SOVIET PHYSICS JETP VOLUME 2, NUMBER 1 JANUARY, 1956

Determination of Transverse Relaxation Times in the Magnetic Resonance of Atomic Nuclei in Weak High Frequency Magnetic Fields

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(Submitted to JETP editor May 22, 1954)
J. Exper. Theoret. Phys. USSR 29, 227-236 (August, 1955)

A method of measurement of the transverse relaxation times is developed, based on the determination of the interval of time between the extrema of the dispersion signal in a weak high-frequency magnetic field in the case of a nonadiabatic transition through resonance. Relations are obtained (and plotted graphically) which allow the determination of the time of transverse relaxation. As an experimental verification of the method, the dependence of the transverse relaxation time on the concentration of paramagnetic ions was measured in aqueous solutions of copper sulfate and ferric nitrate.

1. STATEMENT OF THE PROBLEM

UP to the present, the problem of the experimental determination of relaxation times in the magnetic resonance of atomic nuclei is still not completely clarified in the literature.

In the present work, a method is set forth for the determination of the times of transverse relaxation T_2 in a weak high-frequency magnetic field, i.e., when the following condition is satisfied[1]

$$\gamma^2 H_1^2 / a \ll 1, \qquad (1)$$

where γ is the gyromagnetic ratio of the nucleus under consideration, H_1 is the half amplitude of the high-frequency magnetic field, a is a quantity proportional to the velocity modulation of the longitudinal magnetic field H_z.

$$a = |\gamma| \frac{dH_z}{dt}. \qquad (2)$$

The theory of magnetic resonance for sufficiently small amplitude H_1 was given in references 1 and 2, where it was shown that in the case of linear modulation, when a = const, the dispersion of the signal $u(t)$ and the absorption $v(t)$ changed with time in the following manner:

$$u(t) = |\gamma| H_1 M_0 \times \int_{-\infty}^{t} e^{-(t-t')/T_2} \sin \frac{a}{2}(t^2 - t'^2)\, dt'; \qquad (3)$$

$$v(t) = -|\gamma| H_1 M_0 \times \int_{-\infty}^{t} e^{-(t-t')/T_2} \cos \frac{a}{2}(t^2 - t'^2)\, dt', \qquad (4)$$

where M_0 is the static magnetization.

If the process of transition through the resonance is carried out non-adiabatically, then the dispersion of the signal and the absorption have the form of curves with several extrema[2].

As has been shown[1,3], the signals u and v are related by the expression

$$\frac{du}{dt} + \frac{u}{T_2} + \Delta\omega(t)\, v = 0, \qquad (5)$$

where

$$\Delta\omega(t) = at.$$

The proposed method of measuring T_2 is based on the determination of the interval of time between the first two extrema of the signal of dispersion u upon entering into the resonance region. At the instant when u has an extremum, $du/dt = 0$; therefore, for such times, we have, by Eq. (5),

$$aT_2 t = -u(t)/v(t). \qquad (6)$$

If we now introduce the new variables $\eta = \sqrt{(a/2)\,t'}$ and denote

$$r = \sqrt{\frac{a}{2}}\, t, \quad \alpha = \frac{1}{T_2}\sqrt{\frac{2}{a}}, \qquad (7)$$

[1] S. D. Gvozdover and A. A. Magazanik, J. Exper. Theoret. Phys. USSR 20, 705 (1950)

[2] B. A. Jacobson and R. K. Wangsness, Phys. Rev. 73, 942 (1948)

[3] F. Bloch, Phys. Rev. 70, 460 (1946)

then Eq. (6) takes the form

$$\frac{2r}{\alpha} = \operatorname{tg}[r^2 - \psi(\alpha, r)], \quad (8)$$

where

$$\operatorname{tg}\psi(\alpha, r) = Q/P; \quad (9)$$

$$Q = \int_{-\infty}^{r} e^{\alpha\eta}\sin\eta^2 d\eta; \quad (10)$$

$$P = \int_{-\infty}^{r} e^{\alpha\eta}\cos\eta^2 d\eta; \quad (11)$$

$$Q = Q_0(\alpha) + Q_1(\alpha, r); \quad (12)$$

$$P = P_0(\alpha) + P_1(\alpha, r);$$

$$Q_0(\alpha) = \int_0^{\infty} e^{-\alpha\eta}\sin\eta^2 d\eta \quad (13)$$

$$= \sqrt{\frac{\pi}{2}}\left\{\cos\frac{\alpha^2}{4}\left[0.5 - C\left(\frac{\alpha^2}{4}\right)\right] + \sin\frac{\alpha^2}{4}\left[0.5 - S\left(\frac{\alpha^2}{4}\right)\right]\right\};$$

$$P_0(\alpha) = \int_0^{\infty} e^{-\alpha\eta}\cos\eta^2 d\eta$$

$$= \sqrt{\frac{\pi}{2}}\left\{\cos\frac{\alpha^2}{4}\left[0.5 - S\left(\frac{\alpha^2}{4}\right)\right] - \sin\frac{\alpha^2}{4}\left[0.5 - C\left(\frac{\alpha^2}{4}\right)\right]\right\};$$

S and C are Fresnel integrals;

$$Q_1(\alpha, r) = \int_0^{r} e^{\alpha\eta}\sin\eta^2 d\eta, \quad (14)$$

$$P_1(\alpha, r) = \int_0^{r} e^{\alpha\eta}\cos\eta^2 d\eta.$$

In this way the positions of the extrema of the signal u are determined by the transcendental Eq. (8), and for the determination of the time of transverse relaxation T_2 it is necessary to find the values of the roots of this equation.

2. DETERMINATION OF Q AND P FOR $r = \pm\alpha/2$

In the general case, the integrals Q and P [*Eqs.* (10) and (11)] are not expressed by tabulated functions. However, these integrals can be determined for two particular values $r = \pm\alpha/2$.

We multiply Eq. (10) by $j = \sqrt{-1}$ and add to it Eq. (11):

$$F = P + jQ = e^{j\alpha^2/4}W, \quad (15)$$

where

$$W = \int_{-\infty}^{r} \exp\left\{j\left(\eta - j\frac{\alpha}{2}\right)^2\right\} d\eta. \quad (16)$$

Noting that r is a function of α, we differentiate W with respect to α and integrate $dW/d\alpha$ for the conditions $r = \pm\alpha/2$, obtaining

$$[F]_{r=\pm\alpha/2} \quad (17)$$

$$= e^{j\alpha^2/4}\left[C_0 + \tfrac{1}{2}(-j \pm 1)\int_0^{\alpha} e^{\pm\eta^2/2} d\eta\right].$$

To determine the constant of integration C_0 in Eq. (17), we set $\alpha = 0$ and make use of Eq. (13):

$$[F]_{\substack{\alpha=0\\ r=0}} = \sqrt{\frac{\pi}{8}}(1 + j) = C_0.$$

Thus, we find from Eq. (17)

$$[P(\alpha, r)]_{r=-\alpha/2} \quad (18)$$

$$= \left[\cos\frac{\alpha^2}{4} - \sin\frac{\alpha^2}{4}\right]\sqrt{\frac{\pi}{8}}\left\{1 - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2}}\right)\right\};$$

$$[Q(\alpha, r)]_{r=-\alpha/2}$$

$$= \left[\cos\frac{\alpha^2}{4} + \sin\frac{\alpha^2}{4}\right]\sqrt{\frac{\pi}{8}}\left\{1 - \operatorname{erf}\left(\frac{\alpha}{\sqrt{2}}\right)\right\},$$

where

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^{z} e^{-t^2} dt$$

is the error integral,

$$[P(\alpha, r)]_{r=\alpha/2} = \frac{1}{2}\left\{\left(\sin\frac{\alpha^2}{4} + \cos\frac{\alpha^2}{4}\right) \quad (19)$$

$$\times \int_0^{\alpha} e^{\eta^2/2} d\eta + \sqrt{\frac{\pi}{2}}\left(\cos\frac{\alpha^2}{4} - \sin\frac{\alpha^2}{4}\right)\right\},$$

$$[Q(\alpha, r)]_{r=\alpha/2} = \frac{1}{2}\left\{\left(\sin\frac{\alpha^2}{4} - \cos\frac{\alpha^2}{4}\right)\right.$$

$$\times \int_0^\alpha e^{\eta^2/2} d\eta + \sqrt{\frac{\pi}{2}}\left(\cos\frac{\alpha^2}{4} + \sin\frac{\alpha^2}{4}\right)\Big\}.$$

3. SEPARATION OF THE ROOTS OF THE TRANSCENDENTAL EQUATION (8) FOR $\alpha = 0$

To separate the roots of Eq. (8), we consider first the case $\alpha = 0$, i.e., by Eq. (7), $T_2 = \infty$. Rewriting Eq. (8) in the form

$$2r = \alpha \operatorname{tg}[r^2 - \psi(\alpha, r)],$$

we find that there is a minimum root, for which, from Eq. (9),

$$\text{for} \quad \alpha = 0, \quad r_{0,-1} = 0, \quad [\psi]_{\substack{\alpha=0\\ r=0}} = \pi/4. \tag{20}$$

Moreover, there exist a succession of roots for finite $r_{o,m}$ for which

$$\text{for} \quad \alpha = 0, \; r_{0,m}^2 - \psi(0, r_{0,m}) \tag{21}$$

$$= \frac{\pi}{2} + \pi(m-1), \quad m = 1,2,3,\ldots$$

The root of interest to us, in this case, nearest to $r_{0,-1}$, will correspond to $m = 1$ in view of Eq. (21).

$$r_{0,1}^2 - \psi(0, r_{0,1}) = \pi/2 \tag{22}$$

or, by Eqs. (9), (13), (14),

$$-\operatorname{ctg} r_{0,1}^2 = \operatorname{tg}\psi(0, r_{0,1}) \tag{23}$$

$$= \left[\tfrac{1}{2}\sqrt{\pi/2} + \int_0^{r_{0,1}} \sin\eta^2 d\eta\right]$$

$$\times \left[\tfrac{1}{2}\sqrt{\pi/2} + \int_0^{r_{0,1}} \cos\eta^2 d\eta\right]^{-1}$$

Representing Eq. (23) in the form

$$-\operatorname{tg}(r_{0,1}^2) = \frac{0.5 + C(r_{0,1}^2)}{0.5 + S(r_{0,1}^2)},$$

and making use of the tables of Fresnel integrals[4], we find, by the method of iteration,

$$r_{0,1}^2 = 2.327, \tag{24}$$

whence

$$r_{0,1} = 1.525.$$

[4] E. Jahnke and F. Emde, *Tables of Functions*

4. DETERMINATION OF THE FIRST ROOT OF r_{-1} IN THE GENERAL CASE

For the determination of the position of the first minimum root r_{-1} in the general case ($\alpha \neq 0$), we can, in conformity with Eq. (8), carry out the iteration according to the scheme

$$r_{-1}^{(n)} = \frac{\alpha}{2}\operatorname{tg}[(r_{-1}^{(n-1)})^2 - \psi(\alpha, r_{-1}^{(n-1)})], \tag{25}$$

where by $r_{-1}^{(n)}$ are meant the successive approximations of the desired root. We begin the iteration from the values determined by Eq. (20):

$$r_{-1}^{(1)} = \frac{\alpha}{2}\operatorname{tg}\left[0 - \frac{\pi}{4}\right] = -\frac{\alpha}{2}, \tag{26}$$

$$r_{-1}^{(2)} = \frac{\alpha}{2}\operatorname{tg}\left[\frac{\alpha^2}{4} - (\psi)_{r=-\alpha/2}\right]. \tag{27}$$

But, by Eqs. (9) and (18),

$$[\operatorname{tg}\psi]_{\substack{\alpha\\ r=-\alpha/2}} = \operatorname{tg}\left[\frac{\pi}{4} + \frac{\alpha^2}{4}\right],$$

i.e.,

$$[\psi]_{r=-\alpha/2} = \frac{\pi}{4} + \frac{\alpha^2}{4},$$

whence, in accord with Eq. (27),

$$r_{-1}^{(2)} = -\alpha/2. \tag{28}$$

Inasmuch as the two successive approximations (26) and (28) coincide, the exact value of the first extremum is

$$r_{-1} = -\alpha/2. \tag{29}$$

Thus, the first extremum of the signal sets in for negative values of r, i.e., up to the onset of resonance, which corresponds to $r = 0$.

5. DETERMINATION OF THE POSITION OF THE SUBSEQUENT ROOT IN THE GENERAL CASE

To determine the position of the subsequent root in Eq. (8) for the case $\alpha \neq 0$, we must know the values of the integrals Q and P [Eqs. (10) and (11)]. We introduce the quantity

$$I = P_1(\alpha, r) + jQ_1(\alpha, r) \tag{30}$$

$$= e^{j\alpha^2/4}\int_0^r \exp\left\{j\left(\eta - j\frac{\alpha}{2}\right)^2\right\} d\eta.$$

and the new variable

$$t = \sqrt{j}\left(\eta - j\frac{\alpha}{2}\right) = x + jy, \tag{31}$$

where

$$x = \left(\eta + \frac{\alpha}{2}\right)/\sqrt{2}, \quad y = \left(\eta - \frac{\alpha}{2}\right)/\sqrt{2},$$

we get

$$I = \exp\left\{j\left(\frac{\alpha^2}{4} - \frac{\pi}{4}\right)\right\}\int_{t_1}^{t_3} e^{t^2}dt. \qquad (32)$$

The limits of integration have the following values:

for $\eta = 0, \quad x_1 = \alpha/2\sqrt{2}, \quad y_1 = -\alpha/2\sqrt{2};$ (33)

for $\eta = r, \quad x_2 = \left(r + \frac{\alpha}{2}\right)\Big/\sqrt{2}, \quad y_2 = \left(r - \frac{\alpha}{2}\right)\Big/\sqrt{2}.$

We carry out the integration over two intervals of the complex variable:

$$\int_{t_1}^{t_2} e^{t^2}dt = \int_0^{t_2} e^{t^2}dt - \int_0^{t_1} e^{t^2}dt. \qquad (34)$$

From Eq. (34), we see that in the integration from zero to t_1,

$$t = (1 - j)\,x.$$

Denoting

$$2x^2 = \zeta^2,$$

we get

$$\int_0^{t_1} e^{t^2}dt = e^{-j(\pi/4)}\int_0^{\alpha/2} e^{-j\zeta^2}d\zeta. \qquad (35)$$

Upon substitution of Eqs. (34) and (35) in Eq. (32), I takes the form

$$I = \exp\left\{j\left[\frac{\alpha^2}{4} - \frac{\pi}{4}\right]\right\}\int_0^{t_2} e^{t^2}dt + je^{j\alpha^2/4} \times \int_0^{\alpha/2} e^{-j^2\zeta}d\zeta = I_2 + I_1, \qquad (36)$$

where I_2 and I_1 denote the first and second terms, respectively. The integral which enters into I_1 is calculated with the aid of the Fresnel integrals

$$\int_0^{\alpha/2} e^{-j\zeta^2}d\zeta = \sqrt{\frac{\pi}{2}}\left\{C\left(\frac{\alpha^2}{4}\right) - jS\left(\frac{\alpha^2}{4}\right)\right\}.$$

The real and imaginary parts of I_1 have the form

$$\operatorname{Re} I_1 = \sqrt{\frac{\pi}{2}} \times \left[S\left(\frac{\alpha^2}{4}\right)\cos\frac{\alpha^2}{4} - C\left(\frac{\alpha^2}{4}\right)\sin\frac{\alpha^2}{4}\right], \qquad (37)$$

$$\operatorname{Im} I_1 = \sqrt{\frac{\pi}{2}} \times \left[S\left(\frac{\alpha^2}{4}\right)\sin\frac{\alpha^2}{4} + C\left(\frac{\alpha^2}{4}\right)\cos\frac{\alpha^2}{4}\right].$$

The integral entering into I_2 is calculated with the aid of tables of the real and imaginary parts of the probability integral of complex argument[5]

$$\int_0^{t_2} e^{t^2}dt = \frac{\sqrt{\pi}}{2}\{e^{t_2^2}[V(x_2, y_2) - jU(x_2, y_2)] + j\}, \qquad (38)$$

where U and V denote, respectively, the real and imaginary parts of the probability integral.

Expressing x_2 and y_2 in terms of α and r and substituting Eq. (38) in Eq. (36), we find the expression for I_2

$$I_2 = \frac{\sqrt{\pi}}{2}\left[e^{\alpha r}\exp\left\{j\left(r^2 - \frac{\pi}{4}\right)\right\}(V - jU) + j\exp\left\{j\left(\frac{\alpha^2}{4} - \frac{\pi}{4}\right)\right\}\right]. \qquad (39)$$

Separating the real and imaginary parts, we have

$$\operatorname{Re} I_2 = \frac{\sqrt{\pi}}{2\sqrt{2}}\left\{e^{\alpha r}[(V - U)\cos r^2 + (V + U)\sin r^2] + \left[\cos\frac{\alpha^2}{4} - \sin\frac{\alpha^2}{4}\right]\right\}; \qquad (40)$$

$$\operatorname{Im} I_2 = \frac{\sqrt{\pi}}{2\sqrt{2}}\left\{e^{\alpha r}[(V - U)\sin r^2 \; (V + U)\cos r^2] + \left[\cos\frac{\alpha^2}{4} + \sin\frac{\alpha^2}{4}\right]\right\}.$$

Making use of Eqs. (12), (13), (30), (37) and (40), we find the final expressions for the integrals Q and P:

$$Q = \int_{-\infty}^{r} e^{\alpha\eta}\sin\eta^2\, d\eta = \frac{1}{2}\sqrt{\frac{\pi}{2}}\left\{e^{\alpha r}[(V - U)\sin r^2 - (V + U)\cos r^2] + 2\left[\cos\frac{\alpha^2}{4} + \sin\frac{\alpha^2}{4}\right]\right\}; \qquad (41)$$

$$P = \int_{-\infty}^{r} e^{\alpha\eta}\cos\eta^2\, d\eta = \frac{1}{2}\sqrt{\frac{\pi}{2}}\left\{e^{\alpha r}[(V - U)\cos r^2 + (V + U)\sin r^2] + 2\left[\cos\frac{\alpha^2}{4} - \sin\frac{\alpha^2}{4}\right]\right\}. \qquad (42)$$

Substituting Eqs. (41) and (42) into Eq. (8), we

[5] V. N. Faddeeva and N. M. Terent'ev, *Tables of the Values of the Probability Integral of Complex Argument*, GITTL, 1954

get the equation for the determination of the posi- tions of the extrema of the function $u(t)$:

$$r = \frac{\alpha}{2} \frac{V + U + 2e^{-\alpha r}\left[\sin\left(r^2 - \frac{\alpha^2}{4}\right) - \cos\left(r^2 - \frac{\alpha^2}{4}\right)\right]}{V - U + 2e^{-\alpha r}\left[\sin\left(r^2 - \frac{\alpha^2}{4}\right) + \cos\left(r^2 - \frac{\alpha^2}{4}\right)\right]}; \quad (43)$$

$\alpha/2$	r_{+1}	Δr	$\alpha/2$	r_{+1}	Δr
0.0000	1.525	1.525	1.4142	1.63	3.04
0.1414	1.49	1.63	1.7680	1.78	3.55
0.3535	1.46	1.81	2.4748	2.16	4.63
0.7071	1.45	2.16			

where U and V are functions of x_2 and y_2, determined in accordance with Eq. (33).

In order to find the positions of the subsequent extremum r_{+1}, Eq. (43) is solved by interpolation by the chord method. The results which were computed appear in the Table and are drawn in Fig. 1 in the form of the curve r_{+1}. The curve $\Delta r = r_{+1} - r_{-1} = r_{+1} + \alpha/2$ appears in this same drawing.

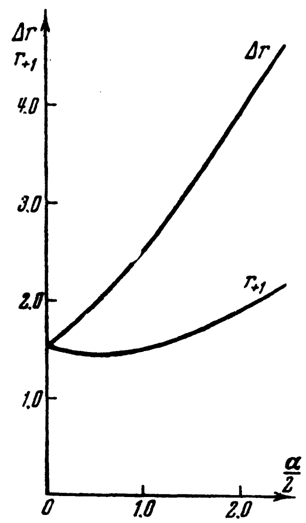

FIG. 1. The position of the second extremum r_{+1} and the distance between the first and second extrema Δr of the signal of dispersion in a weak high-frequency magnetic field for nonadiabatic transition through resonance.

From the graph in Fig. 1 it follows that the position of the extremum r_{+2} depends on $\alpha/2$, and for an increase of $\alpha/2$ from a value ≈ 0.6, the quantity r_{+1} increases monotonically, remaining, however, always less than $\alpha/2$.

6. THE VALUE OF THE ROOT r_{+1} AS $\alpha/2 \to \infty$

For a decrease in the transverse relaxation time T_2 (i.e., as $\alpha \to \infty$) the process of transition through resonance becomes adiabatic, as is known[3], and corresponds to the case of "slow passage". In this case, the signal of dispersion has only two extrema, located at equal distances from the moment of resonance. It can be shown that for $\alpha \to \infty$ the value $r_{+1} \to \alpha/2$. We make use of the iteration scheme (25), given in Sec. 4, and assume that $r_{+1}^{(1)} = \alpha/2$. Then,

$$r_{+1}^{(2)} = \frac{\alpha}{2} \operatorname{tg}\left[\frac{\alpha^2}{4} - (\psi)_{r=\alpha/2}\right]. \quad (44)$$

From Eqs. (9) and (19), it follows that as $\alpha \to \infty$,

$$[\operatorname{tg}\psi]_{\substack{r=\alpha/2 \\ \alpha\to\infty}} = \operatorname{tg}\left(\frac{\alpha^2}{4} - \frac{\pi}{4}\right),$$

i.e.,

$$[\psi]_{\substack{r=\alpha/2 \\ \alpha\to\infty}} = \frac{\alpha^2}{4} - \frac{\pi}{4},$$

while from Eq. (44)

$$r_{+1}^{(2)} = \alpha/2. \quad (45)$$

Since, as $r = \alpha/2, \alpha \to \infty$, Eq. (44) becomes an indentity, the position of the following extremum becomes

$$(r_{+1})_{\alpha\to\infty} = \alpha/2. \quad (46)$$

and, consequently,

$$r_{+1} = |r_{-1}|.$$

7. THE CHANGE OF THE FORM OF THE DISPERSION OF THE SIGNAL

According to theory[1,2], the form of the observed signals depends on the magnitude of the parameter $\sqrt{a}\,T_2$. For values of $\sqrt{a}\,T_2 \geq 1$, the vibrations appear beyond resonance. Keeping in mind that, by Eq. (7)

$$\sqrt{a}T_2 = \sqrt{2}/\alpha,$$

and taking into consideration the graphs in Fig. 1, we find that the form of the dispersion of the signal must change in the following way; upon a decrease in $\alpha/2$, the distance between the first and second extrema (Δr) must decrease and, beginning with $\alpha/2 < 1$, vibrations must appear whose amplitude increases upon further decrease in $\alpha/2$.

Oscillograms of the dispersion of the signal are given in Fig. 2. These illustrate the change in the shape of the signals for change in $\alpha/2$. The oscillograms were obtained for aqueous solutions of different concentrations of copper sulfate and ferric nitrate. All the oscillograms were taken under identical conditions. The corresponding values of $\alpha/2$ are given in the captions for the figures. The oscillograms verify the conclusions of the theory of the dependence of the form of the curves on $\alpha/2$.

8. METHOD OF DETERMINATION OF T_2

In the practical application of this method, the dispersion of the signal is observed in a weak high-frequency magnetic field of frequency ω[6]. To the constant magnetic field H_0, there is added a magnetic field of amplitude H_m and audio frequency ω_m, such that the resultant magnetic field is equal to

$$H_z = H_0 + H_m \sin \omega_m t. \quad (47)$$

If the signal $u(t)$ is located in a linear segment at the center of the oscillogram, then, by Eq. (47),

$$a = |\gamma| H_m \omega_m. \quad (48)$$

Behind the linear part of the modulation there are regions where the velocity of modulation and the time of transition through resonance are constant within the limits of accuracy chosen. For the determination of T_2, it is necessary that the distance between the first and second extrema lie in the linear part, while the remaining part of the signal $u(t)$ can be located in intervals with varying velocity; it is only important that the signal be damped out at the onset of the following resonance, which arises on the reverse path of the development.

Measuring (along the oscillogram) the distance between the first and second extrema and knowing the amplitude and frequency of the modulating field, we can determine the time t. Then, by Eqs. (7) and (48),

$$\Delta r = \sqrt{\frac{a}{2}}\,\Delta t = \sqrt{\frac{|\gamma| H_m \omega_m}{2}}\,\Delta t. \quad (49)$$

We determine $\alpha/2$ graphically for a given Δr from Fig. 1, and compute T_2 from Eq. (7):

$$T_2 = \frac{1}{\sqrt{2a}}\left(\frac{2}{\alpha}\right) = \frac{1}{\sqrt{2|\gamma| H_m \omega_m}}\left(\frac{2}{\alpha}\right). \quad (50)$$

The error in the determination of T_2 arising from the use of Eq. (50), is determined by the errors in the measurement of H_m, ω_m, Δt and the error in the determination of $\alpha/2$ from the graph.

If the above calculated quantities are determined with possible errors H_m: 1%, ω_m: 1%, Δt: 3%, $\alpha/2$: 1%, then for the case that Δr lies at the middle of the graph, i.e., for $2 < \Delta r < 4$, (which ordinarily takes place for suitable choice of the velocity of modulation), the maximum possible error in determining T_2 amounts to 8-15%. For smaller values of Δr, the error in the determination of T_2 increases.

The present method of determining T_2 is sufficiently simple and gives an accuracy not less than the methods described earlier in the literature. The principal advantage of this method is the fact that one can work in a nonstabilized constant magnetic field, which greatly simplifies the arrangement for the measurement of the time of transverse relaxation.

9. EXPERIMENTAL TEST OF THE METHOD

To test the method just described, the values of the transverse relaxation times T_2 were measured for aqueous solutions of different concentrations of copper sulfate ($CuSO_4$) and ferric nitrate [$Fe(NO_3)_3$]. The dependence of T_2 on the concentration of the paramagnetic ions Cu^{++} (curves *1* and *2*) and of the ions Fe^{+++} (curve *3*) in the solution is indicated in Fig. 3. The accuracy of measurement of T_2 was about 10%.

It follows from Fig. 3 that for reduced concentrations the transverse relaxation time approaches a limit whose magnitude is determined by the inhomogeneous constant magnetic field. The amount of inhomogeneity of the constant magnetic field along the specimen ΔH_0 was estimated by the value of T_2, measured for distilled water by the formula

$$\frac{1}{T_2} = \gamma (H_{\text{int}} + \Delta H_0) = \frac{1}{T_{2\,\text{BH}}} + \frac{1}{T_{2\,\text{int}}}, \quad (51)$$

where H_{int} = intensity of the internal field, $T_{2\,\text{int}}$

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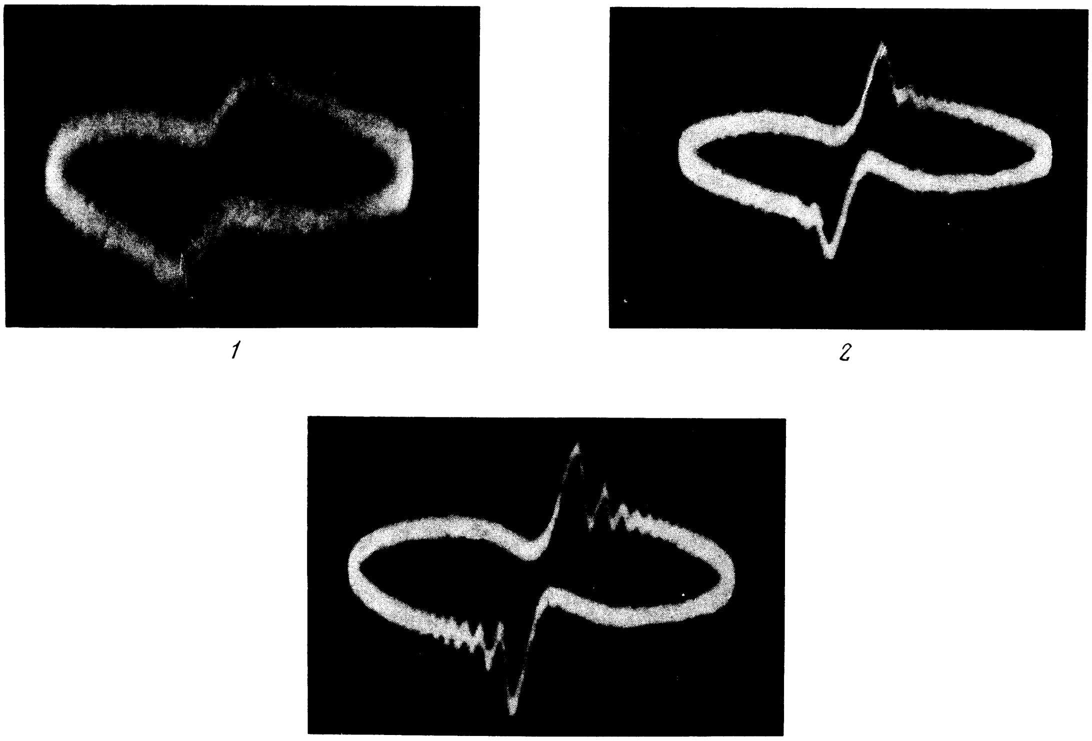
[6] S. D. Gvozdover and N. M. Ievskaia, J. Exper. Theoret. Phys. USSR 25, 435 (1953)



1 2 3

FIG. 2. The change in shape of the signals of dispersion in dependence on $\alpha/2$. The oscillograms were obtained (*1*) from $0.5M$ solution of $Fe(NO_3)_3$; (*2*) from $1M$ solution $CuSO_4$; (*3*) from $0.2M$ solution of $CuSO_4$ under the following conditions: frequency of modulation 50 cps, amplitude of modulation 2.38 oersteds, half amplitude of high-frequency field, 0.03 oersted. Values of $\alpha/2$: 1–1.35; 2–0.85; 3–0.46.

and T_{inhom} are the values of the transverse relaxation times, determined respectively for the internal field and the inhomgeneity. (As is known[7], for distilled water, $T_{2\,int} \gg T_{2\,inhom}$.)

In the results shown in Fig. 3, (curves *4, 5, 6*) the value of the inhomogeneity was respectively equal to 0.072; 0.081; 0.11 oersted, which corresponds to 2.43×10^{-3}; 2.72×10^{-3}; 3.72×10^{-3}% of the value of the constant field.

The dependence of $T_{2\,int}$, determined according to Eq. (51) by taking into account the effect of the inhomogeneity, on the concentration of the paramagnetic ions is shown in Fig. 3 for Cu^{++} ions (curve *7*) and for the Fe^{+++} ions (curve *8*). It follows from the drawing that the dependence of $T_{2\,int}$ on the concentration of the paramagnetic ions is a straight line on the logarithmic scale. The lines for the aqueous solutions of copper sulfate and ferric nitrate are parallel to each other and are inclined at 45° with respect to the abscissa. Such a dependence shows that the transverse relaxation time $T_{2\,int}$ produced by the internal field is inversely proportional to the concentration of the paramagnetic ions.

A comparison of the results of measurements with the data given by Bloembergen[7] (curve *9*) shows that the value of T_2 for aqueous solutions of ferric nitrate, obtained in the present research for large concentrations, where the inhomogeneity is small, agrees with the results of Bloembergen within the limits of experimental error. For small concentrations, the resultant values of $T_{2\,int}$ differ from the T_2 obtained in reference 7 by about

[7] N. Bloembergen, *Nuclear Magnetic Relaxation*, The Hague, 1948

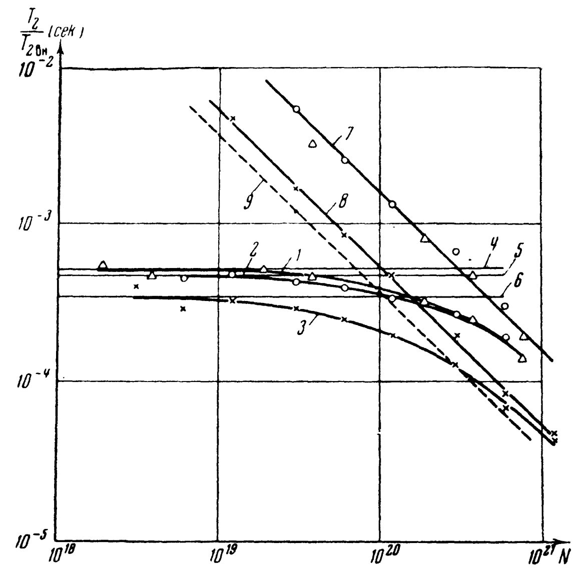


FIG. 3. Dependence of T_2 and $T_{2\,\text{int}}$ on the concentration of paramagnetic ions N for aqueous solutions of $CuSO_4$ and $Fe(NO_3)_3$. The concentration N is expressed in number of ions/cm^3. Measurements for Cu^{++} ions: Δ for conditions when $T_{2\,\text{inhom}}$ is determined by curve *4;* O for conditions when $T_{2\,\text{inhom}}$ is determined by curve *5;* measurements for Fe^{+++} ions under conditions when $T_{2\,\text{inhom}}$ is determined by curve *6.*

40%. This difference can be explained by the fact that the accuracy of the determination of $T_{2\,\text{int}}$ is significantly less as a result of errors in the measurement of $T_{2\,\text{inhom}}$.

Thus, the agreement of the results within the limits of experimental error confirms the possibility of application of the method developed here.

Translated by R. T. Beyer
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