II. The thickness d of the emulsion layer satisfies the condition:

$$d > \Delta = \max S(\theta_0, \Phi)\sin\theta_0.$$

In this case, $\psi_2 = 0$ and $\psi_3 = \pi$ and the expression for σ' has the aspect presented by the next formula:

$$\frac{d}{2}\sigma' = \int\limits_0^{\pi/2} F_1(\theta_0, \Phi)\,d\Phi + \int\limits_{\pi}^{\pi/2} F_1(\theta_0, \Phi)\,d\Phi. \tag{4}$$

III. If the thickness d of the emulsion layer satisfies the condition $\delta < d < \Delta$, then we have either $\psi_2 = 0$; $\psi_3 < \pi$, or $\psi_3 = \pi$ and $\psi_2 > 0$; consequently, the corresponding integrations (quadratures) in Eq. (3) disappear.

In all the enumerated formulas, the functions $M(\theta, \Phi)$ and $N(\theta, \Phi)$ denote, respectively, the following equations:

$$M(\theta, \Phi) = \int f(\vartheta)\,S(\vartheta)\sin^2\theta\,d\theta;$$

$$N(\theta, \Phi) = \int f(\vartheta)\sin\theta\,d\theta.$$

If we approximate $f(\vartheta)$ and $S(\vartheta)$ with polynomials with regard to cos ϑ[2,3], then the calculation of the first four integrations (quadratures) in Eq. (3) can be performed easily and exactly. The last two integrations (quadratures), however, can be calculated by means of any formula of mechanical quadratures.

In this way formula (3) [accordingly, Eq. (4) in the case II], together with formula (1), fully solve the problem of finding the correction $w = 1 - (\sigma'/\sigma)$.

From the cited formulas it is easy to obtain the correction w for the case which was examined earlier[1]. For this purpose we posit $\alpha = 0$, then $\vartheta \equiv \theta$ and $S(\vartheta) \equiv S(\theta)$. The expressions for σ' assume, therefore, the following aspect. In case I:

$$\frac{d}{4}\sigma' = M(\theta_0) - M(0) + d\,\psi_2 N(\theta_0) \tag{5}$$

$$--M(\theta_0)\sin\psi_2 + \int\limits_0^{\psi_2} F(\overline{\theta})\,d\Phi.$$

Accordingly, in case II, we obtain:

$$\sigma' = \frac{4}{d}\,[M(\theta_0) - M(0)]. \tag{6}$$

Formulas (5) and (6) are simpler for practical calculations than the formulas cited earlier[1].

Let us remark that the choice of the layer thickness d and of the angle θ_0 permits us to reduce the majority of practical problems to case II. If, at the same time, $f(\vartheta)$ and $S(\vartheta)$ are given in the form of polynomials with regard to cos ϑ, then the computation of the correction w according to formulas (1) and (4) does not present any difficulties whatever.

I wish to express here my appreciation to Dr. Westmeier for his formulation of the problem and to A. Benet for a certain preliminary analysis.

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Translated by M. I. Weinreich
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An Elementary Derivation of the Formula for the Electromagnetic Energy in a Dispersive Medium

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LET us suppose that a substance of dielectric and magnetic susceptibility $\epsilon(\omega)$ and $\mu(\omega)$, respectively, fills a parallel plate condenser of capacity $C(\omega) = \epsilon(\omega)C_0$ and a thin solenoid of inductance $L(\omega) = \mu(\omega)L_0$, thus forming an oscillatory circuit. The free space values C_0 and L_0 are so chosen that the frequency ω is the natural frequency of the system given by: $\omega^2 = [L(\omega)C(\omega)]^{-1}$. Let us suppose that at $t < 0$ undamped oscillations take place in the circuit. The current through the solenoid, $Ie^{i\omega t}$, and the potential across the condenser, $Ve^{i\omega t}$, due to these oscillations satisfy the well-known condition $V = -i\sqrt{L/C}\;I$.

At $t = 0$ let us insert into the circuit a vanishingly small resistance R; then, at $t > 0$, the oscillations will have a complex frequency $\tilde{\omega}$, determined by the relation:

$$\tilde{\omega}L(\tilde{\omega}) - 1/\tilde{\omega}C(\tilde{\omega}) = iR.$$

It is easy to see that for the case of $R \to 0$, the solution of the above equation is $\tilde{\omega} = \omega + i\delta$, where δ and R are related by the equation:

$$\frac{R}{\delta} = \frac{d}{d\omega}(\omega L) + \frac{1}{\omega^2C^2}\frac{d}{d\omega}(\omega C) \tag{1}$$

$$= \frac{d}{d\omega}(\omega L) + \frac{L}{C}\frac{d}{d\omega}(\omega C).$$

Clearly, the initial energy of the circuit, W, is equal to the total dissipated heat energy.

$$W=\int_0^\infty \frac{1}{2} R\,|I|^2 e^{-2\delta t}\,dt=\frac{R}{4\delta}\,|I|^2.$$

Substituting Eq. (1) into the above relation, and noting that $\frac{L}{C}|I|^2=|V|^2$, we obtain:

$$W=\frac{1}{4}\left[|I|^2\frac{d}{d\omega}(\omega L)+|V|^2\frac{d}{d\omega}(\omega C)\right] \tag{2}$$

$$=\frac{1}{4} L_0\,|I|^2\,\frac{d}{d\omega}(\omega\mu)+\frac{1}{4}C_0\,|V|^2\frac{d}{d\omega}(\omega\varepsilon).$$

However, $L_0|I|^2=\frac{1}{4\pi}|H|^2\tau_m$, $C_0|V|^2=\frac{1}{4\pi}|E|^2\tau_e$, where H is the amplitude of the magnetic vector, τ_m is the volume of the solenoid, E is the amplitude of the electric vector, and τ_e is the volume of the capacitor. Hence, Eq. (2) can be written as follows:

$$W=\frac{|H|^2}{16\pi}\frac{d}{d\omega}(\omega\mu)\,\tau_m+\frac{|E|^2}{16\pi}\frac{d}{d\omega}(\omega\varepsilon)\,\tau_e. \tag{3}$$

From this we can conclude immediately, that the average electric and magnetic energy densities of a sinusoidal field in a dispersive medium are given by:

$$w_e=\frac{|E|^2}{16\pi}\frac{d}{d\omega}(\varepsilon\omega);\quad w_m=\frac{|H|^2}{16\pi}\frac{d}{d\omega}(\mu\omega). \tag{4}$$

These relations were obtained in references 1 and 2 by means of the Fourier integral method.

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The Second Viscosity of Monatomic Gases

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IT is well-known that monatomic gases, which obey Boltzmann statistics and which possess an energy spectrum of the form

$$\varepsilon=p^2/2m, \tag{1}$$

do not have a second viscosity[1]. We show here that this result also takes place if the monatomic gas* obeys quantum statistics (Fermi or Bose), provided that the energy is a power function of the momentum of the particle.

We write the kinetic equation for the distribution function n of the gas under consideration:

$$\frac{\partial n}{\partial t}+\frac{\partial n}{\partial \mathbf{r}}\frac{\partial H}{\partial \mathbf{p}}-\frac{\partial n}{\partial \mathbf{p}}\frac{\partial H}{\partial \mathbf{r}}=I(n). \tag{2}$$

Here $H(\mathbf{p},\mathbf{r})$ is the Hamiltonian of the particle, $I(n)$ is the collision integral. Let there be macroscopic motion of the gas with velocity $\mathbf{u}$, for which $\operatorname{div}\mathbf{u}\neq 0$. In this case the unexcited distribution function is equal to

$$n=\frac{1}{\exp\{(\varepsilon-\mu-\mathbf{pu})/T\}+1} \tag{3}$$

(the sign is negative in the case of Bose statistics, positive for Fermi statistics, $\epsilon(p)$ is the energy of the particle in the quiescent gas, μ is the chemical potential. In the absence of an external field the equality $H=\epsilon(p)$ holds. We substitute the n of Eq. (3) in the left side of the kinetic equation; for simplicity we shall consider the case here for which $\mathbf{u}=0$ (but $\operatorname{div}\mathbf{u}\neq 0$). Moreover, inasmuch as we are interested only in the second viscosity, i.e., in terms in the momentum flux which are porportional to div $\mathbf{u}$, it is reasonable that temperature gradients can be considered absent. For such a case we obtain for the left side of the kinetic equation

$$n'\left\{-\frac{\partial}{\partial t}\frac{\mu}{T}-\frac{\varepsilon}{T^2}\frac{\partial T}{\partial t}-\frac{1}{T}\frac{\partial\varepsilon}{\partial\mathbf{p}}\nabla(\mathbf{p},\mathbf{u})\right\}=I(n). \tag{4}$$

We now consider the equation of continuity for the entropy σ and the density ρ at $\mathbf{u}=0$, which have the following forms:

$$\frac{\partial\rho}{\partial t}+\rho\operatorname{div}\mathbf{u}=0,\quad \frac{\partial\sigma}{\partial t}=0 \tag{5}$$

(adiabatic condition).

In this case, Eq. (4) transforms to

$$n'\left\{\left[\rho\frac{\partial}{\partial\rho}\left(\frac{\mu}{T}\right)_\sigma+\frac{\varepsilon}{T^2}\left(\frac{\partial T}{\partial\rho}\right)_\sigma\rho\right.\right. \tag{6}$$

$$\left.-\frac{1}{3T}p\frac{\partial\varepsilon}{\partial p}\right]\operatorname{div}\mathbf{u}-\frac{1}{2T}\left(\frac{\partial\varepsilon}{\partial p_i}p_k-\frac{1}{3}\delta_{ik}\frac{\partial\varepsilon}{\partial p}p\right)$$

$$\left.\times\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}-\frac{2}{3}\delta_{ik}\frac{\partial u_l}{\partial x_l}\right)\right\}=I(n).$$

The second viscosity is absent in this case if the expression in square brackets on the left side of Eq. (6) vanishes. We show that this takes place if the energy ϵ is proportional to some power of the momentum

$$\varepsilon=ap^n. \tag{7}$$

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* By a monatomic gas we understand a set of particles each of which is characterized by three degrees of freedom (the three component vector momentum p).