case. Thus the calculation of the observed spectrum involves the solution of two algebraic equations of the eighth degree (for Cu^{++}, $S = \frac{1}{2}$, $I = \frac{3}{2}$) which derive from the two ions in the unit cell of the crystal. The solution of the equation in the case for which the dc magnetic field is oriented along the tetragonal axis of the electric field is found easily[3]. The probability for magnetic dipole transitions was also calculated for this case. The solutions of the equation for the second ion were found by an approximation method. The dc magnetic field resonance values were obtained using the following values of the constants which appear in Eq. (1): $A = -8.3 \times 10^{-3}$ cm^{-1}, $B = 4.5 \times 10^{-3}$ cm^{-1}, $Q = 1 \times 10^{-3}$ cm^{-1}, $g_{\|} = 2.47$ and $g_{\perp} = 2.08$.

Our values for these constants differ from those given in reference 8, viz. $A = -10.3 \times 10^{-3}$ cm^{-1}, $B = 3.5 \times 10^{-3}$ cm^{-1}, $Q = 1 \times 10^{-3}$ cm^{-1}, $g_{\|} = 2.45$ and $g_{\perp} = 2.12$. This is probably explained by the fact that in the work mentioned above, the experiments were performed at $T = 20°$ K. If one assumes that the size of the tetragonal component of the electric field should become larger with an increase of temperature, it is possible to explain qualitatively a corresponding change of these constants.

* The experimental work was carried out by N. S. Garif'ianov.

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The Representation of Green's Function in Quantum Electrodynamics in the Form of Continual Integrals

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IN quantum electrodynamics, Green's functions for various problems can be written in the form of the matrix elements of various chronologically T-ordered products of the operators for the electron and photon fields, taken between states of the unperturbed vacuum. Thus, for instance, Green's function for the single electron problem has the following form

$$G_{\alpha\beta}(x, x') = \langle T(\psi_\alpha(x)\bar{\psi}_\beta(x')\, S/S_{\text{vac}}\rangle. \quad (1)$$

Here S_{vac} is the vacuum expectation value of the S-matrix

$$S = \exp\{-i\int H(x)\,dx\}. \quad (2)$$

The symbol T denotes the operation of relativistic invariant time ordering, introduced by Wick[1]. In the case of electrodynamics, the Hamiltonian density $H(x)$ is given by

$$H(x) = -i\bar{\psi}(x)\gamma_\mu A_\mu(x)\psi(x). \quad (3)$$

Analogously, one can write Green's function for other problems.

The aim of the present note is to present an expression for Green's function in the form of continuous integrals in the space of the ψ and A functions*.

According to Hori[3] (see also Anderson[4]), the operation of T-ordering multiplying an expression of the indicated form reduces to multiplication of these expressions by ordering operators for the electric field,

$$e^{\Sigma}, \quad \Sigma = -\frac{1}{4}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' S_{F_{\alpha\beta}} \quad (4)$$
$$\times (x-x')\frac{\delta^2}{\delta\psi_\alpha(x)\,\delta\psi_\beta(x')}$$

and for the photon field

$$e^{\Delta},\quad \Delta=\frac{1}{4}\int\limits_{-\infty}^{\infty}dx\int\limits_{-\infty}^{\infty}dx' D_{F_{\mu\nu}}\times(x-x')\frac{\delta^2}{\delta A_\mu(x)\,\delta A_\nu(x')} \tag{5}$$

where $S_{F_{\alpha\beta}}$ and $D_{F_{\mu\nu}}$ are Green's functions for the free (noninteracting) electron and photon fields, respectively.

In Eq. (4) the spinor functions ψ_α and $\bar{\psi}_\beta$ anitcommute with each other.

Ordering operators act on the products of electron and photon field operators taken in the sense of T-products; i.e., in the products under consideration, in every Hamiltonian function, the operators $\bar{\psi}(x)$ always stand to the left of the operators $\psi(x)$.

Let us introduce a coordinate lattice into four dimensional space, of elementary cell volume ω. We shall denote the lattice points (there are N of them) by Latin letters and shall use the following abbreviated notation

$$\begin{gathered} S_{F_{\alpha\beta}}(x_i-x_k)=S_{\alpha\beta}(ik),\\ D_{F_{\mu\nu}}(x_i-x_k)=D_{\mu\nu}(ik),\\ A_\mu(x_i)=A_\mu(i),\\ \psi_\alpha(x_i)=\psi_\alpha(i). \end{gathered} \tag{6}$$

The introduction of the lattice allows one to use sums instead of integrals and partial derivatives instead of functional derivatives. As an illustration of the method, we shall go into detail in what follows by considering the representation of the vacuum expectation value of the S-matrix in the form of a continuous integral. After this, one can write the formulas for Green's functions analogously. Going over from integrals to sums in Eq. (4) we get

$$S_{\text{vac}}=\left\langle \exp\left\{\frac{1}{4}S_{\alpha\beta}(ik)\frac{\partial^2}{\partial\psi_\alpha(i)\,\partial\bar{\psi}_\beta(k)}\right\}\times\exp\left\{\frac{1}{4}D_{\mu\nu}(ik)\frac{\partial^2}{\partial A_\mu(i)\,\partial A_\nu(k)}\right\}\times\exp\left(-\omega\bar{\psi}_\alpha(i)\,\gamma_\mu^{\alpha\beta}A_\mu(i)\,\psi_\beta(i)\right)\right\rangle. \tag{7}$$

This expression can be rewritten in the form of a $3N$-fold integral. For this we make use of the fact that Eq. (7) is the same as the solution of the diffusion-type equations

$$\frac{\partial S_{\text{vac}}}{\partial t_1}=\frac{1}{4}S_{\alpha\beta}(ik)\frac{\partial^2}{\partial\psi_\alpha(i)\,\partial\bar{\psi}_\beta(k)}S_{\text{vac}}; \tag{8}$$

$$\frac{\partial S_{\text{vac}}}{\partial t_2}=\frac{1}{4}D_{\mu\nu}(ik)\frac{\partial^2}{\partial A_\mu(i)\,\partial A_\nu(k)}S_{\text{vac}}, \tag{9}$$

for values of the time-like variables $t_1=1$ and $t_2=1$ and the vanishing of ψ_α and $A_{\mu\nu}$ (no free electrons or photons), with initial value S_{vac} for $t_1=t_2=0$ given by

$$\exp\left(-\omega\bar{\psi}_\alpha(i)\,\gamma_\mu^{\alpha\beta}A_\mu(i)\,\psi_\beta(i)\right).$$

Equation (8) contains derivatives with respect to the noncommuting spinor functions ψ_α and $\bar{\psi}_\beta$. This situation makes it more difficult to find solutions to the derived equations. In order to avoid this difficulty, we will remove the operator properties of ψ to the matrix elements $S_{\alpha\beta}(ik)$. The operator Σ (4) will be rewritten in the form

$$\Sigma=\check{S}_{\alpha\beta}(ik)\frac{\delta^2}{\delta\psi_\alpha(i)\,\delta\bar{\psi}_\beta(k)}, \tag{10}$$

where ψ_α and $\bar{\psi}_\beta$ are now ordinary functions which commute. The matrix elements of $\check{S}(ik)$ are expressed by means of the operator η_{ik} **

$$\check{S}(ik)=S(ik)\,\eta_{ik}. \tag{11}$$

The operators η_{ik} commute with each other***. If in any product of operators an index occurs more than twice, then the product vanishes. The following multiplication rule holds

$$\eta_{ik}\times\eta_{kl}=\eta_{il}. \tag{12}$$

The operators η_{ii} are merely numbers

$$\eta_{ii}=-1. \tag{13}$$

The elements of the inverse matrix $\check{S}^{-1}(ik)$ are determined analogously

$$\check{S}^{-1}(ik)=-S^{-1}(ik)\,\eta_{ik}. \tag{14}$$

From the above properties of the η_{ik} follows the multiplication rule for the matrix elements $\check{S}(ik)$

$$\check{S}(1i_1)\ldots\check{S}(ni_n)=S(1i_1)\ldots S(ni_n)(-1)^P. \tag{15}$$

The second indices i_α take on values from 1 to n, and P is the parity of the permutation of the second indices.

We must now determine the value of the integral

$$I = \int \exp\{-\bar{\psi}(i)\check{S}^{-1}(ik)\psi(k)\}\,\delta\psi\,\delta\bar{\psi}, \tag{16}$$

$$\delta\psi = \Pi d\psi(i),$$

This type of integral will occur below. We evaluate the integral I as the limit of $I(\alpha)$ for $\alpha \to 0$.

$$I(\alpha) = \int \exp\{-\alpha\bar{\psi}(i)\psi(i)\} \tag{17}$$

$$\times \exp\{-\bar{\psi}(i)\check{S}^{-1}(ik)\psi(k)\}\,\delta\psi\,\delta\bar{\psi}.$$

Let us calculate $I(\alpha)$. To do this we expand $\exp(-\bar{\psi}(i)S^{-1}(ik)\psi(k))$ in the integral in Eq. (17) into a series. We get

$$I(\alpha) = \int \exp\{-\alpha\bar{\psi}(i)\psi(i)\} \tag{18}$$

$$\times \sum \frac{1}{n!}(\bar{\psi}(i)\check{S}^{-1}(ik)\psi(k))^n\delta\psi\,\delta\bar{\psi}.$$

Further, after a simple integration, taking into account Eqs. (14) and (15), we get

$$I(\alpha) \tag{19}$$

$$= \frac{\pi^N}{\alpha^N}\sum(-1)^{n+P}\frac{1}{\alpha^N}S^{-1}(1i_1)\ldots S^{-1}(ni_n)$$

$$= \frac{\pi^N}{\alpha^N}\operatorname{Det}\left|\delta_{ik} + \frac{S^{-1}(ik)}{\alpha}\right|$$

The solution of the system of Eqs. (10) and (11), satisfying initial conditions (12), can be written in the form

$$S_{\text{vac}} = \frac{1}{\pi^{N/2}\sqrt{\operatorname{Det} D_{\mu\nu}}} \tag{20}$$

$$\times \int \exp\{-\bar{\psi}(i)\check{S}^{-1}(ik)\psi(k) - A(i)D^{-1}(ik)A(k)$$

$$-\omega\bar{\psi}(i)\check{A}(ik)\psi(k)\}\,\delta\psi\,\delta\bar{\psi}\,\delta A$$

$$\times\left(\int \exp\{-\psi(i)S^{-1}(ik)\psi(k)\}\,\delta\psi\,\delta\bar{\psi}\right)^{-1},$$

where

$$A_{\alpha\beta}(ik) = \gamma_\mu^{\alpha\beta}A_\mu(i)\,\delta_{ik}\,\eta_{\alpha\beta}.$$

Equation (20) is the expression for S_{vac} in quantum electrodynamics in the form of a continuous integral. Equation (20) can also be put in the form

$$S_{\text{vac}} = \frac{\left\{\int \exp\left[i\int L(x,\psi,A)\,dx\right]\delta\psi\,\delta\bar{\psi}\,\delta A\right\}}{\left\{\exp\left[i\int L_0(x,\psi,A)\,dx\right]\delta\psi\,\delta\bar{\psi}\,\delta A\right\}} \tag{21}$$

where L is a Lagrange function depending on spinor field functions with no field interaction term ****.

In Eq. (20) the integration with respect to one of the fields can be carried out. The integration with respect to the electron field functions is carried out as follows:

$$S_{\text{vac}} = \frac{1}{\pi^{N/2}\sqrt{\operatorname{Det} D_{\mu\nu}}}\,\frac{\alpha^N}{\operatorname{Det}\left|\delta_{ik} + \frac{1}{\alpha}S^{-1}(ik)\right|} \tag{22}$$

$$\times \int \operatorname{Det}\left|\delta_{ik} + \frac{1}{\alpha}S^{-1}(ik) + \frac{1}{\alpha}A(ik)\,\omega\right|$$

$$\times \frac{1}{\alpha^N}\exp(-A(i)D^{-1}(ik)A(k))\,\delta A.$$

From this we finally get[5]

$$S_{\text{vac}} = \frac{1}{\pi^{N/2}\sqrt{\operatorname{Det} D_{\mu\nu}}}\exp\{\operatorname{Sp}\lg|\delta_{ik} \tag{23}$$

$$+\,\omega S(ik)\gamma_\mu A_\mu(k)|\}$$

$$\times(-A(i)D(ik)A(k))\,\delta A.$$

The integration in Eq. (20) can likewise be carried out with respect to the photon field functions. Equations analogous to Eq. (20) can be easily derived for Green's functions in any problem. Thus, for the one electron problem, according to Eq. (1),

$$G(l,m) = \frac{1}{\pi^{N/2}\sqrt{\operatorname{Det} D_{\mu\nu}}}\int\bar{\psi}(l)\psi(m)\,\eta_{lm} \tag{24}$$

$$\times \exp\{-\bar{\psi}(i)\check{S}^{-1}(ik)\psi(k) - A(i)D^{-1}(ik)A(k)$$

$$-\psi(i)\check{A}(ik)\psi(k)\,\omega\}\,\delta\psi\,\delta\bar{\psi}\,\delta A$$

$$\times\left(\int\exp\{-\psi(i)\dot{S}^{-1}(ik)\psi(k)\}\,\delta\psi\,\delta\bar{\psi}\right)^{-1}S_{\text{vac}}^{-1}$$

Here S_{vac} is given by Eq. (20). By simple differentiation, one arrives at the following expression

$$G(ik) = S(ik) - \frac{\delta}{\delta S^{-1}(ik)}\lg S_{\text{vac}}. \tag{25}$$

In taking the functional derivative with respect to $S^{-1}(ik)$, S_{vac} is considered a functional of $S^{-1}(ik)$. After differentiation, the quantities $S^{-1}(ik)$ take on the given values (see Anderson[4]).

In conclusion, I express my gratitude to Acad. L. D. Landau and B. L. Ioffe for participation in the investigation of this question.

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* This representation in the case of a Bose field was first derived by Gel'fand and Minlos[2].

** For simplicity, the spinor indices are suppressed and included in the indices i and k.

*** An exception is the case when the corresponding indices in two η_{ik} coincide; such η_{ik} anticommute. As will be shown in the following, however, this situation cannot be ignored, since terms containing combinations of identical operators drop out on integration.

**** An equation of the form (21) was discussed in the literature[2] by analogy with the corresponding formula for a Bose field. In the present case, on account of the anticommutation of the spinors, such an analogy is not valid.

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Gas Bubble Chamber -- A Possible Recorder of the Elementary Act of Interaction of Ionizing Radiation with Matter *

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THE experimentally sucessful[1,2] attempt to record the tracks of ionizing particles in superheated liquid do not exhaust all possibilities of using the bubbles formed along the track to detect the tracks in the liquid. For example, it is possible to record the tracks of ionizing particles by using a supersaturated solution of gas in the liquid: the instantaneous supersaturation produced by the rapid decrease in gas pressure over the surface of the liquid makes the liquid with the gas dissolved in it internally unstable with respect to the formation of centers for the new phase, namely gas bubbles. The passage of the ionizing particle, causing accumulation of ion centers along the track, local heating of the liquid and break-up of the molecules will contribute to the formation of center volumes inside the liquid and convert them into gas bubbles, which produce the track image (a leading role in the initial stage of formation of the volume is played apparently by the initiating repulsion eruption) of closely located molecular complexes of equal charge, gathered by the ions produced by the ionizing particles. The dead time of the work and the diffusion inertia of the growth of the bubble in the "gas" bubble chamber, associated with the local "impoverishment" of the solution can apparently be reduced considerably by selecting the proper operating conditions and using mixture components having a higher mutual solubility (physical or chemical solution of gas in liquid).

The thermodynamic working conditions of the "gas" bubble chamber are more suitable than the thermodynamic working conditions of the "vapor" bubble chamber: "gas" operating conditions do not require that the liquid be heated to increase its vapor pressure, and the use of these conditions will apparently be advantageous for liquid with low surface tension at those temperatures for which the saturated-vapor pressure of the liquid is insufficient to break away center volumes.

The temperature diffusivity of the "gas" working conditions for the bubble chamber and the possibility of independently varying the components of the working mixture, apparently facilitate the transition to large effective working volumes and to the optimum working mixture.

If the idea of "gas" bubble chamber can be put into practice, it will contribute to the creation of a universal, stable and simple instrument for recording the elementary act of interaction of highly-penetrating radiation with matter.

* Author's own summary of a report prepared in 1953 at the Institute for Physical Chemistry of the Academy of Sciences, USSR.

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On the Problem of the Negative π-meson Decays

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1. IT is reported[1] that when photographic emulsions were exposed to slow negative π-mesons, 18 π-μ meson decays were observed among 40,000 meson track endings. Convincing arguments are cited to show that it is a question of π-meson decays and not some concomitant background event.

The observed events[1] are characterized by the presence of an appreciable energy distribution spread of μ mesons from $\pi\mu$-decays ($\delta E_\rho \approx 0.5$ Mev), which indicates the motion of π mesons at the