case of non-steady motion, since the disappearance of the surface energy σ' (the same applies as well to σ) cannot follow instantaneously upon the stopping of the body, and, apparently, metastable "discontinuities" can exist corresponding to zero velocity of relative motion between the wall and the helium II. The problem of the mechanism and the relaxation time τ, and also of the nature of such "discontinuities" (these appear to be strata in which the Ψ-function is perturbed relative to the corresponding lowest state), remains unclear Thus, as regards the influence of the energy σ' upon nonsteady flow, it is difficult to make any but purely qualitative statements. For example, in determining the viscosity of helium II from the damping of the oscillations of a disk, this damping may for $v < v_c$ be explained partially by the formation of discontinuities at the surface of the disk. Here, in the quasistationary case, an energy $4\sigma' S$ must be expended on the formation of discontinuities during the period θ of the oscillations, S being the surface area of the disk. Actually, however, in the experiments[8,9] the damping corresponding to $\sigma \sim 10^{-2}$ is 4 to 5 orders of magnitude smaller than the indicated quasistationary value, which demonstrates that the inequality $\tau \gg \theta \sim 10$ sec is fulfilled. Nevertheless, it is not impossible that the contribution to the damping associated with the formation of discontinuities is substantial, and that with it is to be connected the disparity between the results of the experiments[8,9] with oscillating disks and the measurements of the viscosity from the moment developed in rotating two coaxial cylinders relative to one another[10] (in the latter case the process is stationary, and the damping must be due solely to the viscosity); in reference 10 lower values are obtained for the damping at a low temperature, than in references 8 and 9, which accords with what has been said. The effect of the discontinuities may also be responsible for the peculiarities in the damping of a disk at large amplitudes[9]. In virtue of what has been stated, it seems to us that discontinuites in the flow velocity of helium II in the vicinity of walls deserve close attention.

The author is obliged to Academician L. D. Landau and to Professor E. M. Lifshitz for their discussion of this problem.

* At the same time, the hypothesis of the possible existence of velocity discontinuities within the bulk helium II becomes especially likely when it is considered that such discontinuities are known to exist in the vicinity of the wall.

** We note that in the theory of superconductivity[6] the surface energy σ_{ns} at the boundary between the superconducting and normal phases can be successfully evaluated from similar considerations. Thus, assuming that the thickness of the transition layer between the phases δ_0/κ (cf reference 6), we obtain

$$\sigma_{ns} \sim \frac{\hbar^2 n_s (\delta_0/\varkappa)}{2m(\delta_0/\varkappa)^2} \sim \frac{H_{km}^2 \delta_0}{2\pi\varkappa},$$

which agrees with more exact calculations[6] (for the symbols, reference 6, noting that $n_s = mc^2/4\pi e^2 \delta_0^2$ and $\kappa^2 = (2e^2/\hbar^2 c^2) H_{km}^2 \delta_0^4$).

*** If σ' and σ depend on v_s [for example, if $\sigma' = \sigma'(v_s \to 0) + bv_s^2$], then, in principle, it is possible for critical processes to develop, in conjunction with the fact that for $v \geq v_c$, $\sigma' \geq \sigma$. We note further that if we set $\sigma = \propto u^2$ (cf. reference 4), then $v_c = 0$. This circumstance, together with a number of others, indicates that (provided that all of the ideas under consideration are correct) the surface energies σ and σ' tend to a limit different from zero as $v_s \to 0$.

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The Effective Density of Rotating Liquid Helium II

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LANDAU and Lifshitz have shown[1] that in the rotation of a vessel containing He II, the normal part of the helium rotates as a whole, while

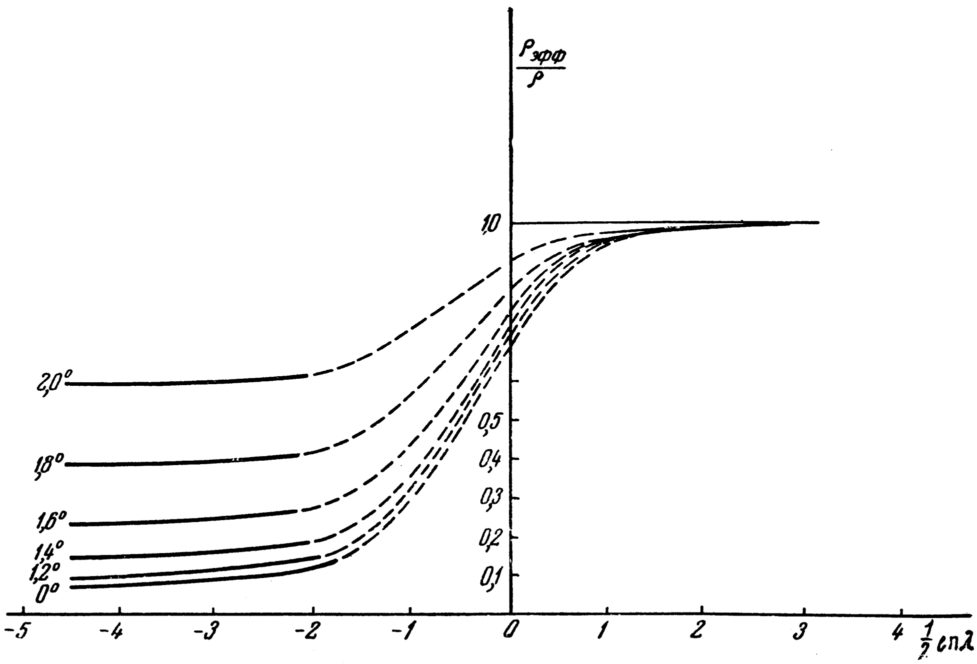

FIG. 1. Dependence of ρ_{eff}/ρ on the angular velocity Ω for various temperatures (the numbers on the curves are in degrees K).

for the superfluid, the cylindrical volume is divided into a series of coaxial cylindrical layers, in each of which superfluid motion takes place with a velocity distribution according to the law

$$v_s^{(i)} = \frac{b_i}{r}\ ;\ b_i = \frac{\Omega}{2}\,\frac{r_i^2 - r_{i+1}^2}{\ln(r_i/r_{i+1})} \tag{1}$$

We make use of the notation of the research just mentioned. The values of the radius of the boundaries of a section r_i are defined in reference 1 for two limiting cases:

a) slow rotation ($\lambda = \rho_S\,\Omega^2 R^3/8\alpha \ll 1$):

$$\sqrt{\xi_{i+1}}\ln\frac{1}{\sqrt{\xi_{i+1}}} = \frac{1}{2}\sqrt{\lambda x_i^3}, \tag{2}$$

$$\xi_{i+1} = \frac{x_{i+1}}{x_i},\quad x_i = \frac{r_i}{R}\ ;$$

b) fast rotation ($\lambda \gg 1$):

$$\Delta = \frac{r_i - r_{i+1}}{R} = \left(\frac{3}{8\lambda}\right)^{1/3}. \tag{3}$$

The latter formula is correct for layers located not too close to the axis of the cylinder.

The fact that the superfluid component takes part in the rotation leads to the unusual dependence of the momentum of the helium on the angular velocity (we recall that it was first believed that only the normal component rotated). This dependence is obtained by substituting, in the expression for the momentum per unit length of the vessel containing the helium,

$$M = 2\pi\int_0^R (\rho_n v_n + \rho_s v_s)\, r^2\, dr \tag{4}$$

the values $v_n = \Omega r$ and v_s in accord with Eqs. (1):

$$M = \frac{\pi\Omega R^4}{2}\left\{\rho_n + \rho_s \sum_j \frac{(x_j^2 - x_{j+1}^2)^2}{\ln(x_j/x_{j+1})}\right\}. \tag{5}$$

If we make use of the latter expression, it is natural to define the effective density of the rotating helium in the following manner:

$$M = \frac{\pi\Omega R^4}{2}\rho_{eff},\ \rho_{eff} = \rho_n + \rho_s \sum_j \frac{(x_j^2 - x_{j+1}^2)^2}{\ln(x_j/x_{j+1})}. \tag{6}$$

In the limiting cases, we have

a) slow rotation [in accord with Eq. (2)]:

$$\rho_{eff} = \rho_n + \rho_s\frac{1}{\ln(1/x_1)}, \tag{6'}$$

$$\sqrt{x_1}\ln\frac{1}{\sqrt{x_1}} = \frac{\sqrt{\lambda}}{2}\quad (\lambda \ll 1);$$

b) fast rotation [in accord with Eq. (2)]:

$$\rho_{eff} = \rho_n + \rho_s\left[1 - \frac{1}{3}\left(\frac{3}{8\lambda}\right)^{1/3}\right]\quad (\lambda \gg 1). \tag{6''}$$

The dependence of the isotherms ρ_{eff}/ρ on the angular velocity is shown in the Figure. The values of ρ_n/ρ are taken from the review of Lifshitz[2] (Fig. 210, p. 404).

Experimental investigation of the dependence of $\rho_{eff}(\Omega)$ provides, it seems to us, a definite possibility of finding the stratification of He II predicted by Landau and Lifshitz. In this case, in order that the meniscus effect does not distort the results, it is necessary that the liquid fill the vessel completely.

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ERRATUM

Khartman, Leont'eva, Siniaviskii and Vasil'ev, Soviet Phys. 1, 537-545 (1955). The two halves of the circuit diagram have been transposed. The half that appears on page 541 should be at the left; the half on page 540 should be at the right.