TABLE

Trial	No. of cases of fission of U per cm^2	No. of cases of stopping of π^- mesons per cm^2	No. of U nuclei per cm^2 $\times 10^{-17}$	No. of cases of capture of π^- mesons by U nuclei per cm^2	Probability of U fission
I	25	7050	6.2 ± 0.6	98 ± 10	0.25 ± 0.05
II	21	5040	5.7 ± 0.2	64 ± 5	0.33 ± 0.08

super-centrifuge (20,000 revolutions per minute). After 20 minutes of centrifuging, the grains of AgBr settled almost completely, covering the walls of the container. The resulting precipitate of grains of AgBr and the leftover gelatin were analyzed for uranium content. The results lead to the conclusion that all the uranium penetrating into the emulsion during the soaking is found in the gelating, and that the adsorption of uranium on the surface of AgBr crystals does not occur. Therefore, in determining the number of π^- mesons interacting with uranium nuclei, it is necessary to count only those mesons which stop in the gelatin ($\sim$42% from our data). Assuming further, in agreement with Fermi[5], that the probability of capture of a slow π^- meson in a chemical bond is proportional to the atomic number Z, it is possible to write the following expression for the number of π^- mesons captured by the nuclei of uranium atoms adsorbed by the gelatin, under the condition that the π^- mesons are stopped with uniform distribution throughout the depth of the emulsion:

$$N = 0.42 N_0 \frac{N_U Z_U}{N_U Z_U + \sum_i N_i Z_i}$$

Here N_0 is the number of π^- mesons stopped per square centimeter of the layer of emulsion; Z_i, N_i are the charge and number of nuclei of the elements composing the gelatin per square centimeter of the layer of emulsion.

Two groups of measurements were carried out. The data, reduced to tabular form, are for a layer of emulsion 100 microns thick.

Thus the results of the present work lead to the conclusion that on capture of π^- mesons, uranium nuclei undergo fission in 30% of the cases. However, it should be noted that this conclusion is valid only if the assumption is correct that the probability of capture of π^- mesons by the different nuclei in the mixture is proportional to the atomic number.

The results of the present work were obtained in December, 1953. There are available at the present time in the literature[6] determinations of the probability of fission of the uranium nuclei upon capturing π^- mesons which differ somewhat from our values. In the referenced work the probability of fission is determined to be 0.18 ± 0.06.

In conclusion, the authors wish to express their appreciation to Professor N. A. Perfilov for valuable instruction and discussion of the results of the work.

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The Scattering of Fast Neutrons by Non-Spherical Nuclei II

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OUR previous article[1] contains the computation of the effective cross section for scattering of fast neutrons by the black nucleus, which has the form of an ellipsoid of revolution and a spin equal to zero. We shall examine the established results under the assumption that before the scattering interaction the nucleus is found in a particular state.

The differential cross section for an excited rotational state of the nucleus $Y_{lm}(\chi, \phi)$ is determined by the formulas (6), (7) of reference 1:

$$\sigma_{lm}(\theta) = \frac{4}{\pi^2}\frac{(kb)^4}{k^2} I^2_{lm}(\theta), \tag{1}$$

$$I_{lm}(\theta) = \frac{1}{8}\sqrt{(2l+1)\frac{(l-m)!}{(l+m)!}}$$

$$\times \int_{-1}^{1}\int_{0}^{2\pi} dx d\varphi \xi(x) P_l^m(x) \frac{J_1(t)}{t}\cos m\varphi;$$

$$t = kb\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi};$$

$$\xi(x) = \sqrt{z^2 + (1-z^2)x^2}.$$

From this point it is not difficult to obtain the following expression: $\sigma_{lm}(\theta) \neq 0$, provided both l and m are even; $\sigma_{lm}(\theta) = 0$ in the remaining cases; in the case of a spherical nucleus, the rotational states are not excited:

$$\sigma_{lm}(\theta) = 0, \quad z = 1, \quad l \neq 0, \quad m \neq 0.$$

The differential cross section for elastic scattering is obtained from formula (1) with $l = m = 0$:

$$\sigma_0(\theta) = \frac{4}{\pi^2}\frac{(kb)^4}{k^2} \tag{2}$$

$$\times \left[\int_0^1\int_0^{\pi/2} dx d\varphi \xi(x) \frac{J_1(kb\theta\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi})}{kb\theta\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi}}\right]^2.$$

In the case of a spherical nucleus ($z = 1$) this gives rise to the well-known expression[2]

$$\sigma_0(\theta) = \frac{(kR)^4}{k^2}\left[\frac{J_1(kR\theta)}{kR\theta}\right]^2.$$

In accordance with Eqs. (9) and (11)[1], the summed cross section for scattering into angle θ with distinct excited rotational states, including elastic scattering, is determined by the formula:

$$\sigma_s(\theta) = \frac{2}{\pi}\frac{(kb)^4}{k^2} \tag{3}$$

$$\times \int_0^{\pi/2}\int_0^1 d\varphi\, dx \xi^2(x) \left[\frac{J_1(kb\theta\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi})}{kb\theta\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi}}\right]^2.$$

For a spherical nucleus, $\sigma_s(\theta) = \sigma_0(\theta)$.

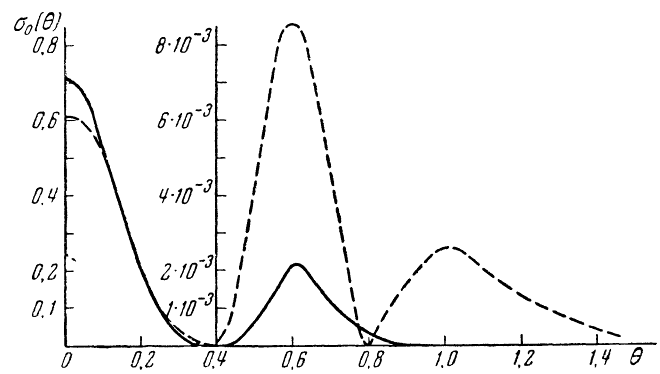

FIG. 1. The function $\sigma_0(\theta)$ for $z = 1$ (dotted line) and $z = 2$ (solid curve); $kR = 10$.

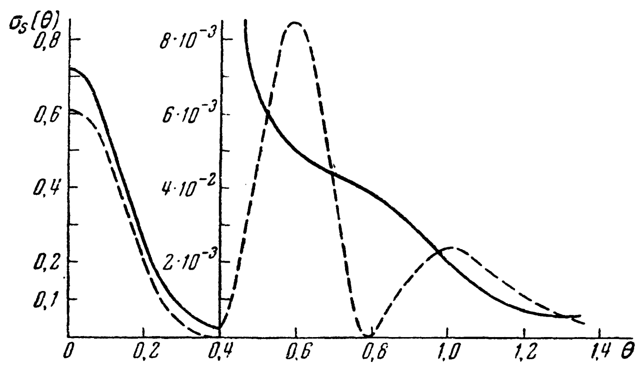

FIG. 2. The function $\sigma_s(\theta)$ for $z = 1$ (dotted line) and $z = 2$ (solid curve); $kR = 10$.

In Fig. 1 is shown the angular distribution of $\sigma_0(\theta)$ for neutrons with energy $kR = 10$ elastically scattered by a non-spherical nucleus ($z = 2$). For comparison there is shown the angular distribution of $\sigma_0(\theta)$ for a spherical nucleus ($z = 1$) of equal volume. From Fig. 1, it can be seen that deformation of the nucleus does not make any radical change in the angular distribution for elastic scattering. As in the case of scattering by a spherical nucleus, the angular distribution of $\sigma_0(\theta)$ has a series of maxima and minima, where, furthermore, the cross section becomes zero at the minima.

Figure 2 shows the summed cross section $\sigma_s(\theta)$ for the non-spherical nucleus ($z = 2$) for energy of the incident neutron $kR = 10$. For comparison there is shown a curve of $\sigma_s(\theta)$ for a spherical nucleus ($z = 1$) of equal volume. From Fig. 2 it can be seen that deformation of the nucleus changes the function $\sigma_s(\theta)$ significantly. In the case of a spherical nucleus, the angular distribution of $\sigma_s(\theta)$ retains the series of maxima and minima, where, furthermore, the cross section $\sigma_s(\theta)$ becomes zero at the minima. But in the presence of observable deformation, the angular distribution of $\sigma_s(\theta)$ no longer has maxima or minima, and does not become zero. In the case of significant deformation, the function $\sigma_s(\theta)$ decreases monotonically with increasing θ.

From the last it follows that the probability of excitation of the lth rotational level of the nucleus decreases rapidly with increasing l. Therefore, the effective energy spread of the neutrons undergoing scattering with excitation of distinct rotational states is found to be of the order $\Delta\epsilon = \hbar^2 l\,(l+1)/2I$, where $l \sim 2$. For heavy nuclei, with $z \approx 2$, $\Delta\epsilon \sim 100$ kev. The angular distribution of $\sigma_s(\theta)$ is convenient for comparison with experimental investigations of angular distribution of elastic scattering, provided that the energy resolution of the experimental detector is of the order of $\Delta\epsilon$.

In accord with formulas (9)-(11) of the previous article[1], the total cross sections σ_t, σ_c, σ_s are found to be independent of the energy, whereupon

$$\sigma_t = 2\sigma_s; \quad \sigma_c = \sigma_s; \quad \sigma_s = \pi R^2 \alpha(z); \tag{4}$$

$$\alpha(z) = \frac{z^{-2/3}}{2}\left(1 + \frac{z^2}{\sqrt{z^2-1}} \arcsin \frac{\sqrt{z^2-1}}{z}\right), \quad z \geqslant 1;$$

$$\alpha(z) = \frac{z^{-2/3}}{2}\left(1 + \frac{z^2}{\sqrt{1-z^2}} \operatorname{Arsh} \frac{\sqrt{1-z^2}}{z}\right), \quad z \leqslant 1,$$

where R is the radius of a spherical nucleus of equal volume.

Given Eq. (2), the total cross section for elastic scattering is determined by the expression:

$$\sigma_0 = \frac{8}{\pi}\frac{(kb)^4}{k^2}\int_0^\infty \theta d\theta \left[\int_0^1 \int_0^{\pi/2} dx d\varphi \xi(x) \frac{J_1\left(kb\theta\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi}\right)}{kb\theta\sqrt{\xi^2(x)\cos^2\varphi + \sin^2\varphi}}\right]^2. \tag{5}$$

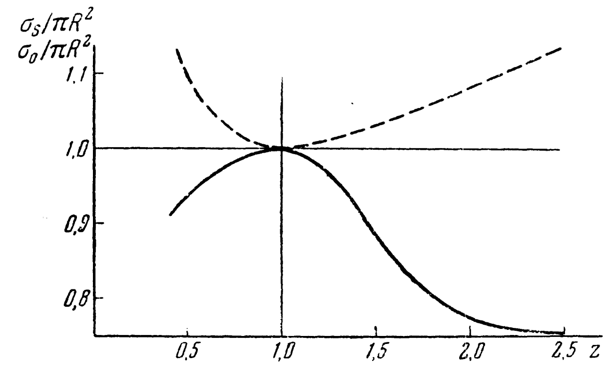

FIG. 3. Total cross section σ_s (dotted line) and σ_0 (solid curve).

The cross section σ_0 is practically independent of energy. Figure 3 shows the total cross section σ_s and σ_0 for different deformations z of the nucleus. The total cross section for excitation of the lth rotational level of the nucleus differs from zero only for even l and diminishes quickly with increasing l. For deformations of the nucleus which are not too great and for $l \geq 2$,

$$\sigma_l \sim 2\pi b^2 \frac{a_l^2}{2^{2l} l},$$

where a_l is the coefficient of resolution

$$\xi(x) = \sum_{l=0}^{\infty} a_l x^l.$$

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The Scattering of Fast Neutrons by Non-Spherical Nuclei. I

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ONE examines the scattering of fast neutrons by the black nucleus, which has the form of a body of revolution and spin equal to zero. The solution of the problem of scattering in the adiabatic approximation[1] may be obtained in the following manner. The ψ-function of the system satisfies the Schroedinger equation:

$$(H_0 + T)\psi = E\psi. \tag{1}$$

where H_0 is the Hamiltonian operator of the system consisting of the neutron and the fixed target nucleus; T is the operator for the rotational energy of the nucleus. It is assumed that the energy of the incident neutron is significantly greater than the rotational energy of the nucleus. For this reason, it is possible , in the zero approximation in Eq. (1), to drop the operator T and set up the ψ-function in the form:

$$\psi \approx u_{\mathbf{k}}(\mathbf{r}, \omega)\, \varphi_{n_0}(\omega), \tag{2}$$

where $H_0 u_{\mathbf{k}} = E_k u_{\mathbf{k}}$; $T\phi_n = \epsilon_n \phi_n$; E_k is the energy of the incident neutron; ϵ_n represents the rotational energy levels of the nucleus. The operator H_0 operates only on the radial coordinate $\mathbf{r}$ of the nucleus, and does not operate on the angular coordinate ω, which determines the orientation of the nucleus. By this means one examines, in the adiabatic approximation, neutron scattering on the fixed target nucleus, which was in the rotational state $\phi_{n_0}(\omega)$ with energy ϵ_{n_0} prior to the scattering interaction.

The operator for the rotational energy of the nucleus has the form[2]

$$T = -(\hbar^2/2I)\,\Delta_\omega,$$

where $\Delta\omega$ is the Laplace operator on the unit sphere, I is the moment of inertia of the nucleus with respect to the principal axis, parpendicular to the axis of symmetry. The eigenfunctions $\phi_n(\omega)$, describing the rotation of the nucleus, appear as the spherical functions $Y_{lm}(\omega)$; the rotational levels of the nucleus are determined by the formula:

$$\varepsilon_l = \frac{\hbar^2 l(l+1)}{2I}; \tag{3}$$

When r approaches ∞, the ψ-function (2) has the form:

$$\psi \sim \left[e^{ikr} + \frac{e^{ikr}}{r} f(\omega, \Omega) \right] \varphi_{n_0}(\omega), \tag{4}$$

where $f(\omega, \Omega)$ is the scattering amplitude on the fixed target nucleus in the direction Ω, dependent on the orientation ω of the nucleus. Resolving the quantity $f(\omega, \Omega)\phi_{n_0}(\omega)$ into a series by functions $\phi_n(\omega)$ we obtain:

$$\psi \sim e^{ikr}\varphi_{n_0}(\omega) + \frac{e^{ikr}}{r} \sum_n F_{nn_0}(\Omega)\, \varphi_n(\omega), \tag{5}$$

where

$$F_{nn_0}(\Omega) = \int d\omega\, \varphi_n^*(\omega)\, f(\omega, \Omega)\, \varphi_{n_0}(\omega). \tag{6}$$

The ψ-function (5) describes the system before the scattering interaction as well as the scattering processes, as a result of which the nucleus is left in distinct rotational states $\phi_n(\omega)$. Energy is not conserved in the approximation being used, because the particles, scattered with distinct excited rotational states, all have the identical wave vector $\mathbf{k}$.