\varepsilon_0^2}\left\{\left(\frac{1}{24} - \frac{kT}{80\hbar\omega}\right) \times \left(\sqrt{\frac{\mu_+}{m}} - \sqrt{\frac{\mu_-}{m}}\right)\right\}^{-1}. \quad (22)$$

Here R (in the approximation used) depends weakly on H for $\hbar\omega > \frac{1}{2}\mu\gamma^2$.

Thus in the case of a semiconductor without impurities ($N_- = N_+$) the Hall coefficient R has the following form:

1) for $\hbar\omega_0 > kT$, and with increasing magnetic field, R varies as $\sim \exp\{\hbar\omega_0/2kT\}$ up to the point where the current $|j_x^+ + j_x^-| \sim$

$\exp\{-\hbar\omega_0/2kT\}$ is no longer less than the current $|j'^{+}_y + j'^{-}_y|$ for a certain $H = H_0$.

2) For $H > H_0$ and $\hbar\omega > \frac{1}{2}\mu\gamma^2$, R depends weakly on H ($\mu'^{\pm}_e$, $\mu_\pm$ depend weakly on H).

3) R depends on T fundamentally as $N(T) \sim \exp(\Phi_{chem}/kT)$ (we can show that all the results obtained are valid for $|\Phi_{chem}| > \hbar\omega_0/2$).

For semiconductors with composite conductivity and an excess of one type of carrier, R is determined by Eq. (20). The same equation follows from kinetic theory[4]. However, while it is valid in our case for $|j^+_y + j^-_y| > |j^+_x + j^-_x|$, in the kinetic theory it holds for $|j^+_x| > |j^+_y|$ and $|j^-_x| > |j^-_y|$, i.e., for $|j^+_x| + |j^-_x| > |j^+_y| + |j^-_y|$.

3. HALL EFFECT IN A POLARIZED SEMICONDUCTOR

We make use of the results of reference 5 and the calculation of the Hall coefficient by the method of stationary states for polarized semiconductors. As before, we must find an expression for $\hat{v}_y$. In this case we separate the translational $\hat{v}^{(1)}_y$ and fluctuating $\hat{v}^{(2)}_y$ parts:

$$\hat{v}_y = (\hat{v}^{(1)}_y + \hat{v}^{(2)}_y), \tag{23}$$

$$\hat{v}^{(1)}_y = \frac{\hbar}{i\mu}\frac{\partial}{\partial\lambda_2} + \omega_0\lambda_1, \quad \hat{v}^{(2)}_y = \omega_0 q_1,$$

where $\mathbf{r} = \mathbf{q} + \vec{\lambda}$, $\mathbf{q}(q_1 q_2 q_3)$ represents the translational part[5,6] of the coordinates of the electron and $\vec{\lambda}$ its fluctuating part.

In zeroth approximation[5] the states of the system are characterized by the wave functions

$$\Psi^0 = \exp\{(i/\hbar)(P_y q_2 + P_z q_3)\} \tag{24}$$

$$\times\ H_n(q_1 - q_{10})\,\psi_N(\vec{\lambda})\prod_f \Theta_f(n_f),$$

where $q^0_1 = -eE/\mu\omega^2_0$, n is the quantum number of the "magnetic" oscillator, N is the quantum number of the fluctuating motion in the polarization potential hole of the polaron, in accord with the ground state of the polaron, $\mathbf{P}$ is the total momentum of the polaron. Here $\mathcal{H}_1 = \omega_0 q_1(p_{\lambda_2} + \mu\omega_0\lambda_1)$ [reference 5, Eq. (4)] is considered as a perturbation, since $(\mathcal{H}_1)_{nn'} \sim (q_1)_{nn'} \sim \sqrt{\hbar/\mu_{xx}\omega_0} \sim \epsilon \ll 1$[5].

With the help of Eq. (24), we average $\hat{v}_y$ and obtain*

$$\bar{v}_y = \omega_0(\bar{q}_1 + \bar{\lambda}_1) = \omega_0 q^0_1 + \omega_0\bar{\lambda}_1 \tag{25}$$

$$= -\gamma + \omega_0(\lambda_1)_{NN},$$

where

$$(\lambda_1)_{NN} = \int \Psi^*_N \lambda_1 \Psi_N (d\lambda).$$

As is seen from references 5 and 7, the quantities $(\lambda_1)_{NN}$ for the weakly perturbed states N of the discrete spectrum of the polaron whose potential well coincides with the ground state Ψ_0, are almost one and the same (in order of magnitude).

The states of the more perturbed part of the spectrum are not generally taken into account in the calculation of $\langle(\lambda_1)_{NN}\rangle$, since the corresponding Boltzmann factors are much smaller[8]. Therefore, we make the approximation

$$\langle(\lambda_1)_{NN}\rangle \approx (\lambda_1)_{00} = 2\pi\alpha_2 R^{21}_{10}, \tag{27}$$

where $R^{21}_{10} = \int \psi_{1s}(\lambda)\,\lambda_1\,\psi_{2p}(\lambda)(d\lambda)$. It was shown in reference 5 that α_2 is coupled with the polarization of the polaron Γ_0 in its ground state: $\alpha_2 R^{21}_{10} = -(\Gamma_0/2e)E$. Consequently, we get from Eqs. (25) and (27):

$$j^-_y = -eN\langle\bar{v}_y\rangle = eN_-\gamma(1 - \nu_- r^-_0 H^2), \tag{28}$$

where

$$\nu_- = \Gamma^-_0/2e^2, \quad r^-_0 = e^2/\mu_- c^2.$$

(For the holes,

$$j^+_y = -eN_+\gamma(1 - \nu_+ r^+_0 H^2),$$

where $\nu_+ = \Gamma^+_0/2e^2$, $r^+_0 = e^2/\mu_+ c^2$.)

The calculation in reference 5 of the spectrum of the polaron in crossed fields $E \perp H$ was carried

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* In the calculation of $\bar{v}_y$ in the following approximations, we must take a more accurate wavefunction.

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[5] M. I. Klinger, J. Exper. Theoret. Phys. USSR 26, 168 (1954).

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[7] S. I. Pekar and M. F. Deigen, J. Exper. Theoret. Phys. USSR 18, 481 (1948); Iu. S. Perlin, J. Exper. Theoret. Phys. USSR 20, 274 (1950).

[8] S. I. Pekar, *Investigation of the electronic theory of crystals*, GTTI, 1951.

out only for such E and H which produce slight changes in the polaron well[$H < H_k$ $= \left(\frac{2}{\chi_0} | J [\Psi_0] \right)^{1/2}$, reference 5, Eq. (17)]. It is easy to see in such a case that, from Eq. (28) the polarization part of the current is $| \Delta j_y^{\pm} | = eN_+ \gamma \nu_{\pm} r_0^+ H^2 < | j^{0\pm}_y | = eN_{\pm}\gamma$. In accord with Eqs. (20) and (28), for $| j_y | > | j_x |$,

$$R_{\pm} \approx -\frac{1}{j_y^{\pm}} \frac{E}{H} = -\frac{1}{N_{\pm} ec (1 - \nu_{\pm} r_0^{\mp} H^2)}. \quad (29)$$

In the case of polaron semiconductors with composite conductivity there can be two cases which we consider separately.

1) $| j_y^+ + j_y^- | > | j_x^+ + j_x^- |$:

$$R \approx -\frac{1}{ec} [N_- - N_+ - (\nu_- N_- r_0^- - \nu_+ N_+ r_0^+) H^2]^{-1}. \quad (30)$$

We note that for $H < \tilde{u}_{\pm} / \nu_{\pm} c r_0^+$ ($\tilde{u}_{\pm}$ is the mobility of the current carrier in the field $H \perp E$), i.e., for $| \Delta j_y^{\pm} | < | j_x^+ | = eN_{\pm} \tilde{u}_{\pm} E$, Eq. (30) is valid if $| N_- - N_+ | > (H/c) | N_- \tilde{u}_- + N_+ \tilde{u}_+ |$. If $H_k > H \gtrsim \tilde{u}_{\pm} / \nu_{\pm} c r_0^{\pm}$, i.e., $| \Delta j_y^{\pm} | > | j_x^{\pm} |$, then Eq. (30) is valid even for $N_- = N_+ = N$. In that case,

$$R \approx -1 / NecH^2 (\nu_- r_0^- - \nu_+ r_0^+). \quad (31)$$

For $N_- = N_+$ the sign of R is determined by the ratio of the effect mass of the electron μ_- and of the hole μ_+, since

$$\nu_- r_0^- - \nu_+ r_0^+ = B \frac{e^2}{mc^2} \left[\left(\frac{\mu_-}{m} \right)^2 - \left(\frac{\mu_+}{m} \right)^2 \right],$$

where B is a constant which is different for different materials.

2) For $H \ll \tilde{u}_{\pm} / \nu_{\pm} c r_0^{\pm}$ and $| N_- - N_+ | < (H/c) | N_- \tilde{u}_- + N_+ \tilde{u}_+ |$, $| j_y^+ + j_y^- | < | j_x^+ + j_x^- |$, and R is determined from the formula

$$R \approx -\frac{c}{H^2} \frac{[N_- - N_+ - H^2 (\nu_- N_- r_0^- - \nu_+ N_+ r_0^+)]}{e (N_+ \tilde{u}_+ + N_- \tilde{u}_-)^2}, \quad (32)$$

or, for $N_- = N_+ = N$,

$$R = -\frac{c (\nu_- r_0^- - \nu_+ r_0^+)}{eN (\tilde{u}_+ (H) + \tilde{u}_- (H))^2}. \quad (33)$$

CONCLUSION

In the present paper the Hall coefficient has been calculated for ionic semiconductors, considered both in the weak coupling approximation and in the adiabatic approximation. The method of stationary states was used, i.e., the calculation was carried out without use of the kinetic equation. The difference between our equations and those following from kinetic theory are most marked for $N_- = N_+$, i. e., in particular, in the case of a semiconductor without impurity. For example, in in this case, for a polaron semiconductor, R is determined by the ratio of the polarizabilities of the electron and hole polarons.

APPENDIX I

We find the equilibrium distribution in P_z - $f(P_z)$ with the aid of a Gibbs grand ensemble; if the energy of the system is given by the expression $E_c = E_c (P_z, P_y, n_0, n_f)$, then

$$f(P_z) = \sum_{P_y, n_0, n_f} \exp\left(-\frac{E_c}{kT}\right) \times \left\{ \sum_{P_z, P_y, n_0, n_f} \exp\left(-\frac{E_c}{kT}\right) \right\}^{-1}. \quad (I.1)$$

If we substitute in (I.1) the equation for E_c, computed in reference 2 with accuracy to ϵ^2 [Eq. (2.5)], we obtain (keeping the notation of reference 2):

$$f(P_z) = D \exp\left(-\frac{P_z^2}{2\mu_1 kT}\right) \overline{W}_1 \overline{W}_2, \quad (I.2)$$

where

$$\overline{W}_1 = \sum_{\ldots n_f \ldots} \exp\left\{-\frac{1}{kT}\left[\Sigma \mathcal{E}' (f) n_f + \frac{\hbar^2}{2\mu} \Sigma f_{1z} f_{2z} n_{f_1} n_{f_2} + \varepsilon^2 \Sigma \alpha_2 (f) n_f + \varepsilon^2 P_z^2 \Sigma C_2 (f) n_f\right]\right\};$$

$$\overline{W}_2 = \sum_{n_0} \exp\left\{-\frac{n_0}{kT} [\hbar\omega_0 + \varepsilon^2 C_1 P_z^2 + \varepsilon^2 \alpha_1]\right\}; \quad (I.3)$$

$$\mu_1 = \mu(1 + 2\mu\varepsilon^2 C_3)^{-1}; \quad D = \exp\left\{-\frac{\hbar\omega_0}{2kT}\right\} \times \left[\sum_{P_y, P_z, n_0, n_f} \exp\left(-\frac{E_c}{kT}\right)\right]^{-1}.$$

The quantities α_1, $\alpha_2(f)$; C_1, $C_2(f)$, C_3 are obtained by an expansion of ΔE in reference 5 [Eq. (2.5)] in powers of P_z (with accuracy to P_z^2):

$$\Delta_2 E = \varepsilon^2 \Sigma\alpha_2(f) n_f + \varepsilon^2 P_z^2 \Sigma C_2(f) n_f, \quad \text{(I.4)}$$

$$\Delta_1 E = (\Delta_1 E)_{P_z=0} + \varepsilon^2 C_3 P_z^2,$$

$$\Delta_3 E = \varepsilon^2 n_0 \alpha_1 + \varepsilon^2 n_0 C_1 P_z^2.$$

We return to the calculation of $\overline{W}_1$; inasmuch as $\epsilon \ll 1$, we can decompose $\overline{W}_1$ in powers of ϵ:

$$\overline{W}_1 = Z_\Gamma\left(1 + P_z^2 \frac{\overline{W}_1'}{Z_\Gamma} + \frac{\overline{W}_1''}{Z_\Gamma}\right), \quad \text{(I.5)}$$

where

$$Z_\Gamma = \sum_{\ldots n_f \ldots} \left\{-\frac{1}{kT}\left[\Sigma\mathcal{E}'(f) n_f + \frac{\hbar^2}{2\mu}\Sigma f_{1z} f_{2z} n_{f_1} n_{f_2}\right]\right\}$$

is the statistical sum of the system of quasiparticles $\mathcal{E}'(f)$.

$$\frac{\overline{W}_1'}{Z_\Gamma} = -\frac{\varepsilon^2}{kT}\Sigma C_2(f)\bar{n}_f; \quad \frac{\overline{W}_1''}{Z_\Gamma} = -\frac{\varepsilon^2}{kT}\Sigma\alpha_2(f)\bar{n}_f, \quad \text{(I.6)}$$

since, by definition,

$$\bar{n}_f = \frac{1}{Z_\Gamma}\sum_{\ldots n_f \ldots} n_f \exp\left\{-\frac{1}{kT}\left[\Sigma\mathcal{E}'(f) n_f + \frac{\hbar^2}{2\mu}\Sigma f_{1z} f_{2z} n_{f_1} n_{f_2}\right]\right\}$$

is the equilibrium value of the occupation number n_f, computed in reference 5. Consequently,

$$\overline{W}_1 = Z_\Gamma \exp\left\{-\frac{\varepsilon^2}{kT}[P_z^2 \Sigma C_2(f)\bar{n}_f + \Sigma\alpha_2(f)\bar{n}_f]\right\}.$$

By analogy,

$$\overline{W}_2 = (1 - e^{-\hbar\omega_0/kT})^{-1} \exp\left\{-\frac{\varepsilon^2\bar{n}_0}{kT}(P_z^2 C_1 + \alpha_1)\right\}. \quad \text{(I.7)}$$

Here we introduce the equilibrium number of oscillator quanta $\bar{n}_0 = [\exp\frac{\hbar\omega_0}{kT} - 1]^{-1}$. Then it follows from Eqs. (I.1)-(I.7) that

$$f(P_z) = D_1 Z_\Gamma \exp\left(-\frac{P_z^2}{2\mu_e' kT}\right), \quad \text{(I.8)}$$

where

$$D_1 = D(1 - e^{-\hbar\omega_0/kT})^{-1} \exp\left\{-\frac{\varepsilon^2}{kT}(\alpha_1\bar{n}_0 + \Sigma\alpha_2(f)\bar{n}_f)\right\};$$

$$\mu_e' = \mu\{1 + 2\mu\varepsilon^2(C_1\bar{n}_0 + \Sigma C_2(f)\bar{n}_f + C_3)\}^{-1}$$

(μ_e' differs from the μ_e in reference 5 by the fact that n_0 is replaced by $\bar{n}_0$).

Making use of the expressions for $\bar{n}_f(T)$, computed in reference 5 [Eqs. (4.5), (4.6)], we calculate Z_Γ:

$$Z_\Gamma = \exp\left[-\frac{\pi^2 k^2 T^2}{12\hbar a_0^2 |P_z - a_z|}\right] \text{ for } 0 < T < T_1,$$

$$Z_\Gamma = \exp\left[-\frac{\pi^2 k^{5/3}\mu^{1/3}T^{5/3}}{10\hbar(\omega a_0 \ln 2)^{1/3}}\right] \text{for } T_1 < T \ll \frac{\hbar\omega}{k},$$

where $a_z = \Sigma\hbar f_z \bar{n}_f(T)$.

Consequently, for $0 < T < T_1$,

$$f(P_z) = D_1 \exp\left[-\frac{P_z^2}{2\mu_e' kT} - \frac{\pi^2 kT}{12\hbar a_0^2 |P_z - a_z|}\right] \quad \text{(I.9)}$$

and for $T_1 < T \lesssim \hbar\omega/k$,

$$f(P_z) = (2\pi\mu_e' kT)^{-1/2} \exp\{-P_z^2/2\mu_e' kT\}. \quad \text{(I.10)}$$

APPENDIX II

In the method of stationary states, we initially calculate the currents j_x, j_y in an infinite gyrotropic medium (taking into account the quantization of the spectrum of the system in the crossed fields $E \perp H$), and then these j_x, j_y are substituted in the expression which connects the currents j_x, j_y with the Hall coefficient R and the resistance

in a transverse magnetic field, ρ. In this case, we introduce the tensors of electrical conductivity σ_{ik} and electrical resistance ρ_{ik} in a gyrotropic medium ($i, k = 1, 2$) and employ the local ratios of the fields E_x, E_y and the currents j_x, j_y, applying them initially to the case of a crystal bounded along OY ($j_y = 0$), in order to establish the connection of ρ and R with ρ_{ik} and σ_{ik}; then we consider the case of an unbounded gyrotropic crystal ($E_y = 0$), to connect R and ρ with the currents j_x and j_y computed with account of the quantization of the energy of the system in the field $E \perp H$. This can be done since the relations

$$E_i = \sum_{k=1}^{2} \rho_{ik} j_k \text{ and } j_i = \sum_{k=1}^{2} \sigma_{ik} E_k \quad (i, k = 1.2)$$

are local relations.

As is shown below, the ordinary expressions for ρ and R ($\rho = j_1 E_1 / j_1^2 + j_2^2$), $R = -j_2 E_1 / (j_1^2 + j_2^2)$ are obtained only if the Onsager relations are satisfied: $\sigma_{12}(H) = +\sigma_{21}(-H)$, $\rho_{12}(H) = \rho_{21}(-H)$. There are cases for which the Onsager relations are not satisfied, e.g., if σ_{12} and σ_{21} (and also ρ_{21}, ρ_{12}) depend not only on H but also on the external field E_1 (the local relation between j_i and E_i remains valid as before). In such a case, because of the axial symmetry of the system in the field H, $\rho_{11} = \rho_{22} \equiv \rho$, $\sigma_{11} = \sigma_{22} \equiv \sigma$, but $\rho_{12} \equiv \rho' \neq -\rho_{21}$ and $\sigma_{12} = \sigma' \neq -\sigma_{21}$. In Sec. 2 of this paper we considered such a case ($\hbar\omega < \frac{1}{2}\mu\gamma^2$).

We can set $\rho_{21} = -\rho' + \bar{\rho}$, $\sigma_{21} = -\sigma' + \bar{\sigma}$, where $\bar{\rho} = \rho_{12} + \rho_{21}$, $\bar{\sigma} = \sigma_{12} + \sigma_{21}$.

Then

$$E_1 = \rho j_1 + \rho' j_2, \quad E_2 = (-\rho' + \bar{\rho}) j_1 + \rho j_2; \quad \text{(II.1)}$$

$$j_1 = \sigma E_1 + \sigma' E_2, \quad j_2 = (-\sigma' + \bar{\sigma}) E_1 + \sigma E_2 \quad \text{(II.2)}$$

We first return to the conclusion of the expression for R: for $j_2 = 0$, $E_2 = +(-\rho' + \bar{\rho}) j_1 = RHj_1$, i.e., $RH = -\rho' + \bar{\rho}$.

It follows from (II.2) that

$$RH = -\rho' + \bar{\rho} = \frac{\sigma' - \bar{\sigma}}{\sigma^2 + (\sigma' - \sigma)^2 + \sigma'\bar{\sigma}}, \quad \text{(II.3)}$$

but for $E_2 = 0$ it follows from (II.2) that

$$\sigma = j_1 / E_1, \quad \sigma' - \bar{\sigma} = -j_2 / E_1. \quad \text{(II.4)}$$

Therefore, (II.3) reduces to the form

$$R = -\frac{j_2 E_1}{H} \{j_1^2 + j_2^2 - \bar{\sigma} j_2 E_1 + \bar{\sigma}^2 E_1^2\}^{-1} \quad \text{(II.5)}$$

This is the general formula for R.

If $\bar{\sigma} = \epsilon^2 \bar{\sigma}_0$ (as is the case in the present work for $\hbar\omega < \frac{1}{2}\mu\gamma^2$) with accuracy to $\epsilon^2 \ll 1$ we obtain inclusively [for $|j_2| > |j_1|$ $R_\pm \approx R_0^\pm \equiv \pm (N_\pm ec)^{-1}$ with accuracy to ϵ^4]:

$$R \approx -\frac{j_2 E_1}{H(j_1^2 + j_2^2)} \left(1 + \varepsilon^2 \frac{\bar{\sigma}_0 j_2 E_1}{j_1^2 + j_2^2}\right). \quad \text{(II.6)}$$

Consequently, for $|j_1| < |j_2| = |j_2^0 + \epsilon^2 j_2'|$

$$R \approx -E_1 / j_2^0 H = R_0. \quad \text{(II.7)}$$

For $|j_1| > |j_2|$

$$R \approx -(E_1 j_2^0 / j_1^2 H) - \varepsilon^2 E_1 j_2' / j_1'. \quad \text{(II.8)}$$

We begin consideration of the expression for the resistance in a transverse magnetic field $H \perp E_1$. It follows from Eqs. (II.1) and (II.2) that

$$\rho = \frac{\sigma}{\sigma^2 + (\sigma' - \bar{\sigma})^2 + \sigma'\bar{\sigma}} \quad \text{(II.9)}$$

$$= \frac{j_1 E_1}{j_1^2 + j_2^2} \left(1 - \frac{\bar{\sigma} j_2 E_1 - \bar{\sigma}^2 E_1^2}{j_1^2 + j_2^2}\right)^{-1}$$

If $\bar{\sigma} \sim \epsilon^2 \ll 1$, we obtain (with accuracy to ϵ^2 inclusively):

for $|j_2| > |j_1|$:

$$\rho \approx \rho^0 \left(1 + \varepsilon^2 \frac{\bar{\sigma}_0 E_1}{j_2}\right),$$

for $|j_1| > |j_2|$:

$$\rho \approx \rho^0 \left(1 + \varepsilon^2 \frac{\bar{\sigma}_0 j_2 E_1}{j_1^2}\right),$$

whence

$$\rho^0 = j_1 E_1 / (j_1^2 + j_2^2).$$

It should be noted that if the Onsager relations

are satisfied and $\overline{\rho} = \overline{\sigma} = 0$, then, in correspondence to the above,

$$R = -j_y E_x (j_x^2 + j_y^2)^{-1}, \quad \rho = j_x E_x (j_x^2 + j_y^2)^{-1}.$$

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